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GAIN-SCHEDULING OPTIMISATION OF PITCH HOLD MODE BASED ON GENETIC ALGORITHMS

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Abstract. *This paper presents an algorithm that was developed to select the best controller gains. The method is based on solving robust optimization problem for family of gain-scheduled PID-controllers. Parametric robust optimization provided by using genetic algorithm approach. This technique is useful to obtain flexibility and robustness in the controller automatic design.*

Keywords: gain-scheduled controller; genetic algorithm; PID-controller; robustness; unmanned aerial vehicle

1. Introduction

The aircraft studied in the paper is the Unmanned aerial vehicle (UAV) Aerosonde for which the Aerosim MATLAB toolbox was developed. Such UAVs are subjected to various disturbances within the flight envelope. Due to gain scheduling control significally depends on the flight regime and PID- controller can effectively provide pitch hold mode. To satisfy the UAV usage range, it is proposed to use gain-scheduled PID-controller instead of singular one.

The genetic algorithms (GAs) are powerful widely applicable tool in many control engineering problems. According to the GA-based approach, the task of optimization design of the dynamic PID-controller is reduced to the problem of the research for global minimum of a multivariable function. The GA optimization procedure is adopted to ensure the control system has the best disturbance rejection performance and robustness for any perturbed model of the closed loop system.

2. Analysis of recent research and publications

The gain-scheduled (GS) control problem has been widely discovered both from theoretical and practical viewpoint, see, for example, [1]. An effective approach to solve the nonlinear control problem is using gain scheduling with linear controllers. For example in an aircraft flight control system, the altitude and Mach number might be the scheduling variables with different linear controller parameters available for various combinations of these two variables [2,3]. In the present paper proposed alternative scheduling parameter, which is the dynamic pressure (DP) $\overline{q}$. The main advantage of DP, that this parameter is related with both flight altitude and speed values.

3. Problem statement

As the above mentioned the DP $\overline{q} = \dfrac{\rho v^2}{2}$ is chosen as scheduling value. To prove the efficiency of the proposed technique, the pitch hold mode is considered as a case study. For the reason to find the compromise between the disturbance rejection performance and robustness, it is used a multi-objectives optimization problem, based on insert several objectives in one cost function and try to satisfy them at the same time [4,5]. The GA algorithm provides a multi-directional random search and an informational exchange among best solutions. Mentioned properties ensure generation of search directions in order to avoid local minima.

4. Gain-scheduled PID-controller design based on GA

A PID-controller which is one of the most powerful but complex controller mode operations combines the proportional, integral, and derivative modes. The analytical expression of PID-controller [6] is given as,

$$W_{C\vartheta q}(z) = P_{pid}(\overline{q}) + I_{pid}(\overline{q}) \cdot T\frac{z}{z-1} + D_{pid}(\overline{q})\frac{z-1}{z}$$

The scheme of a pitch rate feedback loop is shown on fig.1. Accordingly, the controller gain matrix has the following form $K = [P_{pid}, I_{pid}, D_{pid}, K_q, K_\theta]$.

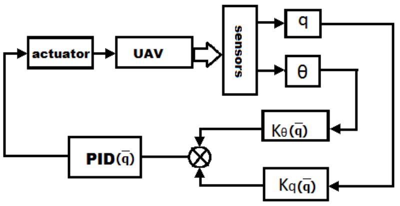

Fig. 1. The scheme of a pitch hold feedback loop

A gain scheduling control system design takes following steps:

1. Obtain a plant model for each operating region. It needs to linearize the plant at several equilibrium operating points.
2. Design a family of linear PID-controllers for the obtained plant models.
3. Implement a scheduling mechanism. It means that the controller coefficients are changed based on the values of the scheduling variables.
4. Provide optimization procedure to family of controllers.
5. Assess control performance with simulation.

5. GA optimization approach

The space of all possible solutions are sets of the controller parameters (chromosomes). In other words we have subspace with five dimensions:

$$(P_{pid_{MIN}}, P_{pid_{MAX}}) \times (I_{pid_{MIN}}, I_{pid_{MAX}}) \times (D_{pid_{MIN}}, D_{pid_{MAX}}) \times$$
$$\times (K_{q_{MIN}}, K_{q_{MAX}}) \times (K_{\theta_{MIN}}, K_{\theta_{MAX}})$$

First of all, it is necessary to generate the start parent population of chromosomes and their fitness function calculation. Each chromosome represents a string of five controller parameters: proportional, integral, derivative, pitch and pitch rate gains. The cost function (fitness function) values are calculated for each string. If any of chromosomes achieves the best value, than without any change it moved to the next generation. Next a new „reproduction“ group of chromosomes selected either according to their fitness values, or randomly selected, or selected combining both methods, etc. are used for crossover and mutation operations. After each step we have a new completed parent population. The algorithm exit condition can be following: the fitness function of the best string in some population fulfills the predefined condition or until the predefined number of populations is put into life.

