AIRPORTS AND THEIR INFRASTRUCTURE

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Svetlana Talakh[1]
Oleksandr Dubik[2]

COMPUTATIONAL INVESTIGATION OF COMPOSED MULTILAYERED HALF-SPACE STRENGTH UNDER PAVEMENT

[1,2]National Aviation University
1, Kosmonavta Komarova avenue, Kyiv, 03680, Ukraine
E-mails: [1]svetlanatalah@gmail.com; [2]saschadubik@ukr.net

Abstract. *The article presents mathematical model of non-linear subgrade half-space on the basis of the von Mises advanced yieldcriterion. We have performed computational investigations of non-rigid pavement deflected mode in plane problem statement under simulation of subgrade half-space active zone including the layers of pavement. The results receiveds how the plastoelastic deformation development influences the strength of non-rigid pavement.*

Keywords: finite element method; limiting state;non-rigid pavements; multilayered subgradehalf-space; plastoelastic deformation; strength calculation

1. Introduction

In the increasingly intensive traffic and loads on traffic-bearing surface there step up the requirements with respect to traffic operation conditions of highways. Usage of roads over the recent years shows that calculation methods for non-rigid pavements shall take into account and naturally predict both elastic and plastic deformations. So far, however, neither theoretically based solutions have been obtained nor regularities discovered with regard to the processes of development and accumulation of permanent deformation in the layers of pavements and active zone of subgrade half-space.

The active zone of pavement subgrade soil may reach 6 m and more depending on ground layer properties. As an example of inhomogeneous subgrade soils serves loess-type grounds being wet, and which occupy over 70 % of the Ukraine's territory. Their distinctive feature is that being in tension under the influence of wheel load and self-weight of pavement and ground under wetting they deliver additional settling and can reduce their structural strength [1-3].

In design of pavements which belong to the sections with second type hydrogeological conditions having loessial soils to prevent ground wetting, it is provided for organisation of damp proof course under the pavement. But in practical terms it is often that these measures prove to be ineffective.

2. Statement of Problem

To solve this problem and improve calculation of non-rigid pavements with the above specified inclusions in the ground active mass and inhomogeneity of subgrade soil, we offer modelling of composed multilayered structure by a discrete inhomogeneous half-space taking into account state equations for discrete and local areas for anisotropic material, equivalent to the real-word loessial soil.

We consider the plane problem statement of a nonlinear deformation solid taking into account geometrical and physical nonlinearity with the use of correlations of themomental scheme of finite element (MSFE) [1] under multilinear approximation of displacement functions [2].

When determining limit deformations of the whole multilayered structure of the half-space fragment under analysis including the ultimate strength of the ground to shearing strength and permissible tensile stress in the layers of artificial subgrade the development of local plastic deformations is taken into account.

When simulating half-space a considerable inhomogeneity of soil layers is expected, and also inclusions which simulate the road structure elements that determine existence of stress concentration and necessity to investigate half-space in its limiting state.

The soil limiting state in elementary neighbourhood under study is to that extent equivalent to the stress condition, when additional little exposure can upset a balance. Such stress condition is also characterised by the fact that shear strength in the elementary region has to be equal to limiting one with respect to the soil in question. Such condition belongs to the second stage of soil limiting state under extensive development of shear deformations in the soil mass. In this case, computational solution of the soil masses stability problem is obtained on the basis of the methods offered in [1, 4] including some adjustment of the yield criterion for subgrade half-space.

3. Solution of the problem

Balance of the elementary volume of any continuous medium, regardless of its mechanical and physical properties is described by the variational equation:

$$\int_v \left(\widehat{\sigma}' + \underset{4}{\widehat{C}} \cdot\cdot \widehat{\gamma} \right) \cdot\cdot \delta\widehat{\gamma} dv + \int_v \rho \ddot{\vec{u}} \cdot \delta\vec{u} dv -$$

$$- \int_v \vec{p} \cdot \delta\vec{u} dv - \int_s \vec{q} \cdot \delta\vec{u} ds = 0, \qquad (1)$$

where $\widehat{\sigma}'$ is the second tensor of initial Piola-Kirchhoff stress; $\widehat{\gamma}$ – Cauchy-Green incremental finite deformation tensor, caused by equilibrium (reference) configuration disturbance C^t and its transition to the configuration $C^{t+\Delta t}$; $\vec{u}, \ddot{\vec{u}}$ – incremental displacement and acceleration vectors; $\vec{p}$, $\vec{q}$ – generalized vectors of mass and surface forces, acting on the body and referred to the configuration C^t; ρ – continuous medium material density.

