on the controllability gramian [2,3]. Let the aircraft dynamics of the aircraft is described by standard vector -matrix differential equation

$$\frac{dx}{dt} = AX + BU + B_1F,$$
$$Y = CX + DU, \tag{1}$$

where A and B are the matrices of the aircraft's stability and controllability derivatives respectively, the state vector for the lateral motion (LAM) has the form $X = [\beta, p, r, \phi, \varphi]'$ with the following components: sidesleep angle β,roll p and yaw r rates,roll ϕ and yaw ψ angles and for longitudinal motion (LOM) it is $X = [u, h, w, \theta, q]'$,, where u, h, w stand for the deviations of the horizontal velocity u, height h and vertical velocity w from their steady-state values,θ,q are pitch angle and rate respectively; control vector for LAM $U = [\delta_a, \delta_r]'$,,where δ_a, δ_r stand for ailerons and rudder displacements respectively, and the same vector for LOM has components: δt- throttle control lever and δe - elevator displacements; f is external disturbance (wind gust, vertical for LOM and lateral for LAM).

Quadratic PI has traditional form of the quadratic cost function (H_2 -norm):

$$J = \int_o^T [X'QX + U'RU]dt,$$

where: X'=[β, p, r,ϕ,ψ] is the state vector, U'=[δa,δr] is the control vector.

Using definition of the specific aerodynamic lateral force [1], which causes the linear lateral acceleration

$$f_s = a_{ij}x_j + b_{ik}u_k, \tag{2}$$

where xj is β for (LAM) and w for (LOM) and uk is ∂r for (LAM) and ∂e for (LOM) it is possible to define contribution , which this force gives to the quadratic PI for the limitation of lateral linear acceleration, using the square of the expression (2) with some weight Qf. In this case the cross product of the fs^2 produces the cross product matrix N and the expression for a H_2-norm would have the form :

$$J = \int_o^T z'Q_1 z dt. \tag{3}$$

In the expression (3) Q_1 is the block matrix $Q_1 = \begin{bmatrix} Q & N \\ N' & Q \end{bmatrix}$ and Z is the vector : $Z' = [X \quad U]$.

As it was noticed before ,the calculation of the PI is proposed to do on the basis of the controllability gramian. For the state space description of the closed-loop control system in the form (1), whose description includes the state variables of the aircraft as well as control variables (outputs of controller) and may be obtained from (1) with the simple substitution of the A,B,C,D matrices by corresponding matrices of the closed loop system A_c, B_c, C_c, D_c; controllability gramian G is the matrix ,which is defined by the solution of the Liapunov equation [2,3] :

$$A_cG + GA_c' + B_cB_c' = 0.$$

Matrices A_c, B_c, C_c, D_c are calculated using known formulas for given matrices of aircraft and autopilot state space description. This operation is easily done in MATLAB by operator «feedback».

In the accordance with the [2] H_2-norm (cost function) in the form (3) could be expressed using controllability gramian G as follows :

$$J = trace(C_{c1}GC_{c1}'). \tag{4}$$

slightly this value to acquire small stability margin, we can consider it as perturbed value a_{ij}^{p} . If $a_{ij}^{p} \cong a_{ij}^{\max}$, it is possible to run only one optimization procedure. If not , it is possible to choose previously defined a_{ij}^{p} as new nominal value and to find new increased perturbed value of a_{ij} from the stability viewpoint. Using these new nominal and perturbed values it would be necessary to repeat optimization procedure, defining new values of FCL parameters for increased stability domain. This sequential procedure could be performed until certain step, when the stability domain could not be increased anymore. In practical cases 2-3 steps of procedure's repetition could cover necessary parameter's tolerances.

3. Achieved results

Application of aforementioned procedure to the robust parametric FCL optimization was made for control of the LAM - dynamics of short-range passenger aircraft in landing mode. Optimization of FCL for aircraft LOM channel was made in the same way, but results are not presented here. Corresponding control law of the lateral channel of autopilot is expressed in terms of Laplas transform as follows :

$$\delta a(s) = \frac{1}{(T_a \cdot s+1)}\left[K_v \cdot v(s) + K_p \cdot p(s) + K_r \cdot r(s) + K_\varphi \cdot \varphi(s) + K_\psi \cdot \psi(s) + (K_y + \frac{K_{iy}}{s}) \cdot y(s)\right]$$

$$\delta r(s) = \frac{1}{(T_a \cdot s+1)}\left[K_{\psi r} \cdot \psi(s) + (\frac{K_W \cdot s}{T_w \cdot s+1} + K_{rr}) \cdot r(s)\right] ,$$

where $\delta a, \delta r$ stand for the deflections of aileron and rudder respectively, T_a, T_w stand for time constants of actuator and wash-out filter (in the rudder channel only), v is the lateral velocity (or sidesleep angle β) ; p, r are the roll and yaw rates; φ, ψ are the roll and yaw angles ; y is lateral displacement from the glideslope (measured by integrated GPS+INS), letters $K_v, \ldots, K_{rr}$ are corresponding gains which have to be determined as a result of optimization procedure. Matrix A_1 of stability derivatives of aircraft 's LAM dynamics in landing mode with «nominal» (unperturbed) parameters for state-space vector $X = [v, p, r, \varphi, \psi, y]^T$ is the following :

$$A_1 = \begin{bmatrix} -0.196 & 4.945 & -246 & 32.2 & 0 & 0 \\ -0.019 & -3.223 & 0.8613 & 0 & 0 & 0 \\ 0.009 & -0.29 & -0.66 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0.131 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