6. Parametric robust optimization

The used method is based on H_∞ / H_2 -robust optimization [7, 8].

1. H_2 -norms of a model in deterministic case:

$$J_d = \sqrt{\sum_{k=0}^{\infty} \left[x_k^T \cdot Q \cdot x_k + u^T \cdot R \cdot u \right]}$$

where x_k – the state space vector, u– the input vector, R,Q – weighting matrices.

2. H_2 -norms of a model in stochastic case:

$$J_d = \sqrt{E_M \sum_{k=0}^{\infty} \left[x_k^T \cdot Q \cdot x_k + u^T \cdot R \cdot u \right]}$$

where E_M mathematical expectation operator.

3. H_∞ -norm: $\|T\|_\infty = \sup_\omega \bar{\sigma}(T(j\omega))$ where σ – singular value of T matrix, $\bar{\sigma}$ – maximum singular value on the current frequency.

Flight various uncertainties could be external and/or internal, structural and/or unstructured, which produce certain deviation from the nominal behavior to perturbed one. The design algorithm is to estimate the performance and robustness of the closed loop system using H_2 -norm of the sensitivity function and H_∞ -norm of the complementary sensitivity function, and then try to find the compromise between those two properties.

For this reason, we use a multi-objectives optimization problem, based on insert several objectives in one cost function and try to satisfy them at the same time [4,9].Therefore the penalty function (PF), restricting location's area of the closed loop system poles in the predefined region M in the complex plan. The penalty function is demonstrated as vertical section through Real axes on the figure 2b .

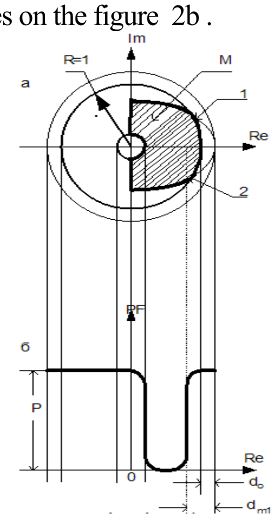

Fig. 2. Penalty function in the complex plan:
a – closed loop system poles; b – $PF(d_m)$ - vertical section through Real axes « PF – Re »

The penalty function is function of this minimum distance d_m:

$$PF(d_m)=\begin{cases}0, \text{if} \quad d_m \ge d_{ml},\\ \dfrac{P}{2}\left[1+\cos\left(\dfrac{\pi(d_m-d_0)}{(d_{ml}-d_0)}\right)\right], \text{if} \quad d_0<d_m<d_{ml},\\ P, \text{if} \quad d_m\le d_0,\end{cases}$$

where P is a big number (for instance, $P=10^4$-10^6).

The expression of cost function is given as,

$$J_\Sigma=\lambda_d J_d^2+\lambda_d^p\left(J_d^p\right)^2+\lambda_s J_s^2+\lambda_s^p\left(J_s^p\right)^2+\lambda_\infty\|T\|_\infty^2+\lambda_\infty^p\|T^p\|_\infty^2+Pf \quad (1)$$

where λ_d, λ_d^p, λ_s, λ_s^p, λ_∞, $\lambda_{\infty p}$ – the LaGrange factors, J_d and J_{dp} define the H_2-norms of the models in deterministic cases for particular DP range. J_s and J_{sp} – define the performances of the stochastic models. $\|T\|_\infty$ and $\|T^p\|_\infty$ are the H_∞-norms and gives the estimation of the robustness of the particular two parametrically disturbed plants. Pf is the penalty function. The cost functions in the expression (1) is the function of controller gain vector $\overrightarrow{C_a}$, that's why the result of optimization procedure will be following $\vec{C}_a^*=\arg\min J_\Sigma(\vec{C}_a)$, $\vec{C}_a\in M_c$, where M_c – stability range of controller gains. The block diagram in fig.3, describes the algorithm of parametric optimization procedure.

7. Case study

The state space vector of the longitude channel is $x=[V_T,\alpha,q,\theta]^T$, where V_T is the true speed, m/s, α is the angle of attack, deg, q is the pitch rate, deg/s, θ is the pitch angle, deg. The control input vector is $u=[\delta_e]^T$ represented by elevator deflections. The nonlinear model of the Aerosonde is linearized for range of operating conditions respected to range of DP from 200 to 650 $\frac{kg}{m\cdot s^2}$ with a granularity of 50 $\frac{kg}{m\cdot s^2}$.