The proposed methods implement the application approach of variation principles and the theory of deformable body boundary state of stress, when the solutions obtained initially involve with division of elastic regions into elastic and non-elastic ones with the developing regions of plastoelastic deformations (shearing ones for the soils). In the process of deformation the reference design finite element model is transformed in accordance with the yield criterion of soil mass and divided into two regions to define the deflected mode (DM): elastic and plastoelastic ones.

In this paper the stability criterion or subgrade half-space yield for a separate local homogeneous isotropic region is proposed in the universal form on the basis of the von Mises advanced yield criterion (on account of inclusion of dependencies from the spherical part of the stress tensor in it) [5] with the use of loading surface by the Mohr–Coulomb criterion [3, 6] and taking into account both the second and third invariants of the deviation tensor of the stress function through the Lode-Nadai invariant [6]. Since the tensor invariants are determined through the components of the spherical and deviation parts of the stress function, and assumptions as for homogeneity and isotropy of the half-space local region provide for their independence from the normal lines of the octahedral planes, the von Mises advanced yield criterion can be presented in the form as follows:

$$f\left(\hat{\sigma}, \hat{S}, \hat{\gamma}^{(p)}, \alpha, \varphi, c \right) =$$

$$= \frac{3}{2} I_1\left(\hat{S}^2 \right) \left(\cos\alpha - \frac{1}{\sqrt{3}} \sin\alpha \cdot \sin\varphi \right)^2 - \qquad (2)$$

$$- \left[\frac{1}{\sqrt{3}} I_1(\sigma) \sin\varphi - \sqrt{3}\, c \cos\varphi \right]^2 = 0,$$

where $\hat{\sigma}, \hat{S}, \hat{\gamma}^{(p)}$ are tensors of total (full) stresses, the stresses of the deviation part and plastic deformations, respectively; α, φ, c – the third invariant of deviator stress through the Lode angle, the angle of internal soil friction, and specific cohesion, respectively; $I_1\left(\hat{S}^2\right)$, $I_1\left(\hat{\sigma}\right)$ – the first invariant of squared stress deviation tensor and the first invariant of the combined stress tensor, respectively. The Lode-Nadai angle α is defined from the formula [6]:

$$\alpha = \frac{1}{3} \arcsin\left(-\frac{3\sqrt{3}}{2} \cdot \frac{I_3}{I_2^{3/2}} \right);$$

– the third and second invariants of stress deviation tensor $\hat{S}$ can be represented as

$$I_3 = I_3\left(\hat{S} \right) = \frac{1}{3} I_1\left(\hat{S}^3 \right); \qquad (3)$$

$$I_2 = I_2\left(\hat{S} \right) = \frac{1}{2} I_1\left(\hat{S}^2 \right).$$

Taking into account (3), we obtain:

$$\alpha = \frac{1}{3} \arcsin\left\{ -\frac{\sqrt{6}}{4\sqrt{2}} \frac{I_1\left(\hat{S}^3 \right)}{\left[I_1\left(\hat{S}^2 \right) \right]^{3/2}} \right\}. \qquad (4)$$

Based on the formula (4) we can ascertain that the Lodi-Nadai parameter α represents the function of the $I_1\left(\hat{S}^2\right)$ and $I_1\left(\hat{S}^3\right)$ invariants.

To solve the problem of the subgrade half-space stability on the basis of the finite element method in the function of basic relations the proposed methods use the balance variational equations (1) and the loading surface equations in the six-dimensional space of total stresses (2), which being scalar take the form as follows:

$$f\left(\overset{*}{\sigma}{}^{ij},\overset{*}{S}{}^{ij}\right)=\frac{2}{3}\overset{*}{S}_{ij}\overset{*}{S}{}^{ij}\left(\cos\alpha-\frac{1}{\sqrt{3}}\sin\alpha\sin\varphi\right)- \quad (5)$$

$$\left[\frac{1}{\sqrt{3}}\overset{*}{\sigma}{}^{ij}G_{ij}\sin\varphi-\sqrt{3}c\cos\varphi\right]^2=0,$$

where $\overset{*}{\sigma}{}^{ij}$, $\overset{*}{S}{}^{ij}$ are the components of full total and deviator stress tensor; α – the Lodi-Nadai parameter, which is the function of the invariants $I_1\left(\hat{S}\right), I_3\left(\hat{S}\right)$; φ, c – the constants of mechanical and physical properties of soil.

