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Valerii Azarskov[1]
Leonid Blokhin[2]
Olga Ermolaeva[3]

STRUCTURE IDENTIFICATION OF NON-LINEAR SYSTEM "MOVING OBJECT AND SERVO DRIVE" UNDER STOCHASTIC DISTURBANCES

National Aviation University
1, Kosmonavta Komarova avenue, Kyiv, 03680, Ukraine
E-mails: [1]azarskov@nau.edu.ua; [3]olgermol@yandex.ru

Abstract. *The article represents an algorithm for dynamics models identification of nonlinear system "moving object and servo drive", taking into account that the stochastic disturbances presented in the real operating mode are acting on it.*

Keywords: dynamics models; frequency response; identification; moving object; nonlinear system; servo drive; stochastic disturbances.

1. Introduction

In the time of solving a wide class of applied problems of controlled processes model construction, identification, state estimation of dynamic objects, filtering, prediction, recognition and control it is often necessary to process data under the initial conditions of uncertainty of both object properties and characteristics of environment with which it has to interact. The significant progress has been made in this area due to the essential results obtained during the last two decades in identification and control theory, intensive development of modern intellectual information technologies, improvement of computer technology, the practical requirements to creation of algorithms operated with insufficient a priori information available at the disposal of the system designer. Meanwhile, many of the previously known methods and algorithms oriented to operation in a priori uncertainty conditions are inoperative in violation of the assumptions on membership sets of object parameters and characteristics of unmeasured disturbances.

The stochastic and guaranteed methods of estimation of unknown variables and processes under conditions of uncertainty, such as Kalman filtering, Bayesian method, ellipsoidal estimation method and others are known. The main disadvantage of the stochastic methods of estimation is the necessity to know the statistical properties of an object. Such information can not always be obtained, or may not reflect the real properties of an object.

Unfortunately, both the stochastic and guaranteed approaches have common disadvantage – they are inoperative in the case if the assumptions do not match the actual properties of the control object. In addition, they are intended mainly for the estimation of the state and parameters of linear objects. This is quite a serious limitation for the real complex control objects because such objects usually have nonlinear dynamics that can not be adequately described by linearized models. Therefore, the problem solution of the development of robust estimation and identification methods which are efficient in violation of a priori hypotheses and minimum a priori information about the control object is really actual one.

Let us consider the structure identification problem of dynamics models of non-linear system "moving object and servo drive" under stochastic disturbances.

The block diagram of the under study system is presented in Fig. 1.

In Fig. 1 it is presented both a servo drive which motion can be described by a system of differential equations transformed by Fourier as

$$x = \mathbf{P}^{-1}\left(\mathbf{M}\cdot\tilde{y}_1 + \mathbf{\Psi_0}\cdot g\right), \quad (1)$$

and a moving object connected with the servo drive which motion can be described by a system of non-linear differential equations transformed by Fourier as

$$\xi = \mathbf{P_0}^{-1}\left(\mathbf{M_0}\cdot x + \mathbf{Y}\cdot g\right). \quad (2)$$

In expressions (1) and (2) the following designations are accepted:

$\mathbf{M}$ and $\mathbf{P}$ are transfer function matrices, that fix the servo drive motion, with sizes $n\times m$ and $n\times n$ respectively; $\mathbf{M}_0$ and $\mathbf{P}_0$ are transfer function matrices, that define the non-linear object motion, each of which has a size $n \times n$; $\mathbf{\Psi_0}$ and $\mathbf{Y}$ are frequency responses of disturbances acting on servo drive and object with size $n \times 1$; g is characteristic of disturbances (it is equal to 1.0 for deterministic disturbances and it is "white noise" designated as Δ for stochastic stationary disturbances).

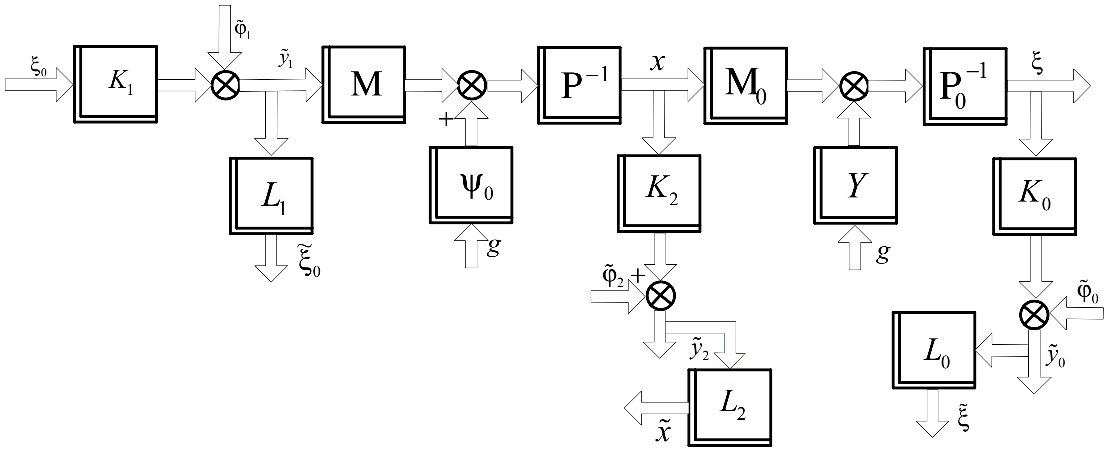

Fig. 1. Block diagram of the non-linear system "moving object and servo drive"

$\mathbf{K_0}$, $\mathbf{K}_1$ and $\mathbf{K}_2$ are transfer function square matrices of vector signals meter ξ, ξ_0 and x, which have size $n \times n$; $\tilde{\varphi}_0$, $\tilde{\varphi}_1$ and $\tilde{\varphi}_2$ are frequency responses of measurement noise of signals vector ξ, ξ_0 and x respectively, received during tests of the system prototypes, with size $n \times 1$; $\tilde{y}_0$, $\tilde{y}_1$ and $\tilde{y}_2$ are estimates of frequency responses of observation signals y_0, y_1 and y_2, received during tests of the system; L_0, L_1 and L_2 are computational algorithms for characteristics estimates calculation of the input and output vectors of the object and servo drive, presented as

$$L_0 = \mathbf{K_0^{-1}}(\tilde{y}_0 - \tilde{\varphi}_0),\ L_1 = \mathbf{K}_1^{-1}(\tilde{y}_1 - \tilde{\varphi}_1),$$
$$L_2 = \mathbf{K}_2^{-1}(\tilde{y}_2 - \tilde{\varphi}_2). \tag{3}$$