Stochastic disturbance is modeled by means of Dryden filter [10]. The state space matrices A and B in general form filled with longitude coefficients and stability derivatives are given below:

$$A=\begin{bmatrix} X_V+X_{Tv}\cos\alpha_e & X_\alpha & -g_0\cos\gamma_e & 0\\ Z_V+Z_{Tv}\sin\alpha_e & Z_\alpha & -g_0\sin\gamma_e & V_T+Z_q\\ 0 & 0 & 0 & 1\\ M_V+M_{Tv} & M_\alpha & 0 & M_q\end{bmatrix};$$

$$B=\begin{bmatrix}X_{\delta e}\\ Z_{\delta e}\\ 0\\ M_{\delta e}\end{bmatrix}$$

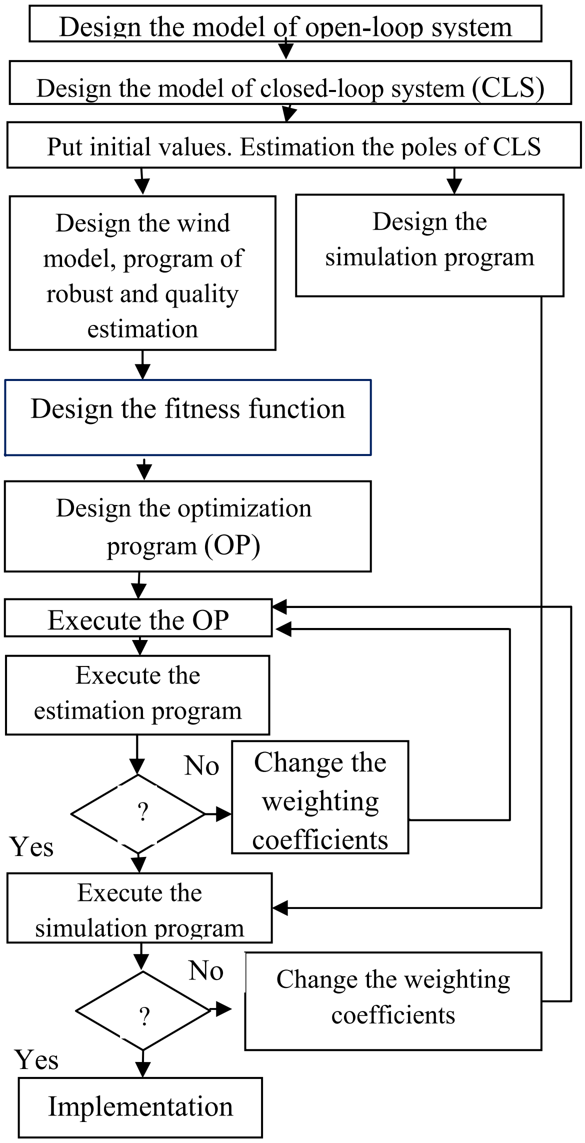

Fig. 3. The block scheme of parametric optimization algorithm

where $X_V = -\frac{\bar{q}S}{mV_T}(2C_L - C_{Dv})$

; $X_\alpha = \frac{\bar{q}S}{m}(C_L - C_{D\alpha})$; $X_{\delta e} = \frac{\bar{q}S}{m}C_{D\delta e}$;

$Z_V = -\frac{\bar{q}S}{mV_T}(2C_L - C_{Lv})$;

$Z_\alpha = -\frac{\bar{q}S}{m}(C_D - C_{L\alpha})$;

$Z_{\delta e} = -\frac{\bar{q}S}{m}C_{L\delta e}$; $Z_q = -\frac{\bar{q}S\bar{c}}{2mV_T}C_{Lq}$;

$M_V = \frac{\bar{q}S\bar{c}}{J_y m V_T}(2C_M + C_{mv})$; $M_\alpha = \frac{\bar{q}S\bar{c}}{J_y}C_{m\alpha}$;

$M_{\delta e} = -\frac{\bar{q}S\bar{c}}{J_y}C_{m\delta e}$;

$M_q = -\frac{\bar{q}S\bar{c}}{J_Y}\frac{\bar{c}}{2V_T}C_{mq}$;

$\gamma = \gamma_e$, $\alpha = \alpha_e$ are equilibrium (steady-state) conditions.

As seen from the description of space matrices coefficients, the aircraft flight dynamic depends on DP changes.

The controller gains after providing optimization procedure are shown in table 1.

