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MATHEMATICAL MODELING OF SELF-EXCITED VIBRATION OF PIPES CONTAINING MOBILE BOILING FLUID CLOTS

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Abstract. *Numerical modeling dynamic behavior of a pipe containing inner nonhomogeneous flows of a boiling fluid has been carried out. The system vibrations at different values of the parameters of the flow nonhomogeneity and its velocity are observed. The possibility of forming stable and unstable flows depending on the character of nonhomogeneity and the velocity of fluid clots has been found.*

Keywords: dynamics; fluid clots; flutter; heat-exchanger; internal flows; non-homogeneous flow; pipe with an initial camber; periods; velocity; vibration.

1. Introduction

The pipe-line of a heat-exchanger containing nonhomogeneous mobile masses of a boiling fluid, vapour and their mixture is one of the main elements of modern heat and nuclear power plants. In its segments possessing an initial camber or taking a curvilinear shape due to their dynamic bending, the centrifugal inertia forces playing the role of active forces and acting in the osculating plane are generated. They are proportional to the pipe curvature, the mass of the moving fluid element and the square of its velocity [1]. In the case of non-steady processes of boiling these forces change in time and lead to the pipe-line vibration.

As experimental studies carried out in connection with analysis of boiling fluid motions in glass tubes heated on the outside testify, at some thermodynamical states and values of geometrical and mechanical parameters of the system there appear the cases of the so-called slug flows. They reside in the fact that in the tube heat-exchanging systems the regimes of fluid boiling are possible, when the generated vapour-water mixture is not homogeneous but consists of some fluid and vapour segments alternating and moving at high velocities. As the mixture flows, the process of boiling continues, thus the lengths of the tube segments filled with a fluid (called fluid clots) are decreasing and the lengths of cavities filled with a vapour (gas slugs) are increasing. In this case their velocities considerably increase.

The observations carried out on heated glass tubes show that the lengths of fluid clots change approximately from 10 internal diameters of the pipe on their formation to a zero on a complete evaporation, and the volume of a fluid, as it evaporates, increases by tenfold. On boiling the volume of gas cavities can change from a zero to 50 diameters of the pipe and then, as a result of clot evaporation, they mix.

The motion of a liquid clot inside a curvilinear channel is accompanied by the action of a centrifugal inertial force on its walls in the direction opposite to the orientation of a principal normal. Besides, as each element of a fluid also takes part in the slewing motion together with a pipe on its vibrations, additional gyroscopic forces of interaction between the fluid and pipe walls are generated. If stiffness of the curved pipe is relatively small, then its interaction with the moving fluid clot can cause noticeable dynamical effects. There are some cases, for example, when due the vibrations by these effects, the holes appear (wear through) in the walls in the sites of the tube contacting with the elements of supporting structures. As a result, the whole heat exchanger unit gets out of order and radioactive heat-transfer agents can find their way into the atmosphere.

For different relations between the lengths of fluid clots and the cavities filled with vapour (vapour slugs), the functioning of such mechanical systems can be accompanied by complex dynamical effects attributed to the possibility of participating the system bodies in several forms of these motions and the presence of gyroscopic interaction between them, like the possibility of static (divergent) loss of stationary motion stability, the appearance of unstable oscillatory motions (of a flutter-type) and parametric resonances [2, 3].

The above-mentioned types of the loss of stability are realized depending on the relations between geometrical and inertial parameters of the system, the velocity of clots as well as the presence or absence of an initial camber, so, if a pipe is curved in its original state, its motion can have the pattern of forced and be accompanied by ordinary resonances. If the initial camber is absent, self-sustained vibrations associated with parametric resonances can be excited in it. They are attributed to the fact that as an nonhomogeneous fluid flows inside the pipe, the internal characteristics of all the system vary all the time that might be an additional cause of the vibrations excitation.

One of the first tasks that triggered off the development of this problem was the task of elimination of considerable vibrations of the Trans-Arabic oil pipe-line [4]. Considering its simplified circuit design, the authors [2, 5] obtained equations for a straight pipe-line dynamics and showed the possibility of losing its stability on attaining critical velocities by the flow.

The paper studies the influence of an initial camber of a pipe, the size of fluid clots and vapour cavities and the velocity of their flow on the character of dynamic loss of a pipe system stability.

2. Statement of the problem

Consider the problem concerning transverse vibrations of an elastic pipe having an initial camber. A nonhomogeneous fluid flows inside the pipe. Let's assume that its nonhomogeneity might be caused, for example, by changing its modular state associated with its heating, boiling and conversion into vapour-and-water mixture. If typical dimensions of liquid clots and vapour cavities dividing them exceed typical dimensions of the pipe-line, for example the diameter of its channel (see Figure 1), one must take into account discontinuities in parameters of density and inner flow velocity. In this case as the pipe-line vibrates, the fluid particles have an accelerated flow both along and transverse the pipe axis, thus forming a dynamical load on the pipe. To calculate inertial forces acting on the pipe elements we assign the law of the fluid clot flow and the vapour-filled cavities motion in its channel proceeding from the condition of preserving the overall vapour-water mixture flow mass rate at the inlet and outlet. Let's form the model of changing the flow parameters of motion assuming that the clots of length a_0 enter the channel at a velocity of V_0. At the inlet a gap between two neighbouring clots is equal to zero. On motion caused by boiling the length of a clot varies as $a_1 = a_0 e^{-kt}$ and decreases at the rate of $\dot{a} = da_1/dt = -ka_0 e^{-kt}$. As a result, the lengths of the spaces (cavities) between clots increase at the rate of $\dot{b} = db_1/dt = cka_0 e^{-kt}$. The volume of vapour in a space is considered to be c times as much as that of a fluid from which it was formed, therefore the relation $\rho_f = c\rho_v$ is performed between the densities of the fluid and the vapour.