Since some parts of the under study system are non-linear and their dynamics models depend on active operating mode (e.g. mode "i"), all elements of the block diagram shown in Fig. 1 and next figures are marked by index "i". The under study operating mode of the system is provided by inputting in the diagram a coefficient β_i which defined and inputted by a researcher (Fig. 2):

$$\tilde{\xi}_0 = L_{1i}\,\mathring{\xi} = \beta_i \tilde{\nu} \tag{4}$$

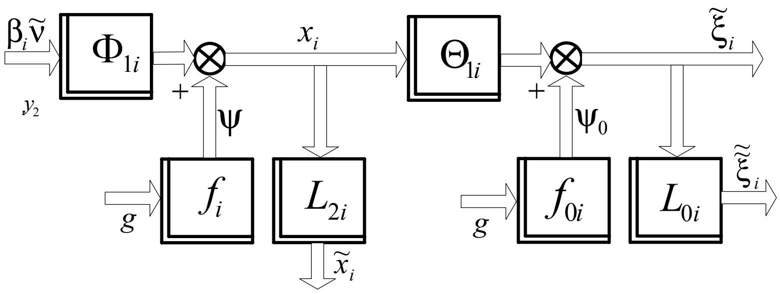

Fig. 2. Transformed block diagram of the system "object and servo drive"

In Fig. 2 the following designations are accepted:

$$\mathbf{P_i^{-1}M_i} = \Phi_{i1},\ \mathbf{P_i^{-1}}\Psi_0 = f_i,\ \mathbf{P}_0^{-1}\mathbf{M}_0 = \Theta_{1i},$$
$$\mathbf{P}_0^{-1}Y_\mathbf{i} = f_{0i}. \tag{5}$$

The frequency response of system output vector has the following form:

$$\tilde{\xi}_i = \Theta_{1i}\Phi_{1i}(\beta_i\tilde{\nu}) + (\Theta_{1i}f_i + f_{0i})g = $$
$$= \Theta_{1i}\Phi_{1i}(\beta_i\tilde{\nu}) + \eta_0. \tag{6}$$

The elements Θ_{1i}, $\mathbf{\Phi}_{1i}$, f_i and f_{0i} of the block diagram have to be identified.

2. Structure identification of servo drive dynamics models

Taking into account the designations (6) introduced above the block diagram of the servo drive in the system has a view presented in Fig. 3.

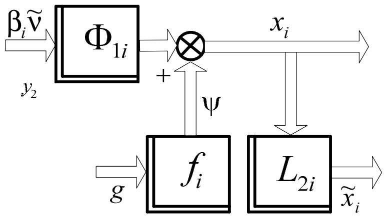

Fig. 3. Block diagram of the identified servo drive

The frequency response of servo drive output vector is

$$x_i = \Phi_{1i}(\beta_i\tilde{v}) + f_i g\,,\ x_{i*} = (\beta_i\tilde{v}_*)\Phi_{1i*} + g_* f_{i*}\,,$$
$$\tilde{x}_i = L_{2i}x_i\,,\ \tilde{x}_{i*} = x_{i*}L_{2i*} \qquad (7)$$

Here it is reasonable to introduce new designations:

$$\Phi_{0i} = (\Phi_{1i}, f_i),\ z_i = \begin{pmatrix}\beta_i\tilde{v}\\ g\end{pmatrix},\ x_i = \Phi_{0i}z_i \qquad (8)$$

If only deterministic disturbances acting on the servo drive, the expressions (7) should be rewritten as

$$\bar{x}_i = \overline{\Phi}_{0i}\bar{z}_i\,,\ \bar{x}_{i*} = \bar{z}_{i*}\overline{\Phi}_{0i*}\,, \qquad (9)$$

and the frequency responses of identification error signals vector should be presented as follows:

$$\varepsilon_{x_i} = \bar{x}_i - \bar{\tilde{x}}_i = \overline{\Phi}_{0i}\bar{z}_i - \bar{\tilde{x}}_i\,,\ \varepsilon_{x_i*} = \bar{z}_{i*}\overline{\Phi}_{0i*} - \bar{\tilde{x}}_{i*}\,. \qquad (10)$$

Functional of servo drive dynamics models identification performance should be written as

$$\bar{I}_x = \frac{1}{2\pi j}\int_{-j\infty}^{j\infty} tr\left(\varepsilon_{x_i}\varepsilon_{x_{i*}}\overline{R}_x\right)ds\,,\ s = j\omega. \qquad (11)$$

The problem of minimization of functional (11) is being solved by Wiener-Kolmogorov method [1].

It is necessary to put the expression (10) into the functional (11) and determine its value

$$\bar{I}_x = \frac{1}{2\pi j}\int_{-j\infty}^{j\infty} tr\left[\left(\overline{\Phi}_{0i}\bar{z}_i - \bar{\tilde{x}}_i\right)\times\right.$$
$$\left.\times\left(\bar{z}_{i*}\overline{\Phi}_{0i*} - \bar{\tilde{x}}_{i*}\right)\overline{R}_x\right]ds = \qquad (12)$$
$$= \frac{1}{2\pi j}\int_{-j\infty}^{j\infty} tr\left[\left(\overline{\Phi}_{0i}\bar{z}_i\bar{z}_{i*}\overline{\Phi}_{0i*} - \overline{\Phi}_{0i}\bar{z}_i\bar{\tilde{x}}_{i*} -\right.\right.$$
$$\left.\left.- \bar{\tilde{x}}_i\bar{z}_{i*}\overline{\Phi}_{0i*} + \bar{\tilde{x}}_i\bar{\tilde{x}}_{i*}\right)\overline{R}_x\right]ds.$$