Based on the plastic flow theory with the use of the loading surface equation (5) we have obtained the linearized constitutive equations of the generalised Hooke's law, which applies to the region of finite deformations to establish links between the incremental stresses and incremental final deformations (under the stage of active loading):

$$\sigma^{ij}=C^{ijkl}_{(e,p)}\gamma_{ij};\qquad C^{ijkl}_{(e,p)}=C^{ijkl}_{(e)}-\beta n^{ij}n^{kl}, \quad (6)$$

where $C^{ijkl}_{(e)}, C^{ijkl}_{(e,p)}$ are the elasticity tensors under the plastoelastic stage of plastoelastic deformation development, respectively; n^{ij} – the components of the second-rank tensor caused by the development of elastic deformations when the function is

$$f\left(\overset{*}{\sigma}{}^{ij},\overset{*}{S}{}^{ij}\right)>0.$$

Description of incremental stress tensor in accordance with the theory of plastic yielding and with the help of the correlations obtained (6) provides for a more accurate calculation of the plastic deformations in the three-dimensional space of the continuous medium region deformation state, which has been demonstrated at solving a number of test problems.

Determination of impact of plastoelastic deformation development on the non-rigid pavements' DM is considered in the context of the combined multilayered structure calculation (non-rigid pavement, artificial and subgrade soil) in plane problem statement.

To compare and analyse the results obtained at numerical computation with the use of the developed methods, we have performed simulation of the carriage frame impact with an adequate tracing to equivalent strip load. At determining limit deformations of the whole multilayered structure of the half-space fragment under analysis including the ultimate strength of the ground to shearing strength and permissible tensile stress in the layers of artificial subgrade the development of local plastic deformations is taken into account.

The non-rigid pavement construction and active subgrade soil are shown on Fig. 1.

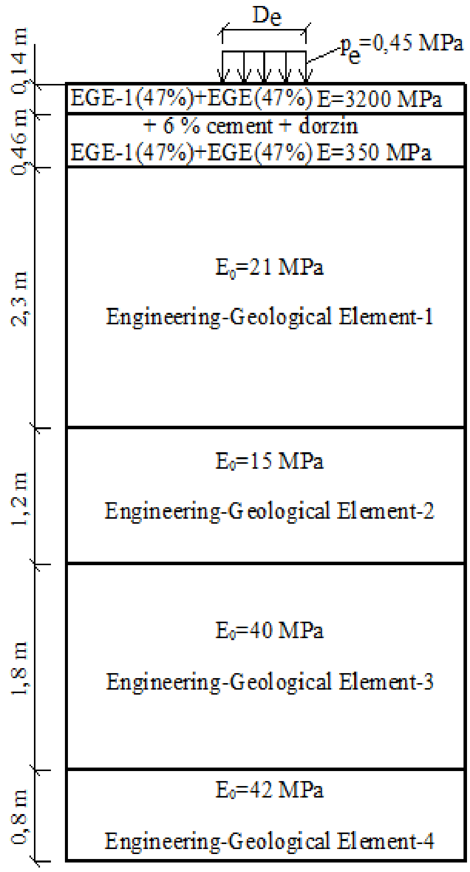

Fig. 1. The engineering-geological section of pavement and active subgrade soil

To obtain an adequate discrete calculation model of the non-rigid pavement including interaction with subgrade soil we have performed the procedures to reduce initial parameters (the geometrical, mechanical and physical, and loading ones) to their equivalent section.

Taking into account the parameters of the equivalent pavement section, artificial subgrade, and the soil multilayered half-space we have built a design diagram for the problem in question including a symmetry plane going through the central axis of the equivalent wheel pattern with dimensions $a \times a$ (Fig. 2). The net domain of the design diagram has a size of: $S_1 = M1 = 2,\ S_2 = M2 = 19,\ S_3 = M3 = 23$, i.e. $2 \times 19 \times 23$, which corresponds to 396 finite elements (FEs), quantity of FE nodes (874), and quantity of resolving equations $k = 2622$. The boundary conditions of the fragment under analysis are as follows.