As the volume the space of a cavity increases, the velocity V_{i+1} of the $i+1$-th clot increases relative to camber. A nonhomogeneous fluid flows inside the pipe. Let's assume that its nonhomogeneity might be caused, for example, by changing its modular state associated with its heating, boiling and conversion into vapour-and-water mixture. If typical dimensions of liquid clots and vapour cavities dividing them exceed typical dimensions of the pipe-line, for example the diameter of its channel (fig. 1), one must take into account discontinuities in parameters of density and inner flow velocity. In this case as the pipe-line vibrates, the fluid particles have an accelerated flow both along and transverse the pipe axis, thus forming a dynamical load on the pipe. To calculate inertial forces acting on the pipe elements we assign the law of the fluid clot flow and the vapour-filled cavities motion in its channel proceeding from the condition of preserving the overall vapour-water mixture flow mass rate at the inlet and outlet. Let's form the model of changing the flow parameters of motion assuming that the clots of length a_0 enter the channel at a velocity of V_0. At the inlet a gap between two neighbouring clots is equal to zero. On motion caused by boiling the length of a clot varies as $a_1 = a_0 e^{-kt}$ and decreases at the rate of $\dot{a} = d\,a_1/d\,t = -k\,a_0 e^{-k\,t}$. As a result, the lengths of the spaces (cavities) between clots increase at the rate of $\dot{b} = d\,b_1/d\,t = c\,k\,a_0 e^{-k\,t}$. The volume of vapour in a space is considered to be c times as much as that of a fluid from which it was formed, therefore the relation $\rho_f = c\rho_v$ is performed between the densities of the fluid and the vapour (fig. 1).

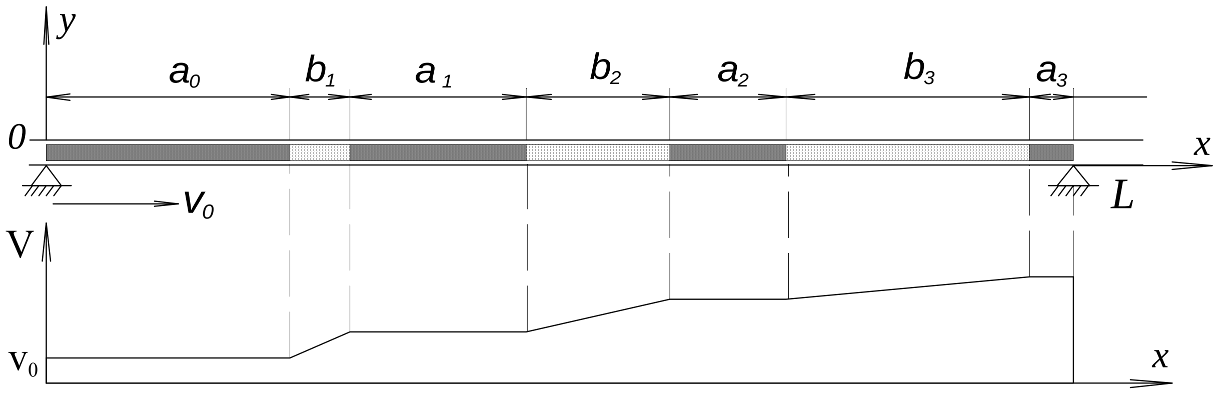

Fig. 1. The diagram of fluid clot flows and changes in internal flows velocity

As the volume the space of a cavity increases, the velocity V_{i+1} of the $i+1$-th clot increases relative to the previous one as $V_{i+1}=V_i(c-1)\dot a$. The velocity of vapour in the cavity between clots is assumed to be distributed linearly (see Figure 1). We investigate the influence of the pipe initial camber on the character of excited vibrations and their stability.

In studying the dynamical interaction between an elastic pipe and an inner flow T. B. Benjamin [6] showed that viscous friction forces occurring during flow appeared to be relatively small. As these forces are directed along the axis line of a pipe, they may be neglected in investigation of its transverse vibration. Thus, we consider the fluid to be perfect and while investigating its influence on the dynamics of the tube we will take into consideration only its inertial properties. In this connection stating the problem on vibrations of a pipe with an inner nonhomogeneous flow, we'll consider the motion of a fluid element along a vibrating and dynamically bending pipe-line. Calculate its acceleration in the direction perpendicular to the pipe axis and determine the inertial force acting on the fluid element and transferring to the pipe walls.

Let a fluid element of the mass m moves along the vibrating pipe at predetermined velocity $V(x)$ (see Figure 1). Considering its motion in the transverse direction, we write the equation $m\,d^2y_f/dt^2-N=0$. Here y_f is the displacement of the fluid element together with the pipe in the direction of the Oy axis, N the force with which the pipe acts upon the element. In this equation we must turn from the function $y_f(t)$, determining the fluid element coordinate, to the deflection function $y(x,t)+y_0(x)$ of the pipe with an initial camber $y_0(x)$ at the point x, where the element is located. To do so, we should take into consideration that the fluid element takes a new position in the pipe at each instant of time, therefore its velocity in a vertical direction is determined not only by the velocity of the pipe point in which the element is located, but also by the fact that the element moves to a neighbouring point in the pipe with another coordinate y and velocity $\dot y$

$$\frac{dy_f}{dt}=\frac{\partial y}{\partial t}+\left(\frac{\partial y_0}{\partial x}+\frac{\partial y}{\partial x}\right)\frac{\partial x}{\partial t}=\dot y+\left(y_0'+y'\right)V. \qquad (1)$$

Differentiating once more both members of (1) with respect to t, one finds the vertical component of absolute acceleration of the fluid element in the vibrating pipe with an initial camber

$$\begin{aligned}\frac{d^2y_f}{dt^2}&=\ddot y+2\dot y'V+V\,y'V'+y''V^2+\\&+y_0''V^2+y'\dot V+y_0'\dot V+V\,y_0'V'.\end{aligned} \qquad (2)$$

This formula can be correlated with the formula of the Coriolis theorem [7] for absolute acceleration of a particle, where $\ddot y$ is the bulk acceleration, $2\dot y'V$ is the Coriolis acceleration, $y''V^2$ the centripetal acceleration, $\dot y'$ the angular velocity of the pipe element, V the relative velocity of the fluid element.

When constructing the equation of transverse vibration of a tubular rod with an inner flow of an nonhomogeneous fluid, we model it by the Eiler-Bernoulli beam and neglect the internal friction forces and the beam friction on interacting with the environment. Such equation of plane transverse vibrations of the pipe can be presented as:

$$EJ\,y^{IV}+\rho_t a_t+\rho_f a_f=0.. \qquad (3)$$

Here EJ is the pipe bending stiffness; ρ_t the tube linear density; ρ_f the linear density of the inner flow; a_t, a_f, are the accelerations along the axis Oy of the tube and fluid elements respectively.

