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APPROACH TO RELIABILITY ANALYSIS OF THE COMPUTER NETWORK FUNCTIONING OF THE MANAGEMENT SYSTEM

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Abstract. *The article considers a mathematical model of information technologies management systems in order to calculate steady-state measures of performance.*

Keywords: arrivals; requests; queuing system; states of the system; steady-state measures of performance; workstations

1. Introduction

For the sake of efficient functioning of a modern school, the management systems based on up-to-date information technologies have been created. They are called information technologies management systems (ITMS).

These systems consist of the other component systems, such as those of functional control, planning, quality control, the help systems and so on. All the systems in the integrated one are complex as well.

The ITMS network in which the arrival of requests or the service time of requests are not regulated may be represented as the queuing system. Such a system may produce queues, or delays, and refusals to provide the service. An important characteristic of ITMS` functioning is a probability of an access to the system from the independent workstations (WS). The access is made through a concentrator in the form of information streams flowing to a main server after they have passed through a common informational data bus.

2. Aim raising.

Let us consider a system comprising a few WSs sending separate requests to the server. The arrival of the requests occurs in a totally random fashion. Randomness here means that the occurrence of an event (e.g., arrival of a request or completion of a service) is not influenced by the length of time that has elapsed since the occurrence of the last event. We suppose that random interarrival time and service time are described quantitatively in queuing models by the exponential distribution, which is defined as follows:

- an interval between successive requests of a user of the i-th WS has an exponential distribution with a following density:

$$f_i(t) = \lambda_i e^{-\lambda_i t} t>0);$$

- the service time of the i-th WS user`s request has the exponential distribution with the density:

$$g_i(t) = \mu_i e^{-\mu_i t} (t>0),$$

where λ - is the rate per unit time at which arrivals are generated,

μ -is the rate per unit time at which departures are generated.

Suppose that the first WS user`s requests have priorities for being operated. They are defined as follows: the first WS user`s request is served immediately having interrupted any other WS` request. The later will be served as soon as service of the first one is finished.

3. System of two workstations users

Let us consider a system of two WSs (*i=2*). Define the possible states of the system at the initial moment of time:

0 – the server is free (no requests);

1 – the first WS user`s request has entered the server, no queue;

2 – the second WS user`s request has entered the free server; the first WS user`s request is absent;

3 – the first WS user`s request has entered the server while the second WS user`s one was being processed; the latter is delayed.

The transition from one state to another is nearly immediate. It takes place at the random moments of arrival of WS users` requests or at the moments when requests` service is finished (departure).

The oriented graph of the states of the system is shown on the fig. 1.

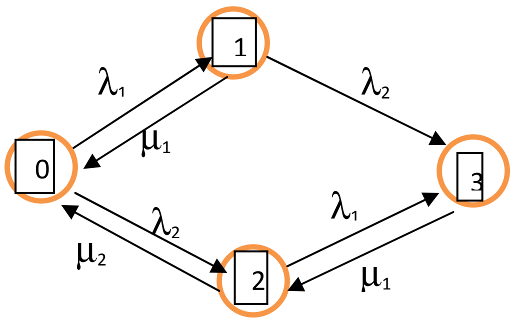

Fig. 1

Let us denote $p_i(t)$ – the probability of the system being in the *i*-state at the moment t.

Using the directed graph, let us write the system of Kolmogorov`s differential equations for the system considered:

$$\frac{dp_0(t)}{dt} = \mu_1 p_1(t) + \mu_2 p_2(t) - (\lambda_1 + \lambda_2) p_0(t);$$

$$\frac{dp_1(t)}{dt} = \lambda_1 p_0(t) - (\mu_1 + \lambda_2)\, p_1(t);$$

$$\frac{dp_2(t)}{dt} = \lambda_2 p_0(t) - (\lambda_1 + \mu_2) p_2(t) + \mu_1 p_3(t);$$

$$\frac{dp_3(t)}{dt} = \lambda_2 p_1(t) + \lambda_1 p_2(t) - \mu_1 p_3(t);$$

$$p_0(0) = 1;\ p_1(0) = p_2(0) = p_3(0) = 0.$$

The system of Kolmogorov`s differential equations makes it real to find all the probabilities of states as functions of time t. The system has been in operation for a sufficiently long time, thus it is interesting to determine the probabilities of states as $t \to \infty$, i.e. steady-state probabilities of states.