The first variation of the functional (12) is the following:

$$\delta\bar{I}_x = \frac{1}{2\pi j}\int_{-j\infty}^{j\infty} tr\left[\left(\overline{R}_x\overline{\Phi}_{0i}\bar{z}_i\bar{z}_{i*} - \overline{R}_x\bar{\tilde{x}}_i\bar{z}_{i*}\right)\delta\overline{\Phi}_{0i*} +\right.$$
$$\left.+ \delta\overline{\Phi}_{0i}\left(\bar{z}_i\bar{z}_{i*}\overline{\Phi}_{0i*}\overline{R}_x - \bar{z}_i\bar{\tilde{x}}_{i*}\overline{R}_x\right)\right]ds. \qquad (13)$$

It is reasonable to introduce the following designations in the variance (13):

$\overline{\Gamma}_{x_*}\overline{\Gamma}_x = \overline{R}_x$; $\bar{z}_i\bar{z}_{i*} \approx \tilde{\overline{D}}_{x_i}\tilde{\overline{D}}_{x_{i*}}$, where $\left|\bar{z}_i\bar{z}_{i*}\right| = 0$, that is this matrix is singular,

$$\overline{T}_x = \overline{T}_{x0} + \overline{T}_{x+} + \overline{T}_{x-} \approx \overline{\Gamma}_x\overline{R}_x\bar{\tilde{x}}_i\bar{z}_{i*}\tilde{\overline{D}}^{-1}_{x_{i*}}$$

Factorization and separation procedures of matrices $\overline{R}_x$, $\bar{z}_i\bar{z}_{i*}$ and $\overline{T}_x$ are being performed by Davis [2] but in the rough.

The condition of approximate equality of the variance (13) to zero is $\overline{\Gamma}_x\overline{\Phi}_{0i}\tilde{\overline{D}}_{x_i} \approx \left(\overline{T}_{x0} + \overline{T}_{x+}\right)$, and the identification algorithm of model structure $\hat{\overline{\Phi}}_{0i}$ has the following form:

$$\hat{\overline{\Phi}}_{0i} \approx \left(\overline{\Gamma}_x\right)^{-1}\left(\overline{T}_{x0} + \overline{T}_{x+}\right)\left(\tilde{\overline{D}}_{x_i}\right)^{-1} \qquad (14)$$

Appling the algorithm (14) the structures $\hat{\overline{\Phi}}_{0i}$ and $\hat{\bar{f}}_i$ are determined approximately.

3. Structure identification of servo drive dynamics models in the system under random disturbances

In this variant of the task the characteristics of random vectors $\overset{\circ}{x}_i$ are

$$\overset{\circ}{x}_i = \overset{\circ}{\Phi}_{0i}\overset{\circ}{z}_i\,,\ \overset{\circ}{x}_{i*} = \overset{\circ}{z}_{i*}\overset{\circ}{\Phi}_{0i*}\,, \qquad (15)$$

where

$$\overset{\circ}{\Phi}_{0i} = \left(\overset{\circ}{\Phi}_{1i}, \overset{\circ}{f}_i\right), \qquad (16)$$

$$\overset{\circ}{z}_i = \begin{pmatrix}\beta_i\overset{\circ}{\tilde{v}}\\ \Delta\end{pmatrix}, \qquad (17)$$

the frequency responses of identification error signals vector $\overset{\circ}{\varepsilon}_{x_i}$ are the following:

$$\mathring{\varepsilon}_{x_i} = \mathring{x}_i - \mathring{\tilde{x}}_i = \mathring{\Phi}_{0i}\,\mathring{z}_i - \mathring{\tilde{x}}_i\,, \qquad \mathring{\varepsilon}_{x_{i*}} = \mathring{z}_{i*}\,\mathring{\Phi}_{0i*} - \mathring{\tilde{x}}_{i*} \tag{18}$$

and the functional of identification performance should be written as

$$\mathring{e}_x = \frac{1}{j}\int_{-j\infty}^{j\infty} tr\left(S'_{\mathring{x}_i\mathring{x}_i}\,\mathring{R}_x \right) ds \tag{19}$$

According to Wiener-Khinchin theorem to spectral density matrices of identification error signals vectors and their components are

$$S'_{\mathring{x}_i\mathring{x}_i} = \left\langle \left(\mathring{\Phi}_{0i}\,\mathring{z}_i - \mathring{\tilde{x}}_i \right)\left(\mathring{z}_{i*}\,\mathring{\Phi}_{0i*} - \mathring{\tilde{x}}_{i*} \right) \right\rangle = \\ = \mathring{\Phi}_{0i}\,S'_{\mathring{z}_i\mathring{z}_i}\,\mathring{\Phi}_{0i*} - \mathring{\Phi}_{0i}\,S'_{\mathring{\tilde{x}}_i\mathring{z}_i} - S'_{\mathring{z}_i\mathring{\tilde{x}}_i}\,\mathring{\Phi}_{0i*} + S'_{\mathring{\tilde{x}}_i\mathring{\tilde{x}}_i}; \tag{20}$$

$$S'_{\mathring{z}_i\mathring{z}_i} = \left\langle \mathring{z}_i\,\mathring{z}_{i*} \right\rangle = \left\langle \begin{pmatrix} \beta_i\,\mathring{\tilde{\nu}} \\ \Delta \end{pmatrix} \left(\beta_i\,\mathring{\tilde{\nu}}, \Delta_* \right) \right\rangle = \beta_i^2 S'_{\mathring{\nu}\mathring{\nu}} + \frac{\sigma_\Delta^2}{\pi};$$

$$S'_{\mathring{\tilde{x}}_i\mathring{z}_i} = \begin{pmatrix} \beta\, S'_{\mathring{x}_i\mathring{\nu}} \\ 0 \end{pmatrix};\; S'_{\mathring{z}_i\mathring{\tilde{x}}_i} = \left(\beta\, S'_{\mathring{x}_i\mathring{\nu}}, 0 \right).$$

The identification problem of the structure $\mathring{\hat{\Phi}}_{0i}$ is being solved by Wiener-Kolmagorov method [1]. It is necessary to put into the functional (19) the matrices (20). In the result the functional (19) takes the following form:

$$\mathring{e}_x = \frac{1}{j}\int_{-j\infty}^{j\infty} tr\left(\left(\mathring{\Phi}_{0i}\,S'_{\mathring{z}_i\mathring{z}_i}\,\mathring{\Phi}_{0i*} - \mathring{\Phi}_{0i}\,S'_{\mathring{\tilde{x}}_i\mathring{z}_i} - \right.\right. \\ \left.\left. - S'_{\mathring{z}_i\mathring{\tilde{x}}_i} + S'_{\mathring{\tilde{x}}_i\mathring{\tilde{x}}_i} \right)\mathring{R}_x \right) ds. \tag{21}$$