Using formulas $a_t = d^2y/dt^2$, $a_f = d^2y_f/dt^2$ and taking into account (2) and (3), we write the equation of pipe vibrations in the sections containing the fluid as follows

$$\begin{aligned} &EJ\,y^{IV} + \rho_f V^2 y'' + (\rho_t + \rho_f)\ddot{y} + \\ &+2V\rho_f \dot{y}' + \rho_f \dot{V} y' = -\rho_f y_0'' V^2 - \rho_f y_0' \dot{V}. \end{aligned} \tag{4}$$

In the sections containing the vapour spaces, $\rho = \rho_v$ and equation (4) takes the form

$$\begin{aligned} &EJ\,y^{IV} + \rho_v V^2 y'' + (\rho_t + \rho_v)\ddot{y} + 2V\rho_v \dot{y}' + \\ &+\rho_v \dot{V} y' + \rho_v V V' y' = -\rho_v y_0'' V^2 - \rho_v y_0' \dot{V} - \\ &-\rho_v y_0' V V'. \end{aligned} \tag{5}$$

The distinguishing feature of the assigned problem described by equations (4) and (5) resides in the fact that when the fluid clots flow, either equation 4 or 5 is alternately used for one and the same points of the pipe. Thus, the chosen mechanical system belongs to the systems with variable parameters (with approximately periodical coefficients and right member). Due to this fact on varying the velocity V both ordinary and parametrical resonance vibrations, typical of such systems, can be excited as the result of the dynamical loss of stability. For the case considered, the problem of studying parametric vibrations is complicated by the presence in equation (4) the component $2\rho_f \dot{y}' V$ describing the internal force referring to the gyroscopic type. Their presence considerably complicates the mode of the pipe motion because its elements start vibrating at different phases.

The second peculiarity of the process studied is that due to the change of inertial properties of the pipe, as fluid clots travel in it, the notions of the frequency spectrum and modes of free vibrations do not exist for it, and natural frequencies in the vicinity of which resonances could be realized are lost. Therefore it's rather difficult to predict a dynamical loss of stability in such systems. And, finally, the difficulty in studying the dynamical system involved increases even more at the cost of the discrete character of the clot flow resulting in the fact that the coefficients of the united set of equations (4) and (5) actually become discontinuous.

The above peculiarities involve difficulties in using analytical methods for studying the dynamic instability of pipes with inner flows, based on the Liapunov and Floquet approaches [8]. Thus this investigation uses the method of direct numerical modelling the system motion at chosen initial disturbances and the assigned velocity V of the flow.

3. Investigation procedure

Let's consider two problems, namely the problems on motion of a nonhomogeneous flow in a straight pipe and in the pipe with an initial camber. To analyze the possibility of self excitation of the straight pipe-line vibrations with the inner flow of a nonhomogeneous fluid, let's impart the system some small initial perturbations in the form of a preset deflection and perform numerical modelling of its dynamical behaviour at various lengths of the clots and different values of the velocity V_0 at the inlet. If the vibrations of the perturbed pipe-line decay, then its initial state is considered to be stable. When the amplitude of vibrations and divergent deflection increase indefinitely, the system is considered to be dynamically unstable. The fluid velocity V_0, at which periodic motion is established in the system, is thought to be critical.

If dynamics of the tube with initial camber should be investigated, it is not necessary to introduce the additional perturbations into the system, as far as the constitutive equations are nonhomogeneous in advance.

When boundary conditions are being preset, it is considered that the pipe-line represents a multispan beam with equal lengths of spans and hinged supports. The system vibrations are modelled by the least power-intensive modes having skew symmetry relative to the support cross-sections. Then it is assumed that the vibrations of neighbor sections of the pipe have opposite phases and in studying them we arbitrarily separate one span of the pipe applying boundary zero conditions to deflections and bending moments at its support points

$$y(0) = y(L) = 0\,, \quad y''(0) = y''(L) = 0\,. \tag{6}$$

If the tube is assumed to have preliminary camber $y_0(x)$, the initial conditions are chosen in the form as follows

$$y(x,0) = 0\,,\ \dot{y}(x,0) = 0\,.$$

If the tube is preliminary straight, the initial conditions are chosen as the initial static perturbation

$$y(x,0) = w_0 \sin \pi x/L, \ \dot{y}(x,0) = 0.$$

Here the coefficient w_0 is considered to be very small.

For numerical integration of the equations at the preset boundary conditions (6) and initial perturbations, let's use the Houbolt implicit finite difference method characterized by the approximation of pinpoint accuracy and stability [9]. In this case for the time t, the time derivatives in equations (4) and (5) are substituted by finite differences in the form of:

$$\dot{y}(x,t) = \dot{y}\big|_t = \begin{bmatrix} 11y_t(x) - 18y_{t-1}(x) + \\ +9y_{t-2}(x) - 2y_{t-3}(x) \end{bmatrix} /6\Delta t$$

$$\ddot{y}(x,t) = \ddot{y}\big|_t = \begin{bmatrix} 2y_t(x) - 5y_{t-1}(x) + \\ +4y_{t-2}(x) - y_{t-3}(x) \end{bmatrix} /\Delta t^2. \qquad (7)$$

Here the following designations are used: $y_t(x) = y(x,t)$, $y_{t-1}(x) = y(x,t-\Delta t)$, $y_{t-2}(x) = y(x,t-2\Delta t)$, $y_{t-3}(x) = y(x,t-3\Delta t)$, Δt is the time numeric integration step.

Taking into account the above relations, equations (4), (5) can be written down as:

$$\begin{aligned} &EJ\frac{d^4y}{dx^4}\Big|_t + \rho_f V_f^2 \frac{d^2y}{dx^2}\Big|_t + + \frac{2(\rho_t+\rho_f)}{\Delta t^2} y\Big|_t + \\ &+\frac{11\rho_f V_f}{3\Delta t}\frac{dy}{dx}\Big|_t = \frac{5(\rho_t+\rho_f)}{\Delta t^2} y\Big|_{t-\Delta t} - \\ &-\frac{4(\rho_t+\rho_f)}{\Delta t^2} y\Big|_{t-2\Delta t} + \frac{(\rho_t+\rho_f)}{\Delta t^2} y\Big|_{t-3\Delta t} + \qquad , \qquad (8) \\ &+\frac{6\rho_f V_f}{\Delta t}\frac{dy}{dx}\Big|_{t-\Delta t} - \frac{3\rho_f V_f}{\Delta t}\frac{dy}{dx}\Big|_{t-2\Delta t} + \\ &+\frac{2\rho_f V_f}{3\Delta t}\frac{dy}{dx}\Big|_{t-3\Delta t} - \rho_f V_f^2 y_0''\Big|_t - \rho_f y_0' \dot{V}_f\Big|_t. \end{aligned}$$