Table 1. Controller gains

DP, $\frac{kg}{m \cdot s^2}$	P_{pid}	I_{pid}	D_{pid}	K_q	K_θ
200	0.5665	0.2186	-0.0721	0.0365	-4.7635
250	0.3691	0.4343	-0.0455	0.0473	-6.3766
300	0.6456	0.2798	-0.1348	0.2108	-2.2564
350	0.6154	0.2849	-0.1676	0.1219	-2.7315
400	0.5277	0.1793	-0.0781	0.3353	-3.0201
450	0.5536	0.186	-0.0805	0.3740	-2.1004
500	0.2244	0.1330	-0.0577	0.4940	-6.4949
550	0.3242	0.1559	-0.0616	0.199	-3.7701
600	0.4180	0.1747	-0.0246	0.2343	-3.6411
650	0.4233	0.2171	-0.0195	0.1934	-5.1648

Performance indices for DP range and standard deviations of the UAV outputs are shown in tables 2 and 3.

Table 2. Standard deviations of the UAV outputs

DP, $\frac{kg}{m \cdot s^2}$	Standard deviations of the UAV outputs in a stochastic case				
	σ_V, m/s	σ_α, deg	σ_q, deg/sec	σ_θ, deg	σ_{el}, deg
200	0.9909	0.2151	0.0221	0.0348	0.0342
250	1.0719	0.2193	0.0298	0.0358	0.0321
300	1.0746	0.2436	0.0524	0.0362	0.0609
350	1.2066	0.2414	0.0275	0.0353	0.0572
400	1.0630	0.1735	0.0221	0.0354	0.0294
450	1.2538	0.2061	0.0159	0.0346	0.0243
500	1.2423	0.2441	0.03	0.0373	0.0550
550	1.0376	0.1763	0.0238	0.0357	0.0296
600	1.1926	0.1879	0.0175	0.0353	0.0226
650	1.1057	0.1609	0.0152	0.0355	0.0225

Table 3. Performance indices for DP range

$H_{2\text{-norm}}$		H_∞ -norm
Deterministic case	Stochastic case	
1.0829	1.2287	3.4165
1.5169	1.6150	3.5608
1.3955	1.1423	2.5850
1.4645	1.6239	3.0458
1.3712	1.1691	2.8391
1.0713	1.1447	1.8986
1.6482	1.7871	2.8533
1.3592	1.2211	2.7598
0.8107	1.1208	2.0588
0.7919	1.2114	2.2432

The values of H_∞–norm are appropriate, the difference between H_2-norm in stochastic and deterministic case is very small.

The pitch angle step plots for the DP 300, 600 $\frac{kg}{m \cdot s^2}$ are shown on the figure 4.

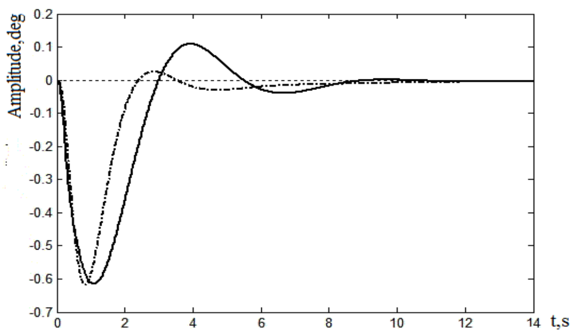

Fig. 4. System response to step input pitch angle with scheduled DP 300 $\frac{kg}{m \cdot s^2}$ (dash-and-dot line) and 600 $\frac{kg}{m \cdot s^2}$ (solid line)

Given step responses show that settling time values and the maximum overshoot values reflect the stability of close-loop system.

8. Conclusion

The design results prove the effectiveness of the proposed control method. The required flight performances are respected as well as the robustness of the control law. It can be seen that the handling quality of the models for each DP range are satisfied. The flight usage range is satisfied within UAV flight envelope.

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О.І. Надсадна. Оптимізація параметрів табличного регулятора стабілізації каналу тангажу із застосуванням генетичних алгоритмів

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Безпілотні літальні апарати останнього покоління розробляються для різних сфер експлуатації і можуть застосовуватися для виконання як військових, так і цивільних цілей. Такі безпілотні літальні апарати функціонують у широкому діапазоні збурень. Процедура оптимізації на основі генетичного алгоритму забезпечує високі показники якості та робастності для збурених моделей динаміки замкнутої системи управління.

Ключові слова: безпілотний літальний апарат; генетичний алгоритм; ПІД-регулятор; робастність; табличний регулятор

О.И. Надсадная. Оптимизация параметров табличного регулятора стабилизации канала тангажа с использованием генетических алгоритмов

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Беспилотные летательные аппараты последнего поколения разрабатываются для различных сфер эксплуатации и могут применяться для выполнения как военных, так и гражданских задач. Такие беспилотные летательные аппараты функционируют в широком диапазоне возмущений. Процедура оптимизации на основе генетического алгоритма обеспечивает высокие показатели качества и робастности для возмущенных моделей динамики замкнутой системы управления.

Ключевые слова: беспилотный летательный аппарат; генетический алгоритм; ПИД-регулятор; робастность;. табличный регулятор

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