The first variation of the functional (21) is

$$\delta\mathring{e}_x = \frac{1}{j}\int_{-j\infty}^{j\infty} tr\left[\left(\mathring{R}_x\,\mathring{\Phi}_{0i}\,S'_{\mathring{z}_i\mathring{z}_i} - \mathring{R}_x\,S'_{\mathring{z}_i\mathring{\tilde{x}}_i} \right)\delta\mathring{\Phi}_{0i*} + \right. \\ \left. + \delta\mathring{\Phi}_{0i}\left(S'_{\mathring{z}_i\mathring{z}_i}\,\mathring{\Phi}_{0i*}\,\mathring{R}_x - S'_{\mathring{\tilde{x}}_i\mathring{z}_i}\,\mathring{R}_x \right) \right] ds. \tag{22}$$

To apply the Davis factorization and separation procedures [2] to the considered matrices it is necessary to include the following designations:

$$\mathring{\Gamma}_{x_*}\,\mathring{\Gamma}_x = \mathring{R}_x;\quad \mathring{D}_{x_i}\,\mathring{D}_{x_{i*}} = S'_{\mathring{z}_i\mathring{z}_i}; \\ \mathring{\mathrm{T}}_x = \mathring{\mathrm{T}}_{x0} + \mathring{\mathrm{T}}_{x+} + \mathring{\mathrm{T}}_{x-} = \mathring{\Gamma}_x\,S'_{\mathring{z}_i\mathring{\tilde{x}}_i}\,\mathring{D}_{x_i}^{-1}. \tag{23}$$

Taking into account the designations (23) the expression for the variation (22) is

$$\delta\mathring{e}_x = \frac{1}{j}\int_{-j\infty}^{j\infty} tr\left[\mathring{\Gamma}_{x*}\left(\mathring{\Gamma}_x\,\mathring{\Phi}_{0i}\,\mathring{D}_x - \mathring{\mathrm{T}} \right)\mathring{D}_{x*}\,\delta\mathring{\Phi}_{0i*} + \right. \\ \left. + \delta\mathring{\Phi}_{0i}\,\mathring{D}_x\left(\mathring{D}_{x*}\,\mathring{\Phi}_{0i*}\,\mathring{\Gamma}_{x*} - \mathring{\mathrm{T}}_* \right)\mathring{\Gamma}_x \right] ds,$$

and condition of equality to zero of the variance (22) is the following $\mathring{\Gamma}_x\,\mathring{\Phi}_{0i}\,\mathring{D}_x = \left(\mathring{\mathrm{T}}_{x0} + \mathring{\mathrm{T}}_{x+} \right)$.

Identification algorithm of servo drive structure $\mathring{\hat{\Phi}}_{0i}$ is

$$\mathring{\hat{\Phi}}_{0i} = \left(\mathring{\Gamma}_x \right)^{-1}\left(\mathring{\mathrm{T}}_{x0} + \mathring{\mathrm{T}}_{x+} \right)\left(\mathring{D}_x \right)^{-1}. \tag{24}$$

Thereby the optimal structures for servo drive $\mathring{\hat{\Phi}}_{0i}$ and $\mathring{\hat{f}}_i$ are determined.

4. Structure identification of dynamics models of non-linear system "object and servo drive" taking into account the already known servo drive dynamics models

The block diagram of the system for identification of object dynamics models is presented in Fig. 4.

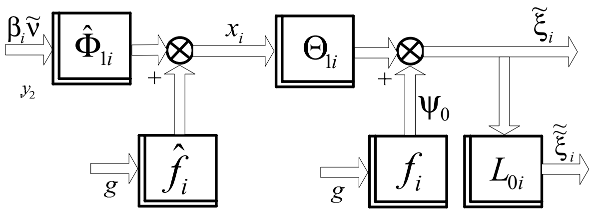

Fig. 4. Transformed block diagram of the system

The designations of the block diagram units (Fig. 4) coincide with the expressions (5), (6) and (8) mentioned above.

The frequency response of the system output vector should be written as

$$\tilde{\xi} = \Theta_{1i}x_i + f_{0i}g = (\Theta_{1i}, f_{0i})\begin{pmatrix}\tilde{x}_i\\ g\end{pmatrix} =$$
$$= (\Theta_{1i}, f_{0i})\begin{bmatrix}(\hat{\Phi}_{0i}(\beta_i\tilde{\nu}))\\ g\\ g\end{bmatrix} = \qquad (25)$$
$$= (\Theta_{1i}, f_{0i})\begin{pmatrix}\hat{\Phi}_{1i}\beta_i\tilde{\nu}+\hat{f}_i g\\ g\end{pmatrix} = \Theta_{0i}\tilde{z}_i, \; .$$

where $\tilde{z}_i = \begin{pmatrix}\hat{\Phi}_{1i}\tilde{\nu}\beta_i + \tilde{f}_i g\\ g\end{pmatrix}.$

The frequency responses of identification error signals vector of the system dynamics models under deterministic disturbances are

$$\overline{\tilde{\varepsilon}}_{0i} = \overline{\tilde{\xi}}_i - \overline{\tilde{\tilde{\xi}}}_i = \overline{\Theta}_{0i}\begin{pmatrix}\hat{\Phi}_{1i}\overline{\tilde{\nu}}\beta_i + \hat{\overline{f}}_i g\\ g\end{pmatrix} - \overline{\tilde{\tilde{\xi}}}_i = \overline{\Theta}_{0i}\overline{\tilde{z}}_i - \overline{\tilde{\tilde{\xi}}}_i;$$
$$\overline{\tilde{\varepsilon}}_{0i*} = \left(\beta_i\overline{\tilde{\nu}}_*\hat{\Phi}_{1i*} + g_*\hat{f}_{i*},\; g_*\right)\overline{\Theta}_{0i*} - \overline{\tilde{\tilde{\xi}}}_{i*} = \overline{\tilde{z}}_{i*}\overline{\Theta}_{0i*} - \overline{\tilde{\tilde{\xi}}}_{i*}. \qquad (26)$$