$$\begin{aligned} &EJ\frac{d^4y}{dx^4}\Big|_t + \rho_v V_v^2 \frac{d^2y}{dx^2}\Big|_t + \frac{2(\rho_t+\rho_v)}{\Delta t^2} y\Big|_t + \\ &+\frac{11\rho_v V_v}{3\Delta t}\frac{dy}{dx}\Big|_t + \rho_v V V' \frac{dy}{dx}\Big|_t = \\ &= \frac{5(\rho_t+\rho_v)}{\Delta t^2} y\Big|_{t-\Delta t} - \frac{4(\rho_t+\rho_v)}{\Delta t^2} y\Big|_{t-2\Delta t} + \\ &+\frac{(\rho_t+\rho_v)}{\Delta t^2} y\Big|_{t-3\Delta t} + \frac{6\rho_v V_v}{\Delta t}\frac{dy}{dx}\Big|_{t-\Delta t} - \\ &-\frac{3\rho_v V_v}{\Delta t}\frac{dy}{dx}\Big|_{t-2\Delta t} + \frac{2\rho_v V_v}{3\Delta t}\frac{dy}{dx}\Big|_{t-3\Delta t} - \\ &-\rho_v V_v^2 y_0''\Big|_t - \rho_v y_0' \dot{V}_v\Big|_t - \rho_v y_0' V_v V_v'\Big|_t. \end{aligned}$$

Considering the states $y_{t-1}(x)$, $y_{t-2}(x)$, $y_{t-3}(x)$ of the system at times $t-\Delta t$, $t-2\Delta t$, $t-3\Delta t$ to be known one can find the state $y_t(x)$ of the system at time t using (8) with appropriate boundary conditions and then turn to determining the system states at times $t+\Delta t$, $t+2\Delta t$, etc. Inasmuch as equations (8) represent the four layer difference scheme and we have only two initial conditions, the first step of the calculational processes is performed with the use of the three layer Newmark difference scheme.

Equations (8) with boundary conditions (6) are solved using the method of the transfer matrix. To do this, the fourth order equations (8) were transformed to the system of the first order equations. For the fist equations of (8) it looks like

$$\frac{dy_1}{dx} = y_2, \ \frac{dy_2}{dx} = y_3, \ \frac{dy_3}{dx} = y_4,$$

$$\begin{aligned} &EJ\frac{dy_4}{dx}\Big|_t = -\rho_f \dot{V}_f y_{2,t} - \rho_f V_f^2 y_{3,t} - \\ &-\frac{2(\rho_t+\rho_f)}{\Delta t} y_{1,t} - \frac{11\rho_f V_f}{3\Delta t} y_{2,t} + \frac{5(\rho_t+\rho_f)}{\Delta t} y_{1,t-1} - \\ &-\frac{4(\rho_t+\rho_f)}{\Delta t^2} y_{1,t-2} + \frac{(\rho_t+\rho_f)}{\Delta t^2} y_{1,t-3} + \qquad (9) \\ &+\frac{6\rho_f V_f}{\Delta t} y_{2,t-1} - \frac{3\rho_f V_f}{\Delta t} y_{2,t-2} + \frac{2\rho_f V_f}{3\Delta t} y_{2,t-3} \\ &-\rho_f V_f^2 y_0''\Big|_t - \rho_f y_0' \dot{V}_f\Big|_t. \end{aligned}$$

Let's write this system in a general form

$$d\vec{y}/dx = A(x)\vec{y} + \vec{f}(x). \qquad (10)$$

Here $\vec{y} = \vec{y}(s)$ is the 4-dimensional vector of the unknown functions; x the independent variable changing within the limits of $0 \le x \le L$; $A(x)$ the known discontinuous matrix-function of the independent variable x; $\vec{f}(x)$ the preset vector of right members determined by the known solution functions at previous steps in time.

The solution to (9) must be subset to boundary conditions (6) in the interval bounds, which are predetermined at the beginning $x = 0$ and at the end $x = L$ of the integration interval.

We represent them in the general form as

$$B\vec{y}(0) = 0, D\vec{y}(L) = 0, \qquad (11)$$

where matrices B and D measure 2×4.

For constructing the solution $\vec{y}(x)$, let's choose such 2 components $y_j(x)$ among $y_i(x)\cdot(i=\overline{1,4})$ components, any values $y_j(0)$ of which don't violate the first equation (11) at zero values of the other components. After renumbering the unknown values $y_i(x)\cdot(i=\overline{1,4})$ in such a way that the index j could take on the values $j=\overline{1,2}$, the solution to problems (10) and (11) can be given as

$$\vec{y}(x)=Y(x)\vec{C}+\vec{y}_0,$$

where $\vec{y}_0$ is the solution to the Cauchy problem for system (10) at zero initial conditions, $Y(x)$ is the matrix 4×2 in size of particular solutions $y_{ij}(x)$ to the homogeneous matrix differential equation

$$dY/dx=A(x)Y \tag{12}$$

with initial conditions $y_{ij}(0)=\delta_i^j$ $\left(i=\overline{1,4}, j=\overline{1,2}\right)$ for independently modified variables, and with initial conditions chosen from the first equation of system (11) for the other variables $y_{ij}(0)$ $\left(j=3{,}4\right)$.

Here δ_i^j is the Kronecker symbol.

As $Y(x)$ is the solution to the homogeneous equation (12), then on choosing initial conditions for the predetermined vectors, we pay special attention to their linear independence. This is achieved by assuming the matrix of initial conditions $Y(0)$ to have the unit elements $y_{11}(0)=1$, $y_{22}(0)=1$. In doing so any pair of vectors $y_{ij}(0)$ are mutually orthogonal that provides their linear independence.

The vector of the constants $\vec{C}=\left(C_1,C_2\right)^T$ is chosen so that the equality

$$DY(L)\vec{C}+D\vec{y}_0(L)=0,$$

following from the second conditions of system (11) could be satisfied.

The construction of the matrix – function $Y(x)$ and the vector-function $\vec{y}_0(x)$ is made by integrating equations (10) and (12) by the fourth order Runge-Kutta method. The peculiarity of using such approach is that due to the presence of large factors in the coefficients of the system (8), it is rigid and there are rapidly growing functions among its particular solutions. Therefore in constructing the matrix of its fundamental solutions, the method of discrete orthogonalization by Godunov is additionally used which makes it possible to obtain a stable computational process by orthogonalizing the vector-solutions to the Cauchy problems in the finite number of argument change interval points. Its essence is in the fact that the integration interval is divided into sections, and the numerical integration of the initial differential equation is carried out on each of these sections in the same way as in using the method of transfer matrix. The lengths of the sections are such that the particular solutions to a homogeneous equation within the limits of one section could remain linearly independent. When passing from one section to another, the matrix of the solutions is subject to linear transformation so that the vectors of particular solutions of the homogeneous and nonhomogeneous equations become orthogonal. Thus it is possible to preserve the linear independence of the equation solutions in the whole interval of integration. To avoid excessive increase of the numerical values of the nonhomogeneous equation solutions, the normalization factor is introduced at the section boundaries.