1) in the global coordinates, along the symmetry plane $Z^{1`}OZ^{3`}$ the constraints are imposed on the displacements and angular deflections in the direction $Z^{2`}$;

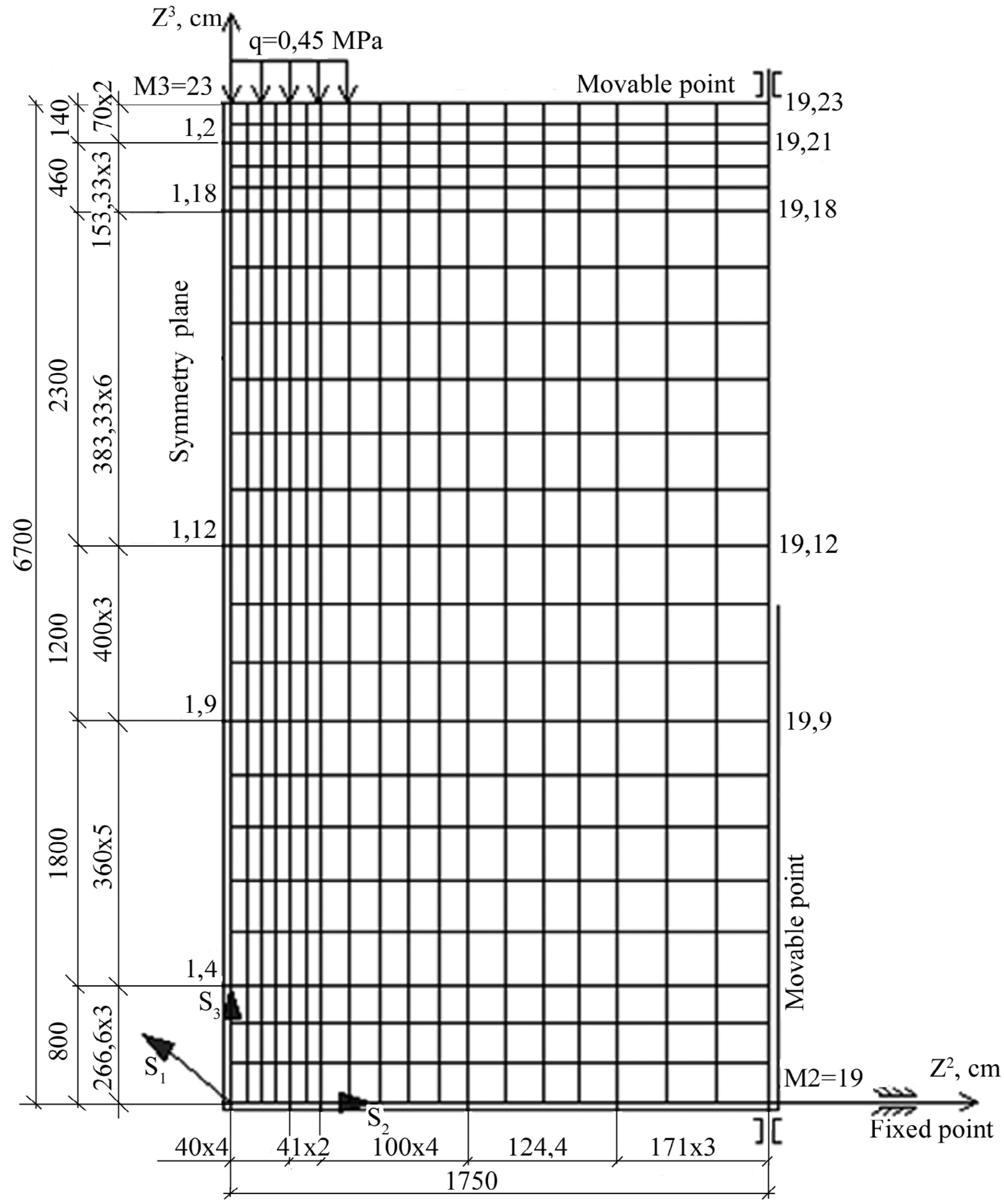

Fig. 2. The design diagram of the multilayered half-space

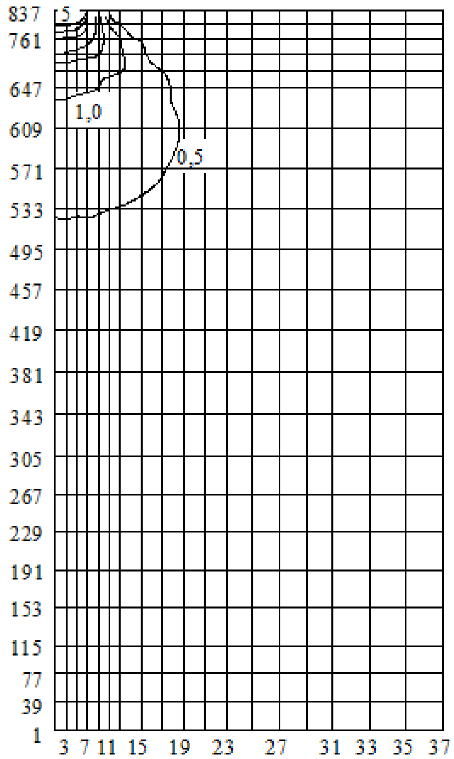

Fig. 3. Isolines of compressive stresses σ^{33} (isobars),kgf/cm^2

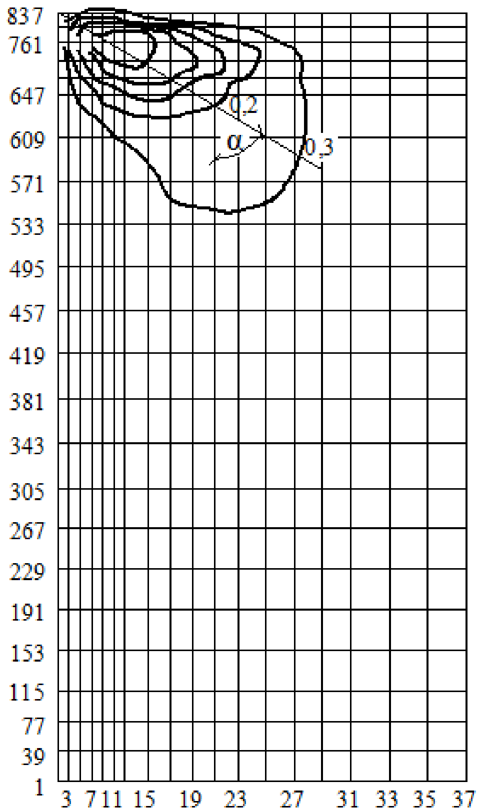

Fig. 4. Shear stress contours σ^{23} (thrust lines),kgf/cm^2

2) along the lower plane of the fragment one can simulate the border of the subgrade soil active zone, in addition the constraints are imposed along the global coordinates directions $Z^{2`}$ and $Z^{3`}$ (fixed point);

3) along the lateral right plane, which is the limit of the design fragment lengthwise along coordinate axes $Z^{2`}$, the constraints are imposed only on the displacements in direction $Z^{2`}$ (movable point).

External load in the form of strip load: along the upper limiting surface of half-space one applies an evenly distributed load with intensity of $q = 0{,}45$ MPa per interval along the upper edge of the design diagram with node grid coordinates $S_3 = 23$; $S_2 = 1 \ldots 4$.

The horizontal layers which define the half-space heterogeneity are simulated in accordance with the shell region with border nodes – the start $N_2 = 1$, $N_3 = 1$ and end $k_2 = 19$, $k_3 = 4$ ones – and inclusions of 1-5 type including the pavement artificial subgrade and equivalent layer of the pavement itself.

4. Analysis of researches