All eigenvalues of matrix A_1 are located in left half-plane. Matrix A_2 of perturbed parametrs differs from matrix A_1 only by element a_{21}, which is the effective dihedral, i.e. rolling moment derivative coefficient with respect the sidesleep angle, which define the damping of both dutch roll and spiral modes. In matrix A_2 this coefficient is equal -0.007 (2.7 times less in absolute value in comparison with the same element of A_1), that's why this matrix has eigenvalue in right half-plane, so LAM of aircraft itself with perturbed parameter a_{21} is unstable. Control derivatives matrix B is the following:

$$B=\begin{bmatrix} 0 & 0.0137 & 0.0017 & 0 & 0 & 0 \\ 0.1476 & 0.0069 & -0.1006 & 0 & 0 & 0 \end{bmatrix}^T.$$

After converting FCL (6) in the state-space form and obtaining state-space description of the closed loop system with aircraft's matrices A_1 (or A_2) and B, defined by (5) and (6), it is possible to apply aforementioned optimization procedure for parametric H_2 –optimization and further robust optimization of FCL. The vector of FCL gains $\vec{K}$ after robust optimization was obtained in the following form:

$$\vec{K}=\left[K_v, K_p, K_r, K_\varphi, K_\psi, K_y, K_{iy}, K_w, K_{\psi r}, K_{rr}\right]=$$

$$=[0.71,\ -8.68,\ 0.054,\ 105,\ 2.7,\ 0.39,\ 0.028,\ -37.2,\ -31.8,\ -33.37].$$

Some components of this vector are large , but it is possible to use conditional optimization procedure with restrictions on the input variables ,if it would be necessary to restrict some gains from the other possible viewpoints.

The results of robust parameter optimization of LAM Control Law , when the effective dihedral a_{21} varies in the tolerances [-0.019, -0.007], are presented at the fig.1 and fig.2

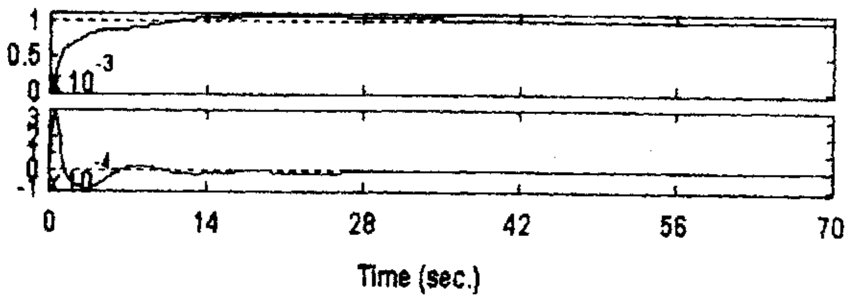

Fig.1

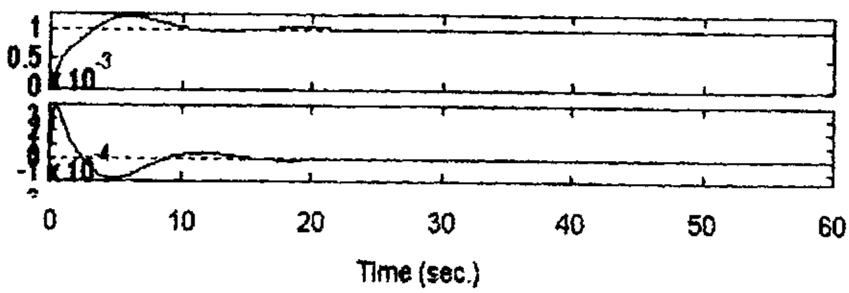

Fig. 2

respectively (transient processes for system with nominal a_{21} = -0.019 and perturbed parameter a_{21} = -0.007). At these figures only 2 first state vector $X = [v, p, r, \varphi, \psi, y]^T$ components are represented: longitudianal velosity v and roll rate p. All other components are not shown her to the limitation of the volume of paper, but all these figures show, that they are similar for the perturbed and unperturbed systems . H_2 - norm, which defines the accuracy of closed -loop system and accordance with airworthiness requirements [6] , decreases at only 10-20% in comparison with results of the pure H_2-optimization. The same difference is between the values of the H_∞-norm. The input disturbance was selected as lateral wind gust, whose shape was defined by FAR-requirements. This procedure could be extended to any other variable parameter, for instance, to the "weathercock stability coefficient" a_{31}. The similar results were obtained for LOM, when "longitudinal static stability coefficient" was changed in the wide range.

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