4. Results and discussions

The calculation algorithms and computer programs for carrying out numeric modelling pipe vibrations at various values of their geometrical parameters were developed on the basis of the above outlined procedure.

To study the influence of the initial camber on the character of vibrations of a pipe system, the cases, when in the initial state the pipe was straight $\left(y_0(x)\equiv 0\right)$ and when its centre line was curved according to the law $y_0(x)=\dfrac{L}{400}\sin\dfrac{\pi x}{L}$ were considered. For the first problem non-trivial solutions may appear as a result of either divergent or flutter bifurcations. The results of the calculations for the above cases are given in Table 1, where L is the length of the pipe, h the thickness of its wall, a_0 the length of the clots at the inlet, k the parameter determining the velocity of fluid evaporation. It was assumed for all the pipes that

$$E=2\cdot 10^{11}Pa;\ \rho_t=\left((R)^2-(R-h)^2\right)\rho;$$

$$\rho=7800\frac{kg}{m^3};\ \rho_f=\pi\,(R-h)^2\rho_w;\ \rho_w=1000\frac{kg}{m^3},$$

$$R=0{,}015\,m;\ c=10.$$

At chosen values of the parameters eight problems were solved (see Table 1) differing in the lengths of the clots at the inlet a_0 and the value k determining the velocity of evaporation of the boiling fluid. Here the value of a_0 were $L/8$ and $L/4$, and the values of k were chosen so that during the flow in the pipe channel a fluid clot could decrease its length by 15–40 %.

For each problem at a fixed value of V_0, dynamics of the pipe at a time interval equal the time of arrival of three hundred and more clots was studied. It was assumed that the pipe was given some initial excitation in the form of a low initial velocity. If then the vibrations were decaying, the initial state was considered to be stable, but if the amplitude of vibrations increased the initial state was unstable. To find resonance flows, the velocity V_0 was varied and modelling the flow was repeated at a new value of V_0. The least value of V_0 at which the amplitude of vibrations began to increase without limit was considered to be critical. The step ΔV_0 of variation V_0 was $\Delta V_0 = 0{,}2\ m/s$. In the vicinity of a critical state the calculations were made specific with the step $\Delta V_0 = 0{,}1\ m/s$.

Table 1. Velocities values and periods of forced vibrations of a straight-line pipe

№	L, m	h, m	a_0	k, s^{-1}	Values of dynamical parameters				
1.	5	0.003	$L/8$	0.1	$V_0 = 1.1\ m/s$	$\boxed{V_{0,cr} = 1.2\ m/s}$	$V_0 = 20\ m/s$	$V_0 = 40\ m/s$	$V_0 = 87.5\ m/s$
					$T_v = 0.244\ s$	$T_v = 0.245\ s$	$T_v = 0.262\ s$	$T_v = 0.289\ s$	$T_v = 1.301\ s$
					$T_c = 0.568\ s$	$T_c = 0.52\ s$	$T_c = 0.031\ s$	$T_c = 0.016\ s$	$T_c = 0.007\ s$
2.	5	0.003	$L/8$	0.5	$V_0 = 1.8\ m/s$	$\boxed{V_{0,cr} = 1.9\ m/s}$	$V_0 = 20\ m/s$	$V_0 = 40\ m/s$	$V_0 = 80\ m/s$
					$T_v = 0.24\ s$	$T_v = 0.242\ s$	$T_v = 0.26\ s$	$T_v = 0.292\ s$	$T_v = 0.783\ s$
					$T_c = 0.347\ s$	$T_c = 0.329\ s$	$T_c = 0.031\ s$	$T_c = 0.016\ s$	$T_c = 0.008\ s$
3.	5	0.003	$L/4$	0.5	$V_0 = 5\ m/s$	$\boxed{V_{0,cr} = 5.1\ m/s}$	$V_0 = 10\ m/s$	$V_0 = 20\ m/s$	$V_0 = 40\ m/s$
					$T_v = 0.246\ s$	$T_v = 0.247\ s$	$T_v = 0.25\ s$	$T_v = 0.262\ s$	$T_v = 0.298\ s$
					$T_c = 0.25\ s$	$T_c = 0.245\ s$	$T_c = 0.125\ s$	$T_c = 0.063\ s$	$T_c = 0.031\ s$
4.	5	0.003	$L/4$	1	$V_0 = 6.9\ m/s$	$\boxed{V_{0,cr} = 7\ m/s}$	$V_0 = 10\ m/s$	$V_0 = 20\ m/s$	$V_0 = 40\ m/s$
					$T_v = 0.245\ s$	$T_v = 0.245\ s$	$T_v = 0.25\ s$	$T_v = 0.262\ s$	$T_v = 0.297\ s$
					$T_c = 0.18\ s$	$T_c = 0.179\ s$	$T_c = 0.125\ s$	$T_c = 0.063\ s$	$T_c = 0.031\ s$
5.	8	0.001	$L/8$	0.1	$V_0 = 0.5\ m/s$	$\boxed{V_{0,cr} = 0.6\ m/s}$	$V_0 = 4\ m/s$	$V_0 = 10\ m/s$	$V_0 = 25\ m/s$
					$T_v = 0.623\ s$	$T_v = 0.63\ s$	$T_v = 0.7\ s$	$T_v = 0.783\ s$	$T_v = 1.518\ s$
					$T_c = 0.2\ s$	$T_c = 1.66\ s$	$T_c = 0.25\ s$	$T_c = 0.1\ s$	$T_c = 0.04\ s$
6.	8	0.001	$L/8$	0.5	$V_0 = 1.2\ m/s$	$\boxed{V_{0,cr} = 1.3\ m/s}$	$V_0 = 4\ m/s$	$V_0 = 10\ m/s$	$V_0 = 20\ m/s$
					$T_v = 0.61\ s$	$T_v = 0.613\ s$	$T_v = 0.66\ s$	$T_v = 0.763\ s$	$T_v = 1.325\ s$
					$T_c = 0.833\ s$	$T_c = 0.77\ s$	$T_c = 0.25\ s$	$T_c = 0.1\ s$	$T_c = 0.05\ s$
7.	8	0.001	$L/4$	0.5	$V_0 = 3\ m/s$	$V_0 = 4.4\ m/s$	$\boxed{V_{0,cr} = 4.5\ m/s}$	$V_0 = 10\ m/s$	$V_0 = 20\ m/s$
					$T_v = 0.66\ s$	$T_v = 0.675\ s$	$T_v = 0.69\ s$	$T_v = 0.768\ s$	$T_v = 1.253\ s$
					$T_c = 0.667\ s$	$T_c = 0.454\ s$	$T_c = 0.444\ s$	$T_c = 0.2\ s$	$T_c = 0.1\ s$
8.	8	0.001	$L/4$	1	$V_0 = 3\ m/s$	$V_0 = 4\ m/s$	$V_0 = 6.1\ m/s$	$\boxed{V_{0,cr} = 6.2\ m/s}$	$V_0 = 10\ m/s$
					$T_v = 0.643\ s$	$T_v = 0.655\ s$	$T_v = 0.668\ s$	$T_v = 0.703\ s$	$T_v = 0.758\ s$
					$T_c = 0.667\ s$	$T_c = 0.5\ s$	$T_c = 0.328\ s$	$T_c = 0.323\ s$	$T_c = 0.1\ s$