The DM computational investigations of non-rigid pavement construction half-space in question in its interaction with the artificial and subgrade soil have been conducted both with consideration for plastic deformations development and without inclusion of the latter.The results showed that the calculation taking into account the development of plastic deformation maximum movement will be more on 1,09мм, that is $\cong 11{,}0\%$, and the maximum stress more on 0,41 kgf/cm^2, that is $\cong 8{,}2\%$

The stress pattern is presented as pressure isolines (isobars in Figure 3) and displacement stress (thrust lines in Figure 4).

Some irregular course ofthe isolines shows heterogeneity of the multilayered soil, but their general pattern obtained in the task in question corresponds to the known classical examples shown in [3, 5, 6]:

i.e. the isobars are strictly symmetric relative to the vertical axial plane, and they are decreased by 10 times at a relative depth $\frac{z}{b} \approx 5{,}5 \ldots 6{,}0$;

i.e. the shear stress values reach 0.5 p (up to 2.5 kgf/cm^2) and the "thrust line" symmetry axis turns at an angle $\alpha = \text{arctg}\,137 \cong 54^\circ$.

5. Conclusion

We have built the mixed mathematical model of non-linear subgrade half-space on the basis of the von Mises advanced yield criterion, including the rate-type constitutive equation in exorbitant condition and which takes into consideration the third invariant of stress deviation tensor. We have developed the discrete model of wheel loads caused by carriage frame of modern vehicles and the one of the rigid half-space with description of multilayered inclusions with the real-world mechanical and physical characteristics of soils on the basis of the geological-engineering sections.