Denoting steady-state probabilities of states as p_i, one may determine them from the system of algebraic equations:

$$\mu_1 p_1 + \mu_2 p_2 - (\lambda_1 + \lambda_2) p_0 = 0;$$

$$\lambda_1 p_0 - (\mu_1 + \lambda_2)\, p_1 = 0;$$

$$\lambda_2 p_0 - (\lambda_1 + \mu_2) p_2 + \mu_1 p_3 = 0;$$

$$\lambda_2 p_1 + \lambda_1 p_2 - \mu_1 p_3 = 0;$$

$$p_0 + p_1 + p_2 + p_3 = 1.$$

Solving the system, one of the equations may be omitted because it is a consequence of the others. Knowing the steady-state probabilities of states of the system one can evaluate the steady-state measures of performance.

The time portion of the second WS user`s waiting in a queue is $p_3$, the time portion when there is no queue in the system is $p_0+p_1+p_2$, the time portion when the users` requests are serviced with no delay is p_1+p_3.

$$P_{queu} = p_3,$$

$Q = 1 - P_{queu} = 1 - P_{queu} = 1 - p_3$ - relative capacity.

4. System of three workstations users. Let us consider the system with three WSs that is operating in the same way as mentioned above.

Suppose that only the first WS user`s requests have the priority. The order of servicing of the second and the third WS users` requests is as follows: first come, first served.

Let us define the possible states of the system:

0 – the server is free (no requests);

1 – the first WS user`s request has entered the free server. There are no other requests;

2 – the second WS user`s request has entered the free server. There are no other requests;

3 – the third WS user`s request has entered the free server. There are no other requests;

4 – the server processes the first WS user`s request, the second WS user`s one is delayed, the request from the third WS is absent:

a) the second WS user`s request entered the server while the first WS user`s one was being processed; the second WS user`s one is delayed;

b) the first WS user`s request entered the server while the second WS user`s one was being processed; the service of the second WS user`s request is interrupted;

5 – the server processes the first WS user`s request, the third WS user`s one is delayed. The request from the second WS is absent:

a) the third WS user`s request entered the server while the first WS user`s one was being processed; the third WS user`s one is delayed;

b) the first WS user`s request entered the server while the third WS user`s one was being processed; the service of the third WS user`s request is interrupted;

6 – the third WS user`s request entered the server while the second WS user`s one was being processed;

7 – the second WS user`s request entered the server while the third WS user`s one was being processed;

8 – the first WS user`s request entered the server while the second WS user`s one was being processed. The request from the third WS is in the queue;

9 – the first WS user`s request entered the server while the third WS user`s one was being processed. The request from the second WS is in the queue.

The oriented graph of the system`s states has such a form:

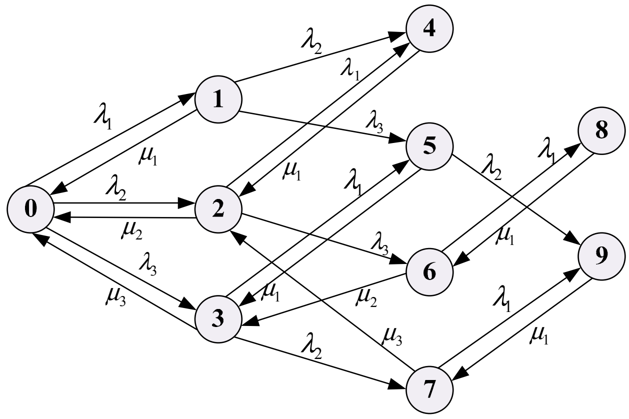

Fig. 2

Having used the oriented graph of the system`s states (fig. 2) and denoted the probability to find the system in the *i*-state at the time moment *t* as $p_i(t)$, one can write the Kolmogorov`s equation for the given system:

$$\frac{dp_0(t)}{dt} = \mu_1 p_1(t) + \mu_2 p_2(t) + \mu_3 p_3(t) - (\lambda_1 + \lambda_2 + \lambda_3) p_0(t);$$

$$\frac{dp_1(t)}{dt} = \lambda_1 p_0(t) - (\mu_1 + \lambda_2 + \lambda_3) p_1(t);$$

$$\frac{dp_2(t)}{dt} = \lambda_2 p_0(t) - (\lambda_1 + \mu_2 + \lambda_3) p_2(t) + \mu_1 p_4(t) + \lambda_1 p_7(t);$$