For the predetermined values of velocities, the values of periods T_c of arrival of clots in to the pipe channel (see Table 1) were calculated, which could be compared with the values of the time T_v between two neighbouring maximum values of the pipe middle point displacement along axis Oy.

Note that for problems 3 ($V_0 = 5\ m/s$, $V_0 = 5{,}1\ m/s$), 7 ($V_0 = 3\ m/s$) and 8 ($V_0 = 3\ m/s$) value T_v is equal to period T_c but for problems 3 ($V_0 = 10\ m/s$, $V_0 = 20\ m/s$), 4 ($V_0 = 10\ m/s$), 8 ($V_0 = 6{,}1\ m/s$) value T_v is approximately a multiple of T_c.

As the results of investigation of dynamics of pipes with an initial camber (see Table 2) show, interaction of forced and parametric vibrations has not led to the displacement of critical velocities values. Values T_v of the curved pipe have not practically been changed. The above- mentioned peculiarities for values T_v of straight pipe vibrations are also characteristic for the pipe with an initial camber. Figure 2 gives vibration graphs for the point $x = L/2$ of the pipe centre line along axis Oy for problem 8 (see Table 2). The associated states of a flow (the arrangement of clots and their velocities) for the instant of time, when a clot arriving at the channel at the velocity of V_0 reaches its full length a_0 and starts separating from its main flow at the point $x = 0$, are shown in Fig. 3.

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Table 2. Velocities values and periods of forced vibrations of a pipe with camber

№	L, m	h, m	a_0	k, s^{-1}	Values of dynamical parameters				
1.	5	0.003	$L/8$	0.1	$V_0 = 1.1 m/s$	$\boxed{V_{0,cr} = 1.2\ m/s}$	$V_0 = 20\ m/s$	$V_0 = 40\ m/s$	$V_0 = 87.5\ m/s$
					$T_v = 0.24\ s$	$T_v = 0.245\ s$	$T_v = 0.261\ s$	$T_v = 0.287\ s$	$T_v = 1.32\ s$
					$T_c = 0.568\ s$	$T_c = 0.52\ s$	$T_c = 0.031\ s$	$T_c = 0.016\ s$	$T_c = 0.007\ s$
2.	5	0.003	$L/8$	0.5	$V_0 = 1.8\ m/s$	$\boxed{V_{0,cr} = 1.9\ m/s}$	$V_0 = 20\ m/s$	$V_0 = 40\ m/s$	$V_0 = 80\ m/s$
					$T_v = 0.245\ s$	$T_v = 0.247\ s$	$T_v = 0.26\ s$	$T_v = 0.293\ s$	$T_v = 0.785\ s$
					$T_c = 0.347\ s$	$T_c = 0.329\ s$	$T_c = 0.031\ s$	$T_c = 0.016\ s$	$T_c = 0.008\ s$
3.	5	0.003	$L/4$	0.5	$V_0 = 5\ m/s$	$\boxed{V_{0,cr} = 5.1\ m/s}$	$V_0 = 10\ m/s$	$V_0 = 20\ m/s$	$V_0 = 40\ m/s$
					$T_v = 0.244\ s$	$T_v = 0.246\ s$	$T_v = 0.25\ s$	$T_v = 0.258\ s$	$T_v = 0.29\ s$
					$T_c = 0.25\ s$	$T_c = 0.245\ s$	$T_c = 0.125\ s$	$T_c = 0.063\ s$	$T_c = 0.031\ s$
4.	5	0.003	$L/4$	1	$V_0 = 6.9\ m/s$	$\boxed{V_{0,cr} = 7 m/s}$	$V_0 = 10\ m/s$	$V_0 = 20\ m/s$	$V_0 = 40\ m/s$
					$T_v = 0.246\ s$	$T_v = 0.247\ s$	$T_v = 0.25\ s$	$T_v = 0.258\ s$	$T_v = 0.297\ s$
					$T_c = 0.18\ s$	$T_c = 0.179\ s$	$T_c = 0.125\ s$	$T_c = 0.063\ s$	$T_c = 0.031\ s$
5.	8	0.001	$L/8$	0.1	$V_0 = 0.5\ m/s$	$\boxed{V_{0,cr} = 0.6\ m/s}$	$V_0 = 4\ m/s$	$V_0 = 10\ m/s$	$V_0 = 25\ m/s$
					$T_v = 0.624\ s$	$T_v = 0.63\ s$	$T_v = 0.73\ s$	$T_v = 0.783\ s$	$T_v = 1.532\ s$
					$T_c = 0.2\ s$	$T_c = 1.66\ s$	$T_c = 0.25\ s$	$T_c = 0.1\ s$	$T_c = 0.04\ s$
6.	8	0.001	$L/8$	0.5	$V_0 = 1.2\ m/s$	$\boxed{V_{0,cr} = 1.3\ m/s}$	$V_0 = 4\ m/s$	$V_0 = 10\ m/s$	$V_0 = 20 m/s$
					$T_v = 0.615\ s$	$T_v = 0.614\ s$	$T_v = 0.66\ s$	$T_v = 0.768\ s$	$T_v = 1.326\ s$
					$T_c = 0.833\ s$	$T_c = 0.77\ s$	$T_c = 0.25\ s$	$T_c = 0.1\ s$	$T_c = 0.05\ s$
7.	8	0.001	$L/4$	0.5	$V_0 = 3\ m/s$	$V_0 = 4.4\ m/s$	$\boxed{V_{0,cr} = 4.5\ m/s}$	$V_0 = 10\ m/s$	$V_0 = 20 m/s$
					$T_v = 0.665\ s$	$T_v = 0.68\ s$	$T_v = 0.683\ s$	$T_v = 0.765\ s$	$T_v = 1.268\ s$
					$T_c = 0.667\ s$	$T_c = 0.454\ s$	$T_c = 0.444\ s$	$T_c = 0.2\ s$	$T_c = 0.1\ s$
8.	8	0.001	$L/4$	1	$V_0 = 3\ m/s$	$V_0 = 4\ m/s$	$V_0 = 6.1\ m/s$	$\boxed{V_{0,cr} = 6.2\ m/s}$	$V_0 = 10\ m/s$
					$T_v = 0.63\ s$	$T_v = 0.63\ s$	$T_v = 0.665\ s$	$T_v = 0.69\ s$	$T_v = 0.75\ s$
					$T_c = 0.667\ s$	$T_c = 0.5\ s$	$T_c = 0.328\ s$	$T_c = 0.323\ s$	$T_c = 0.1\ s$