If the disturbances acting on the system are random, the frequency responses of identification error signals vector have to be the following:

$$\mathring{\tilde{\varepsilon}}_{0i} = \mathring{\Theta}_{0i}\begin{pmatrix}\mathring{\hat{\Phi}}_{1i}\,\mathring{\tilde{\nu}}\,\beta_i + \mathring{\hat{f}}_i\,\Delta\\ \Delta\end{pmatrix} - \mathring{\tilde{\tilde{\xi}}}_i,$$
$$\mathring{\tilde{\varepsilon}}_{0i*} = \left(\beta_i\,\mathring{\tilde{\nu}}_*\,\mathring{\Phi}_{1i*} + \Delta\,\mathring{\hat{f}}_{i*},\; \Delta_*\right) - \mathring{\tilde{\tilde{\xi}}}_{i*}. \qquad (27)$$

The functional of system dynamics models identification performance under deterministic disturbances is

$$\bar{I}_0 = \frac{1}{2\pi j}\int_{-j\infty}^{j\infty} tr\left(\overline{\tilde{\varepsilon}}_{0i}\overline{\tilde{\varepsilon}}_{0i*}\overline{R}_0\right)ds, \qquad s = j\omega, \qquad (28)$$

and the functional of system dynamics models identification performance under random disturbances is

$$\mathring{e}_0 = \frac{1}{j}\int_{-j\infty}^{j\infty} tr\left(S'_{\mathring{\tilde{\varepsilon}}_{0i}\mathring{\tilde{\varepsilon}}_{0i}}\,\mathring{R}_0\right)ds. \qquad (29)$$

On the basis of Wiener-Khinchin theorem it is possible to create a transposed spectral density matrix of identification error signals vector of system dynamics models that is the following:

$$S'_{\mathring{\varepsilon}_{0i}\mathring{\varepsilon}_{0i}} = \left\langle \mathring{\tilde{\varepsilon}}_{0i}\,\mathring{\tilde{\varepsilon}}_{0i*}\right\rangle = \left\langle\left[\mathring{\Theta}_{0i}\begin{pmatrix}\mathring{\hat{\Phi}}_{1i}\,\mathring{\tilde{\nu}}\,\beta_i + \mathring{\hat{f}}_i\,\Delta\\ \Delta\end{pmatrix} - \mathring{\tilde{\tilde{\xi}}}_i\right]\times\right.$$
$$\left.\times\left[\left(\beta_i\,\mathring{\tilde{\nu}}_*\,\mathring{\Phi}_{1i*} + \Delta_*\,\mathring{\hat{f}}_{i*},\, \Delta_*\right)\mathring{\Theta}_{0i*} - \mathring{\tilde{\tilde{\xi}}}_{i*}\right]\right\rangle =$$
$$= \mathring{\Theta}_{0i}\begin{pmatrix}\mathring{\hat{\Phi}}_{1i}\,\mathring{\tilde{\nu}}\,\mathring{\tilde{\nu}}_*\,\mathring{\Phi}_{1i*}\beta_i^2 + \dfrac{\sigma_\Delta^2}{\pi}\mathring{\hat{f}}_i\,\mathring{\hat{f}}_{i*} & \dfrac{\sigma_\Delta^2}{\pi}\mathring{\hat{f}}_i\\ \dfrac{\sigma_\Delta^2}{\pi}\mathring{\hat{f}}_{i*} & \dfrac{\sigma_\Delta^2}{\pi}\end{pmatrix}\mathring{\Theta}_{0i*} -$$
$$-\mathring{\Theta}_{0i}\begin{pmatrix}\mathring{\hat{\Phi}}_{1i}\,S'_{\tilde{\tilde{\xi}}_i\mathring{\tilde{\nu}}}\,\beta_i\\ 0\end{pmatrix} - \left(\beta_i S'_{\tilde{\tilde{\xi}}_i\mathring{\tilde{\nu}}_*}\,\mathring{\Phi}_{1i*},\; 0\right) + S'_{\mathring{\tilde{\tilde{\xi}}}_i\mathring{\tilde{\tilde{\xi}}}_i}, \qquad (30)$$

where

$$S'_{\mathring{\tilde{z}}_{0i}\mathring{\tilde{z}}_{0i}} = \begin{pmatrix}\mathring{\hat{\Phi}}_{1i}\,\mathring{\tilde{\nu}}\,\mathring{\tilde{\nu}}_*\,\mathring{\Phi}_{1i*}\beta_i^2 + \dfrac{\sigma_\Delta^2}{\pi}\mathring{\hat{f}}_i\,\mathring{\hat{f}}_{i*} & \dfrac{\sigma_\Delta^2}{\pi}\mathring{\hat{f}}_i\\ \dfrac{\sigma_\Delta^2}{\pi}\mathring{\hat{f}}_{i*} & \dfrac{\sigma_\Delta^2}{\pi}\end{pmatrix}.$$

5. Structure identification of object dynamics models in the system under deterministic disturbances

Substitution of the expressions (25) and (26) into the functional (30) makes possible to present it as follows:

$$\bar{I}_0 = \frac{1}{2\pi j}\int_{-j\infty}^{j\infty} tr\left[\left(\overline{\Theta}_{0i}\overline{\tilde{z}}_i - \overline{\tilde{\tilde{\xi}}}_i\right)\left(\overline{\tilde{z}}_{i*}\overline{\Theta}_{0i*} - \overline{\tilde{\tilde{\xi}}}_{i*}\right)\overline{R}_0\right]ds =$$
$$= \frac{1}{2\pi j}\int_{-j\infty}^{j\infty} tr\left[\left(\overline{\Theta}_{0i}\overline{\tilde{z}}_i\overline{\tilde{z}}_{i*}\overline{\Theta}_{0i*} - \overline{\Theta}_{0i}\overline{\tilde{z}}_i\overline{\tilde{\tilde{\xi}}}_{i*} - \right.\right. \qquad (31)$$
$$\left.\left. - \overline{\tilde{\tilde{\xi}}}_{i*}\overline{\tilde{z}}_{i*}\overline{\Theta}_{0i*} + \overline{\tilde{\tilde{\xi}}}_i\overline{\tilde{\tilde{\xi}}}_{i*}\right)\overline{R}_0\right]ds.$$