$$\frac{dp_3(t)}{dt} = \lambda_3 p_0(t) + (\lambda_1 + \lambda_2 + \mu_3) p_3(t) + \mu_1 p_5(t) + \mu_2\, p_6(t)\,;$$

$$\frac{dp_4(t)}{dt} = \lambda_2 p_1(t) + \lambda_1 p_2(t) - \mu_1 p_4(t);$$

$$\frac{dp_5(t)}{dt} = \lambda_3 p_1(t) + \lambda_1 p_3(t) - \mu_1 p_5(t);$$

$$\frac{dp_6(t)}{dt} = \lambda_3 p_2(t) - (\lambda_1 + \mu_2) p_6(t) + \mu_1 p_8(t);$$

$$\frac{dp_7(t)}{dt} = \lambda_2 p_3(t) - (\lambda_1 + \mu_3) p_7(t) + \mu_1 p_9(t);$$

$$\frac{dp_8(t)}{dt} = \lambda_1 p_6(t) - \lambda_1 p_8(t);$$

$$\frac{dp_9(t)}{dt} = \lambda_2 p_5(t) + \lambda_1 p_7(t) + \mu_1 p_9(t);$$

$$p_0(0) = 1;\ p_1(0) = p_2(0) = p_3(0) = p_4(0) = p_5(0) = p_6(0) = p_7(0) = p_8(0) = p_9(0) = 0.$$

The steady-state probabilities of states p_i $(i = \overline{0,9})$ can be calculated from the corresponding system of algebraic equations:

$$\mu_1 p_1 + \mu_2 p_2 + \mu_3 p_3 - (\lambda_1 + \lambda_2 + \lambda_3) p_0 = 0;$$

$$\lambda_1 p_0 - (\mu_1 + \lambda_2 + \lambda_3) p_1 = 0;$$

$$\lambda_2 p_0 - (\lambda_1 + \mu_2 + \lambda_3) p_2 + \mu_1 p_4 + \lambda_1 p_7 = 0;$$

$$\lambda_3 p_0 + (\lambda_1 + \lambda_2 + \mu_3) p_3 + \mu_1 p_5 + \mu_2 p_6 = 0;$$

$$\lambda_2 p_1 + \lambda_1 p_2 - \mu_1 p_4 = 0;$$

$$\lambda_3 p_1 + \lambda_1 p_3 - \mu_1 p_5 = 0;$$

$$\lambda_3 p_2 - (\lambda_1 + \mu_2) p_6 + \mu_1 p_8 = 0;$$

$$\lambda_2 p_3 - (\lambda_1 + \mu_3) p_7 + \mu_1 p_9 = 0;$$

$$\lambda_1 p_6 - \lambda_1 p_8 = 0;$$

$$\lambda_2 p_5 + \lambda_1 p_7 + \mu_1 p_9 = 0;$$

$$\sum_{i=0}^{9} p_i = 1.$$

5. Conclusions

In order to analyze the reliability of information technologies management systems for institutions of higher education, the models of the theory of queuing system has been applied, taking all the peculiarities of the given system into consideration. For more detailed investigation of the functioning of similar systems, one should apply semi-Markovian processes. However, the mathematical mechanism will be far more difficult.

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В.С. Коновалюк[1], І.С. Клюс[2]. Підхід до аналізу надійності функціонування комп'ютерної мережі системи організаційного управління

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Для обчислення показників ефективності функціонування системи організаційного управління у вищих навчальних закладах пропонується розглядати комп'ютерну мережу як систему масового обслуговування, в якій заявки поступають з автоматизованих робочих місць, одне з яких має пріоритет на обслуговування.

Ключові слова: запит; заявка; система масового обслуговування; стани системи; стаціонарні показники ефективності

В.С. Коновалюк[1], И.С. Клюс[2]. Подход к анализу надежности функционирования компьютерной сети системи организационного управления

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Для вычисления показателей эффективности функционирования системы организационного управления в высших учебных заведениях предлагается рассматривать компьютерную сеть как систему массового обслуживания, в которой заявки поступают с автоматизированных рабочих мест, одно из которых имеет приоритет на обслуживание.

Ключевые слова: заявка; система массового обслуживания; состояния системы; стационарные показатели эффективности; требования

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