One can notice that at $V_0 = 3{,}3\ m/s$ (see Figure 2) the vibrations are decayed with additional beats. With further increasing the velocity $V_0 = 6\ m/s$ the pipe vibrations are stable in nature and take the mode of beats. In the critical case $V_{0,\,cr} = 6{,}2\ m/s$ the pipe loses its stability in the mode of flutter, but not according to the linear law and with additional vibrations. In the postcritical state ($V_0 > V_{0,cr}$) the elastic system remains unstable and in doing so it begins to vibrate with less frequency. Figure 4 illustrates the modes of the cambered tube plane vibrations which take place for problem 8 during time T_v. The pipe was found to vibrate according to the combination of the first and the second modes of natural vibrations of a pipe without a fluid flow.

In conclusion one may note a peculiarity characteristic of the dynamical process under discussion. The case is that when vibrational motions of a pipe are excited by inner mobile clots, a joint action of two affecting mechanisms is shown up, each of them having its own nature. First, we observe here only a dynamical action of inertial centrifugal forces on an elastic pipe, which in this case play the role of active forces. The action of these forces determines the presence of right member in the constitutive equations and their nonhomogeneity. Second, the effects characteristic of a parametric mechanism of vibration excitations are shown up here.

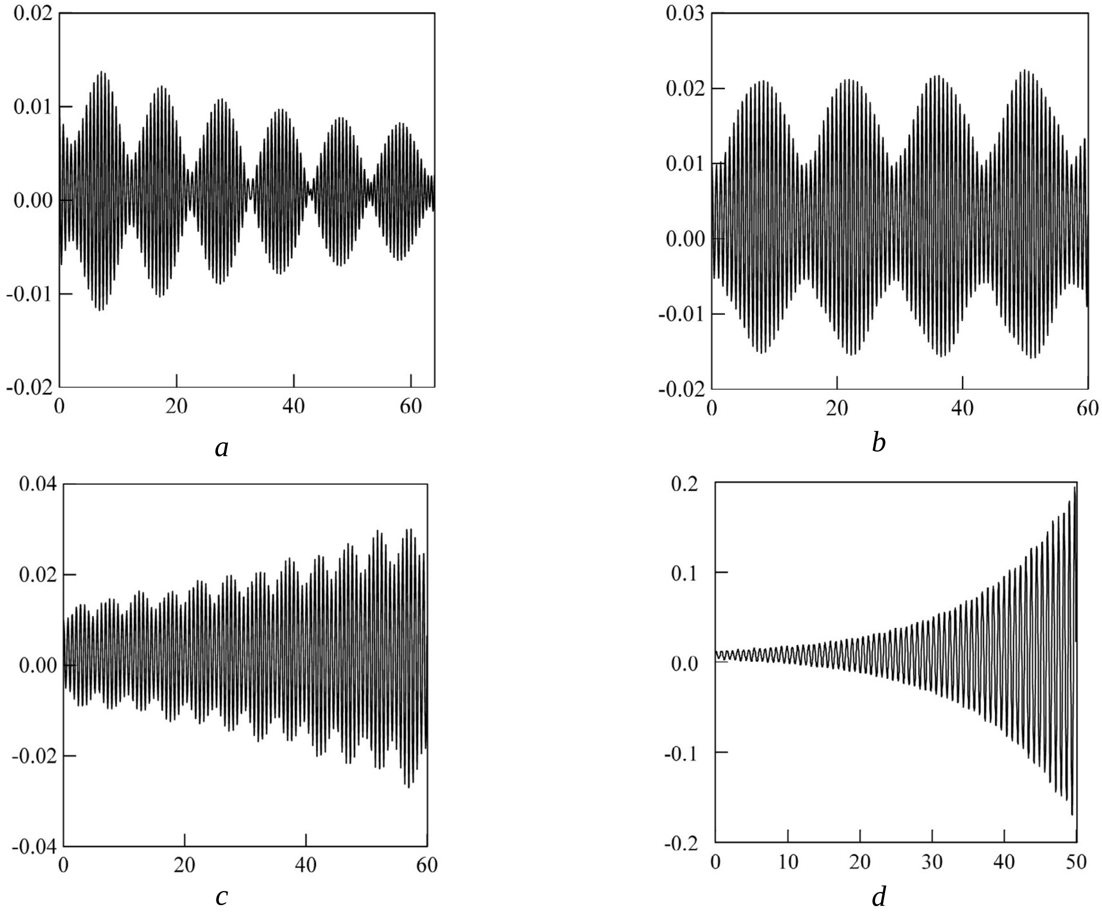
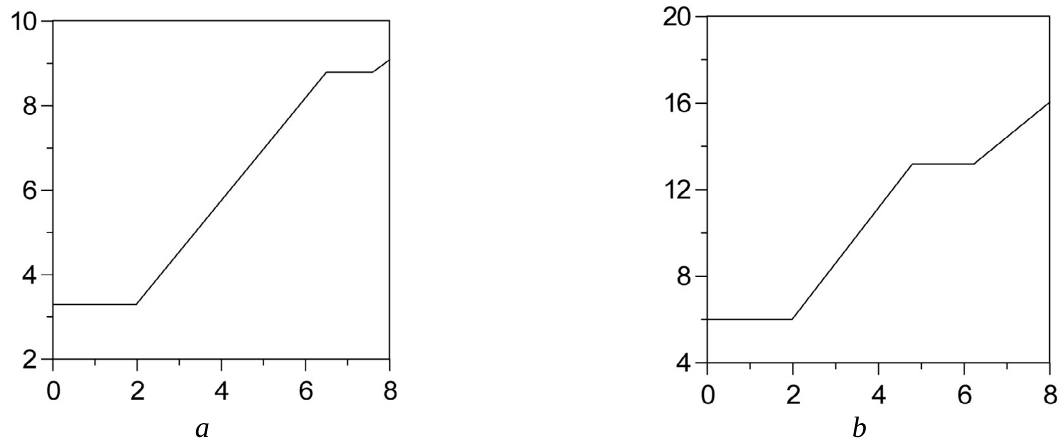