Minimization of the functional (31) is carried out by Wiener-Kolmogorov method [1]. The first variation of the functional is

$$\delta\bar{I}_0 = \frac{1}{2\pi j}\int_{-j\infty}^{j\infty} tr\left[\left(\overline{R}_0\overline{\Theta}_{0i}\overline{\tilde{z}}_i\overline{\tilde{z}}_{i*} - \overline{R}_0\overline{\tilde{\tilde{\xi}}}_i\overline{\tilde{z}}_{i*}\right)\delta\overline{\Theta}_{0i*} + \right.$$
$$\left. + \delta\overline{\Theta}_{0i}\left(\overline{\tilde{z}}_i\overline{\tilde{z}}_{i*}\overline{\Theta}_{0i*}\overline{R}_0 - \overline{\tilde{z}}_i\overline{\tilde{\tilde{\xi}}}_i\overline{R}_0\right)\right]ds. \qquad (32)$$

Taking into account the factorization and separation procedures of matrices [2] the following designations are inserted into the variation (32):

$$\overline{\Gamma}_{0*}\overline{\Gamma}_0 = \overline{R}_0; \quad \tilde{\overline{D}}_0\tilde{\overline{D}}_{0*} \approx \overline{\overline{z}}_i\overline{\overline{z}}_{i*};$$
$$\overline{\mathrm{T}}_0 = \overline{\mathrm{T}}_{00} + \overline{\mathrm{T}}_{0+} + \overline{\mathrm{T}}_{0-} = \overline{\Gamma}_0\overline{\overline{\xi}}\overline{\overline{z}}_{i*}\tilde{\overline{D}}_{0*}^{-1}, \tag{33}$$

where $\left|\overline{\overline{z}}_i\overline{\overline{z}}_{i*}\right| = 0,$ that is this matrix is singular.

The condition of equality to zero of the variance (32) is the following

$$\overline{\Gamma}_0\overline{\Theta}_{0i}\tilde{\overline{D}}_0 \approx \left(\overline{\mathrm{T}}_{00} + \overline{\mathrm{T}}_{0+}\right).$$

Identification algorithm of structure $\hat{\overline{\Theta}}_{0i}$ is

$$\hat{\overline{\Theta}}_{0i} \approx \left(\overline{\Gamma}_0\right)^{-1}\left(\overline{\mathrm{T}}_{00} + \overline{\mathrm{T}}_{0+}\right)\left(\tilde{\overline{D}}_0\right)^{-1}. \tag{34}$$

According to the algorithm (34) the optimized characteristics $\hat{\overline{\Theta}}_{0i}$ and $\hat{\overline{f}}_{0i}$. get known.

6. Particular variant of identification problem solution of structure $\hat{\overline{\Theta}}_{0i}$ when the features of frequency responses of signals vector $\tilde{\overline{\overline{z}}}_i$ are located in the left-half s-plane.

In this variant of identification problem the variance (32) is

$$\delta\overline{I}_0 = \frac{1}{2\pi j}\int_{-j\infty}^{j\infty} tr\left[\overline{\Gamma}_{0*}\left(\overline{\Gamma}_0\overline{\Theta}_{0i}\overline{\overline{z}}_i - \overline{\Gamma}_0\overline{\overline{\xi}}_i\right)\overline{\overline{z}}_{i*}\delta\overline{\Theta}_{0i*} + \right.$$
$$\left. + \delta\overline{\Theta}_{0i}\overline{\overline{z}}_i\left(\overline{\overline{z}}_{i*}\overline{\Theta}_{0i*}\overline{\Gamma}_{0*} - \overline{\overline{\xi}}_i\overline{\Gamma}_{0*}\right)\right]ds, \tag{35}$$

where $\overline{\Gamma}_{0*}\overline{\overline{\xi}}_i = \overline{\mathrm{T}}_{0i}$,
and the condition of equality to zero of the variance (35) is the following:

$$\overline{\Gamma}_0\overline{\Theta}_{0i}\overline{\overline{z}}_i = \left(\overline{\mathrm{T}}_{0i0} + \overline{\mathrm{T}}_{0i+}\right).$$

Identification algorithm of the optimal structure $\hat{\overline{\Theta}}_{0i}$ can be presented as

$$\hat{\overline{\Theta}}_{0i} = \overline{\Gamma}_0^{-1}\left(\overline{\mathrm{T}}_{0i0} + \overline{\mathrm{T}}_{0i+}\right)\left(\overline{\overline{z}}_i\right)^{\#}, \tag{36}$$

where "#" is a sign of pseudoinversion of a vector.

In accordance with Gantmacher [3] the factor $\left(\overline{\overline{z}}_i\right)^{\#}$ is $\left(\overline{\overline{z}}_i\right)^{\#} = A^+ = C^*\left(CC^*\right)^{-1}\left(B^*B\right)^{-1}B^*,$ where the sign "*" at the top is a symbol of transposition;

$$B = \overline{\overline{z}}_i; \quad C = (1{,}0); \quad \left(CC^*\right)^{-1} = 1{,}0;$$

$$\left(B^*B\right) = \left(\overline{\overline{z}}_{1i}, \overline{\overline{z}}_{2i}, \ldots, \overline{\overline{z}}_{vi}, \ldots \overline{\overline{z}}_{ni}\right)\cdot\begin{pmatrix}\overline{\overline{z}}_{1i}\\ \overline{\overline{z}}_{2i}\\ \vdots\\ \overline{\overline{z}}_{vi}\\ \vdots\\ \overline{\overline{z}}_{ni}\end{pmatrix} = \sum_{v=1}^{n}\overline{\overline{z}}_{vi}^2;$$

$$\overline{\overline{z}}^{\#} = A^+ = 1\cdot\frac{1}{\sum_{v=1}^{n}\overline{\overline{z}}_{vi}^2}\overline{\overline{z}}_i^* = \frac{\overline{\overline{z}}_i^*}{\sum_{v=1}^{n}\overline{\overline{z}}_{vi}^2}.$$

So the identified optimal structure $\hat{\overline{\Theta}}_{0i}$ is calculated by the following algorithm:

$$\hat{\overline{\Theta}}_{0i} = \overline{\Gamma}_0^{-1}\left(\overline{\mathrm{T}}_{0i0} + \overline{\mathrm{T}}_{0i+}\right)\frac{\overline{\overline{z}}_i^*}{\sum_{v=1}^{n}\overline{\overline{z}}_{vi}^2}. \tag{37}$$

7. Structure identification of object dynamics models in the system under random disturbances

Block diagram of the system in the under study variant of the task is presented in Fig. 5.