UsingtheMSFEofnon-rigidpavementconstructionwehaveperformedcomputationalinvestigationsinplaneproblemstatementinitsinteractionwiththeofsubgradehalf-spaceactivezoneincludingthelayersofpavement.The results showed that the calculation taking into account the development of plastic deformation maximum movement will be more on $11{,}0\,\%$, and the maximum stress more on $8{,}2\,\%$.

The concentration of plastic deformation observed in the regions adjacent to the loading surface

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С.М. Талах[1], О.М. Дубик[2]. Чисельне дослідження міцності комбінованого багатошарового півпростору під дорожнім покриттям

[1,2]Національний авіаційний університет, просп.. Космонавта Комарова, 1, Київ, Україна, 03680.
E-mails: [1]svetlanatalah@gmail.com; [2]saschadubik@ukr.net

Стаття присвячена актуальним питанням розрахунків на міцність дорожніх одягів нежорсткого типу при взаємодії з активною зоною ґрунтової основи. Запропоновано комбіновану математичну модель багатошарового ґрунтового півпростору на основі розширеного критерію текучості Мізеса, що включає рівняння стану в приростах у заграничному стані і враховує третій інваріант тензора-девіатора напружень. Виконано чисельні дослідження за допомогою монетної схеми скінченних елементів конструкції дорожнього одягу нежорсткого типу в плоскій постановці задачі при взаємодії з активною зоною ґрунтового півпростору. Дослідження напружено-деформованого стану багатошарового півпростору проводилися при розрахунку з урахуванням розвитку пластичних деформацій та без урахування останніх. Результати досліджень показали, що при розрахунках з урахуванням розвитку пластичних деформацій максимальні переміщення збільшуються приблизно на 11%, а максимальні напруження - на 8,2%. Концентрація пластичних деформацій спостерігається в областях, прилеглих до поверхні навантаження.

Ключові слова: багатошаровий ґрунтовий півпростір; граничний стан; дорожні одяги нежорсткого типу; метод скінченних елементів; пружно-пластична деформація; розрахунок на міцність

С.М. Талах[1], А.Н. Дубик[2]. Численное исследование прочности комбинированного многослойного полупространства под дорожным покрытием

[1,2]Национальный авиационный университет, просп. Космонавта Комарова, 1, Киев, Украина, 03680
E-mails: [1]svetlanatalah@gmail.com; [2]saschadubik@ukr.net

Статья посвящена актуальным вопросам прочностных расчетов дорожных одежд нежесткого типа при взаимодействии с активной зоной грунтовой основы. Предложена комбинированная математическая модель многослойного грунтового полупространства на основе расширенного критерия текучести Мизеса, включающая уравнение состояния в приращениях в запредельном состоянии и учитывающая третий инвариант тензора-девиатора напряжений. Выполнены численные исследования с помощью моментной схемы конечных элементов конструкции дорожной одежды нежесткого типа в плоской постановке задачи при взаимодействии с активной зоной грунтового полупространства. Исследования напряжено-деформированного состояния многослойного полупространства проводились при расчете с учетом развития пластических деформаций и без учета последних. Результаты исследований показали, что при расчете с учетом развития пластических деформаций максимальные перемещения увеличиваются приблизительно на 11%, а максимальные напряжения – на 8,2%. Концентрация пластических деформаций наблюдается в областях, примыкающих к поверхности нагружения.

Ключевые слова: дорожные одежды нежесткого типа; метод конечных элементов; многослойное полупространство; предельное состояние; расчет на прочность; упругопластическая деформация

Talakh Svetlana (1962). Candidate of Engineering. Associate Professor.
Department of Reconstruction of Airports and Highways of Airports, National Aviation University, Kyiv, Ukraine.
Education: Ukrainian Institute of Engineers Water Resources, Ukraine (1984).
Research area: optimization of the shape and structure of thin-walled spatial structures.
Publications: 50.
E-mail: svetlanatalah@gmail.com

Dubik Oleksandr (1989). Postgraduate student.
Department of Reconstruction of Airports and Highways of Airports, National Aviation University, Kyiv, Ukraine.
Education: National Aviation University, Kyiv, Ukraine (2012).
Research area: calculation of non-rigid type of road pavements.
Publications: 5.
E-mail: saschadubik@ukr.net