Fig. 2. The forms of vibrations in time of a central cross-section of a pipe with mobile boiling away clots ($L = 8\ m,\ a = L/4$). (*a*) $V_0 = 3{,}3\ m/s$; (*b*) $V_0 = 6\ m/s$; (*c*) $V_0 = 6{,}2\ m/s$; (*d*) $V_0 = 10\ m/s$



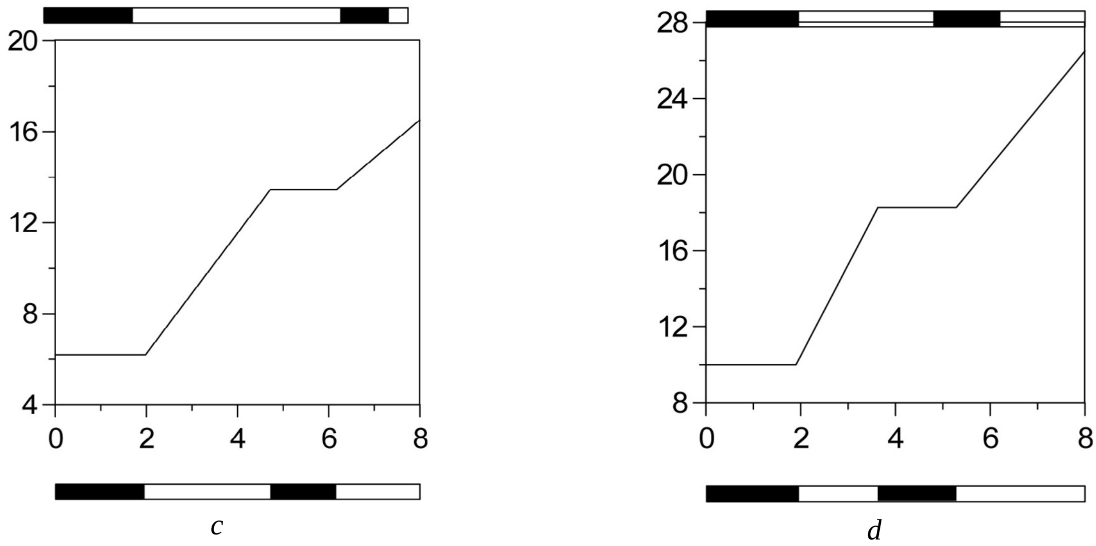


Fig. 3. The diagrams of distributing fluid clot velocities
(*a*) $V_0 = 3{,}3\ m/s$; (*b*) $V_0 = 6\ m/s$; (*c*) $V_0 = 6{,}2\ m/s$; (*d*) $V_0 = 10\ m/s$

5. Conclusions

The purpose of this paper is to carry out the numerical modelling of self-excited vibrations of tubular rods containing inner flows of nonhomogeneous boiling fluid. Straight rods and the rods having initial camber have been considered. The model of dynamics of flow with allowance made for a discontinuous character of the parameters of its density, as well as the fluid clots flow mode in the process of their heating and evaporations is suggested. The action of inertial forces of positional and gyroscopical types is taken into account. The analysis of the results obtained makes it possible to make the following conclusions:

1. Unstable equilibrium states accompanied by self-excitation of vibrations and flutter type loss of stability can arise in a pipe from the action of inertial forces of a nonhomogeneous non-stationary inner flows on the pipe walls. In a number of cases the divergent conditions of losing the straight-line stability were realized in supercritical states.

2. The mechanism of losing straight shape of a pipe results from the action of centrifugal and Coriolis' inner flow inertial forces which can be classified as positional and gyroscopical ones.

3. The nonhomogeneity of an inner fluid flow manifests itself both in the nonhomogeneity of centrifugal inertial forces acting on a pipe in the transverse direction and in the change with time of the system general mass geometry. In this connection purely dynamical and parametrical excitations of vibrations take place.

4. Gyroscopic inertial forces caused by the interaction between slewing movement of pipe elements and linear flows of fluid masses have a marked influence on the dynamic process character. They lead to the system loss of a general motion phase and to essential complication of the modes of the pipe transverse vibrations.

5. The calculations testify that in the general case the transverse motions of a pipe constitute non-stationary vibrations in which one can distinguish a conventional period T_v. As a rule this period doesn't appear to be comparable to the period of arriving fluid clots in the pipe channel although in some cases these values were almost equal or multiple.

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Є. Ю. Толбатов. Математичне моделювання коливань, що самозбуджуються та містять рухомі киплячі згустки рідини

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Представлена задача про чисельне моделювання динамічної поведінки прямолінійною труби, що містить внутрішні неоднорідні потоки киплячій рідини. Оскільки аналітичне рішення цієї проблеми пов'язане зі зміною геометрії мас системи і розривом коефіцієнтів рівнянь, представляється складним, був розроблений метод комп'ютерного моделювання динаміки труби, який ґрунтується на одночасному використанні методів чисельного інтегрування за часом і методу початкових параметрів спільно з процедурою ортогоналізації по просторової змінної .Були виявлені різні види коливань труби, а також можливість встановлення стійких і нестійких режимів руху залежно від характеру неоднорідності та швидкості руху рідинних згустків.

Ключові слова: внутрішні потоки; динаміка; згустки рідини; коливання; неоднорідна рідина; періоди; теплообмінник; труба з початковою опуклістю; флатер, швидкість.

Е. Ю. Толбатов. Математическое моделирование самовозбуждающихся колебаний труб, содержащих подвижные сгустки закипающей жидкости

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Представлена задача о численном моделировании динамического поведения прямолинейной трубы, содержащей внутренние неоднородные потоки кипящей жидкости. Так как аналитическое решение этой проблемы, связанное с изменением геометрии масс системы и разрывом коэффициентов уравнений, представляется затруднительным, был разработан метод компьютерного моделирования динамики трубы, который основывается на одновременном использовании методов численного интегрирования по времени и метода начальных параметров совместно с процедурой ортогонализации по пространственной переменной. Были обнаружены различные виды колебаний трубы, а также возможность установления устойчивых и неустойчивых режимов движения в зависимости от характера неоднородности и скорости движения жидкостных сгустков.

Ключевые слова: внутренние потоки; динамика; колебания; неоднородная жидкость; периоды; сгустки жидкости; скорость; теплообменник; труба с начальной погибью; флаттер.

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