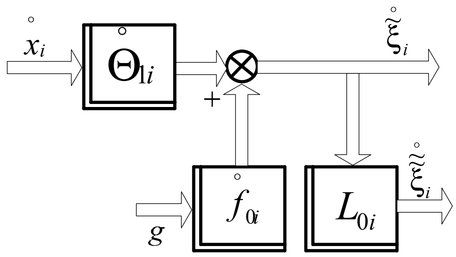

Fig. 5. Block diagram of the system for object identification problem solution under random disturbances

As before the following designations are used here

$$\overset{\circ}{P}{}_{0i}^{-1}\overset{\circ}{M}_{0i} = \overset{\circ}{\Theta}_{1i}; \quad \overset{\circ}{f}_{0i} = \overset{\circ}{P}{}_{0i}^{-1}\overset{\circ}{Y}_i;$$
$$\overset{\circ}{x}_i = \hat{\Phi}_{1i}\left(\beta_i\overset{\circ}{\tilde{\mathrm{v}}}\right) + \hat{\overset{\circ}{f}}_i\Delta; \quad \tilde{\mathrm{v}} = \tau_1\tilde{y}_0;$$
$$\tau_{0i} = K_0^{-1}\left(\tilde{y}_0 - \tilde{\varphi}_0\right).$$

The frequency response of signals vector $\overset{\circ}{\xi}_i$ is

$$\overset{\circ}{\tilde{\xi}}_i = \overset{\circ}{\Theta}_{1i}\overset{\circ}{x}_i + \overset{\circ}{f}_{0i}\Delta = \overset{\circ}{\Theta}_{1i}\left[\hat{\overset{\circ}{\Phi}}_{1i}\left(\beta_i\overset{\circ}{\tilde{v}}\right) + \hat{\overset{\circ}{f}}_i\Delta\right] + \overset{\circ}{f}_{0i}\Delta =$$

$$= \left(\overset{\circ}{\Theta}_{1i},\ \overset{\circ}{f}_{0i}\right)\begin{bmatrix}\hat{\overset{\circ}{\Phi}}_{1i}\left(\beta_i\overset{\circ}{\tilde{v}}\right) + \hat{\overset{\circ}{f}}_0\Delta \\ \Delta\end{bmatrix} = \overset{\circ}{\Theta}_{0i}\overset{\circ}{\tilde{z}}_i. \qquad (38)$$

The frequency response of identification error signals vector of object dynamics models in the system under random disturbances is

$$\overset{\circ}{\tilde{\varepsilon}}_0 = \overset{\circ}{\tilde{\xi}}_i - \overset{\circ}{\tilde{\tilde{\xi}}}_i = \overset{\circ}{\Theta}_{0i}\overset{\circ}{\tilde{z}}_i - \overset{\circ}{\tilde{\tilde{\xi}}}_i,\quad \overset{\circ}{\tilde{\varepsilon}}_{0*} = \overset{\circ}{\tilde{z}}_{i*}\overset{\circ}{\Theta}_{0i*} - \overset{\circ}{\tilde{\tilde{\xi}}}_{i*}, \quad (39)$$

and transposed spectral density matrix of error signals vector $\overset{\circ}{\tilde{\varepsilon}}_0$ is determined by the expression (30). Substitution of the matrices (30) into the functional (29) makes possible to present it as follows:

$$\overset{\circ}{e}_0 = \frac{1}{j}\int_{-j\infty}^{j\infty} tr\left(S'_{\overset{\circ}{\tilde{\varepsilon}}_{0i}\overset{\circ}{\tilde{\varepsilon}}_{0i}}\overset{\circ}{R}_0\right)ds =$$

$$= \frac{1}{j}\int_{-j\infty}^{j\infty} tr\left[\left(\overset{\circ}{\Theta}_{0i}\overset{\circ}{\tilde{z}}_i - \overset{\circ}{\tilde{\tilde{\xi}}}_i\right)\left(\overset{\circ}{\tilde{z}}_{i*}\overset{\circ}{\Theta}_{0i*} - \overset{\circ}{\tilde{\tilde{\xi}}}_{i*}\right)\overset{\circ}{R}_0\right]ds =$$

$$= \frac{1}{j}\int_{-j\infty}^{j\infty} tr\left[\left(\overset{\circ}{\Theta}_{0i}S'_{\overset{\circ}{\tilde{z}}_i\overset{\circ}{\tilde{z}}_i}\overset{\circ}{\Theta}_{0i*} - \overset{\circ}{\Theta}_{0i}S'_{\overset{\circ}{\tilde{\tilde{\xi}}}_i\overset{\circ}{\tilde{z}}_i} - \right.\right. \qquad (40)$$

$$\left.\left. - S'_{\overset{\circ}{\tilde{z}}_i\overset{\circ}{\tilde{\tilde{\xi}}}_i}\overset{\circ}{\Theta}_{0i*} + S'_{\overset{\circ}{\tilde{\tilde{\xi}}}_i\overset{\circ}{\tilde{\tilde{\xi}}}_i}\right)\overset{\circ}{R}_0\right]ds,$$

where matrix $S'_{\overset{\circ}{\tilde{z}}_i\overset{\circ}{\tilde{z}}_i}$ is the following:

$$S'_{\overset{\circ}{\tilde{z}}_i\overset{\circ}{\tilde{z}}_i} = \left\langle \overset{\circ}{\tilde{z}}_i\ \overset{\circ}{\tilde{z}}_{i*}\right\rangle = \left\langle\begin{bmatrix}\hat{\overset{\circ}{\Phi}}_{1i}\left(\beta_i\overset{\circ}{\tilde{v}}\right) + \hat{\overset{\circ}{f}}_0\Delta \\ \Delta\end{bmatrix}\times\right.$$

$$\left.\times\left[\left(\beta_i\overset{\circ}{\tilde{v}}_*\right)\hat{\overset{\circ}{\Phi}}_{1i*} + \Delta\hat{\overset{\circ}{f}}_{0*},\ \Delta_*\right]\right\rangle = \qquad (41)$$

$$= \begin{bmatrix}\beta_i^2\hat{\overset{\circ}{\Phi}}_{1i}\overset{\circ}{\tilde{v}}\overset{\circ}{\tilde{v}}_*\hat{\overset{\circ}{\Phi}}_{1i*} + \frac{\sigma_\Delta^2}{\pi}\hat{\overset{\circ}{f}}_0\hat{\overset{\circ}{f}}_{0*} & \frac{\sigma_\Delta^2}{\pi}\hat{\overset{\circ}{f}}_0 \\ \frac{\sigma_\Delta^2}{\pi}\hat{\overset{\circ}{f}}_{0*} & \frac{\sigma_\Delta^2}{\pi}\hat{\overset{\circ}{f}}_0\hat{\overset{\circ}{f}}_{0*}\end{bmatrix}.$$

The problem of minimization of the object dynamics models identification performance functional (31) under random disturbances is also being solved by Wiener-Kolmogorov method [1]. The first variation of the functional (40) should be written as

$$\delta\overset{\circ}{e}_0 = \frac{1}{j}\int_{-j\infty}^{j\infty} tr\left[\left(\overset{\circ}{R}_0\overset{\circ}{\Theta}_{0i}S'_{\overset{\circ}{\tilde{z}}_i\overset{\circ}{\tilde{z}}_i} - \overset{\circ}{R}_0 S'_{\overset{\circ}{\tilde{z}}_i\overset{\circ}{\tilde{\tilde{\xi}}}_i}\right)\delta\overset{\circ}{\Theta}_{0i*} + \right. \qquad (42)$$

$$\left. + \delta\overset{\circ}{\Theta}_{0i}\left(S'_{\overset{\circ}{\tilde{z}}_i\overset{\circ}{\tilde{z}}_i}\overset{\circ}{\Theta}_{0i*}\overset{\circ}{R}_0 - S'_{\overset{\circ}{\tilde{\tilde{\xi}}}_i\overset{\circ}{\tilde{z}}_i}\overset{\circ}{R}_0\right)\right]ds$$

In the variation of the functional the following designations are inserted:

$$\overset{\circ}{\Gamma}_{0*}\overset{\circ}{\Gamma}_0 = \overset{\circ}{R};\qquad \overset{\circ}{D}_0\overset{\circ}{D}_{0*} = S'_{\overset{\circ}{\tilde{z}}_i\overset{\circ}{\tilde{z}}_i};$$

$$\overset{\circ}{T}_0 = \overset{\circ}{T}_{00} + \overset{\circ}{T}_{0+} + \overset{\circ}{T}_{0-} = \overset{\circ}{\Gamma}_0 S'_{\overset{\circ}{\tilde{z}}_i\overset{\circ}{\tilde{\tilde{\xi}}}_i}\overset{\circ}{D}{}_{0*}^{-1}.$$

The condition of equality to zero of the variation (42) is defined as

$$\overset{\circ}{\Gamma}_0\overset{\circ}{\Theta}_{0i}\overset{\circ}{D}_0 = \left(\overset{\circ}{T}_{00} + \overset{\circ}{T}_{0+}\right),$$

and the identification algorithm of optimal structure $\hat{\overset{\circ}{\Theta}}_{0i} = \left(\hat{\overset{\circ}{\Theta}}_{1i},\ \overset{\circ}{f}_{0i}\right)$ of the object in the system under random disturbances is the following:

$$\hat{\overset{\circ}{\Theta}}_{0i} = \overset{\circ}{\Gamma}{}_0^{-1}\left(\overset{\circ}{T}_{00} + \overset{\circ}{T}_{0+}\right)\overset{\circ}{D}{}_0^{-1}. \qquad (43)$$

Thereby, the assigned task of structure identification of the object in the system under stochastic disturbances has been solved.

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В.М. Азарсков[1], Л.М. Блохін[2], О.В. Єрмолаєва[3]. Структурна ідентифікація нелінійної системи «рухомий об'єкт–сервопривод» при стохастичних впливах
Національний авіаційний університет, просп. Космонавта Комарова, 1, Київ, Україна, 03680
E-mails: [1]azarskov@nau.edu.ua; [3]olgermol@yandex.ru
Пропонується алгоритм ідентифікації моделей динаміки нелінійної системи «рухомий об'єкт-сервопривод» при врахуванні впливів на неї стохастичних збурюючих факторів в реальних умовах експлуатації.
Ключові слова: вектор збурень; ідентифікація; модель динаміки; нелінійна система; рухомий об'єкт; сервопривод; стохастичний вплив; частотна характеристика.

В.Н. Азарсков[1], Л.Н. Блохин[2], О.В. Ермолаева[3]. Структурная идентификация нелинейной системы «подвижный объект–сервопривод» при стохастических воздействиях
Национальный авиационный университет, просп. Космонавта Комарова, 1, Киев, Украина, 03680
E-mails: [1]azarskov@nau.edu.ua; [3]olgermol@yandex.ru
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Azarskov Valerii. Doctor of Engineering. Professor.
Aircraft Control Systems Department, National Aviation University, Kyiv, Ukraine.
Education: Kyiv Civil Aviation Engineers Institute, Kyiv, Ukraine (1968).
Research area: system identification, aircraft control systems, aviation and flight simulators and devices.
Publications: 250.
E-mail: azarskov@nau.edu.ua

Blokhin Leonid. Doctor of Engineering. Professor.
Aircraft Control Systems Department, National Aviation University, Kyiv, Ukraine.
Education: Kiev Polytechnic University, Kyiv, Ukraine (1959).
Research area: system identification, aircraft control systems, aviation and flight simulators and devices.
Publications: 400.
E-mail: azarskov@nau.edu.ua

Ermolaeva Olga. Lecturer.
Aircraft Control Systems Department, National Aviation University, Kyiv, Ukraine.
Education: Kyiv International University of Civil Aviation, Kyiv, Ukraine (1998).
Research area: system identification, aviation simulators and devices.
Publications: 38.
E-mail: olgermol@yandex.ru