## MODERN AVIATION AND SPACE TECHNOLOGY

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# THEORETICAL PRINCIPLES OF CONGESTION CONDITIONS RESEARCH

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**Abstract.** *Recent years the number of vehicles on the roads increases continuously. This paper considers favorable factors of congestion. An example of a mathematical model of a situation of congestion on a regulated crossroad is provided. The dynamics of the queue congestion at crossroads, as well as its length is also showed.*

**Keywords:** allowing cycle; congestion queue; crossroad; traffic; vehicle.

## 1. Introduction

The up-to-date rapid growth of traffic under the current circumstances of a significant lagging in the road network development, usually leads to the imbalance, hence, the excess of demand for road services over the available supply of these services. Within a minor excess, delays increase, consequently reducing the speed of communication, increasing the emission of harmful compounds in the exhaust and badly affecting the technical and economic performance of the road transport rolling stock. A significant excess of traffic over crossroads', roadway potentials or the road network potential in general, leads to congestion on the go.

Formation of the congestion conditions is caused by the combination of following main factors [1]:

– organizational and governing – the scheme of traffic organization and management designed without considering of the traffic formation and distribution peculiarities on this part of the road network;

– deficient – lack of a significant size strip of the roadway;

– unpredictable – formation of an accident with serious consequences, weather conditions, natural disasters, major repairs and construction works at the area of the road network without the use of appropriate measures and traffic control.

Congestion is a state of the traffic flow in the area of road network, in which the speed of the vehicle is close to zero and the density of traffic flow reaches a maximum value, and the time travel along the congestion area of the road increases drastically. This chapter outlines the conditions of formation and the basic characteristics of traffic jams by the following key aspects:

– congestion at controlled intersections – places of alternate transmission of traffic flows and maneuvering vehicles concentration;

– congestion at driving- the transition from free traffic flow to congestion;

– congestion in settlements – as a result of insufficient supply of fleet vehicle with lane roadway.

## 2. Analysis of the research and publications

Congestion problem involved many scientists. Among them are the Kai Nagel, Carlos F. Daganzo, Lighthill, Whitham and others. The old theories of traffic flow considering the free flow as normal state. Now, with the increasing number of cars on the road, these theories need to be revised. One of the leading researchers of traffic flows in our time, Carlos Daganzo offers for mathematical models use two parameters: the length of the queue and the time of vehicle travel between the sensors [2].

## 3. The purpose of the paper

This article discusses the possibility of investigating the traffic congestion by examining the queues.

## 4. Model architecture

Crossroad is a section of the road network, where vehicles maneuvers necessary for changes in the direction of their movement are concentrated and alternate pass of traffic and pedestrian flows is carried out.

Mathematical Model of congestion conditions at the crossroad.

There are three situations in which a traffic jam appears on a regulated intersection:

1. When the cycle of traffic light control, or rather their cycles allowing enough bandwidth for traffic in the *j*-direction of *k*-lanes carriageway [3]:

$$C < \frac{2t_r + T_0}{1-q-r},\ t_{gr} < \frac{q}{q+r}(C-T_0),\ t_{gr} < t_{0jk}, \tag{1}$$

where $C$ – is the cycle of traffic light control;

$t_r$ – driver's reaction time, $t_r \approx 0{,}1\ldots0{,}6$ sec;

$T_0$ – the total length of the intermediate cycles, which should be sufficient for the release of crossroads from *j*-directions, crossing the stop line at the moment of changes allowing the cycle to intermediate:

$r$ and $q$ – dynamic characteristics of traffic flow (harmonic magnitude) respectively to the even and odd directions of the movement;

$t_{gr}$ – the allowing cycle duration, sec/cycle;

$t_{0jk}$ – temporal queue length at the intersection of the stop line on the *j*-direction of the *k*-lane sec/cycle.

2. When the queue size at the stop line and its temporal length is over the one that is designed for traffic light cycle regulation:

$$n_{0j} > \frac{[t_r + T_0 + \beta(C-T_0)]\sum_{i=1}^{4}\lambda_{jk}^i}{1-q},$$

$$t_{0jk} > \frac{[2t_r + T_0 + \beta(C-T_0)]q_0}{1-q_0}, \tag{2}$$

where $n_{0j}$ – size of the queue formed at the stop line on the *j*-direction, veh/cycle;

$\beta = 1 - \alpha$ – a part of the forbidding tact;

$\alpha = \frac{q}{q+r}$ – a part of the allowing tact;

$\lambda_{jk}^i$ – the intensity of the arrival of vehicles to the brake lines of *j*-direction, on *k*-lane, at *i*-intersection, veh/sec;

???– density of the flow (number of vehicles occupying a unit of the road), veh/m.

3. When the maximum capacity is less than the actual intensity of the arrival of vehicles to a stop intersection lines: $\lambda_{\max jk} < \lambda_{jk}$

$$\lambda_{\max jk} < (t_{зел} - t_r)3600/C\tau_{jk}, \tag{3}$$

where $\lambda_{\max jk}$ – the maximum capacity of the crossroad, veh/hour;

$\tau_{jk}$ – the time interval between the front bumpers of vehicles on *j*-direction, of *k*-lane at the moment of crossing the stop line, sec/veh.

If you calculate the length of cycle traffic light regulation by the size of the traffic flow in average values $C\bar{\lambda}_j = n_{0jk}$, and in some periods this balance, i.e. equality, is broken upwards, then results will remain the same, hence the intensity if coming to a stop line is bigger, that the one adopted in the calculation of cycle time, allowing tact and length if the queue, herewith:

$$t_{gr}/\tau_{jk} < C\lambda_{jk} > n_{0jk},\quad n_{0jk} < C\lambda_{jk} \tag{4}$$

where $n_{0jk}$ – size of the queue formed at the stop line on the *j*-direction, of the *k*-lane, veh/cycle.

Accordingly, inequalities (1–4) describes the congestion conditions occurrences at the crossroads. In case when the left parts of the inequalities mentioned above are amount to the right ones, then congestion conditions will be omitted and the traffic light will function normally. If the intensity of arrival at the stop line is less than the average intensity value, embedded in $C$, $t_{gr}$, $n_{0jk}$, undue delays will occur.

## 5. Determination of the congestion time interval in the allowing cycle

The capacity of a single lane of a regulated intersections, is marked with the maximum traffic per hour of traffic light activity for this lane of the road, which is appropriate for the road conditions traffic regimes and regulations determined by the traffic light regulation and can be defined by the specification of the following equation [3]:

$$\lambda_{\max jk} = (t_{зел} - t_r)3600/C\tau_{jk},\ \text{veh/hour}. \tag{4}$$

It should be noted that the reaction of the highly-qualified driver $t_r$ amounts to zero.

When congestion in traffic flow, traffic intensity is greater than the rated capacity for the intersection for $\Delta\lambda_{jk}$. Hence, the right-hand part of equation (5), which reflects the nominal capacity will declined by $\Delta\lambda_{jk}$ than the actual traffic. Hereof the principal value is the cycle of traffic light, which depends linearly on the intensity and maximum throughput.

To avoid congestion conditions, program and methodological aspects of the issue should preserve the balance (the equality), though it is rather a mechanical, than a logical operation.

$$\lambda_{\max jk} + \Delta\lambda_{jk} = \frac{(t_{gr} + \Delta t - t_r)3600}{(C+\Delta C)\tau_{jk}}, \tag{5}$$

where $\Delta\lambda_{jk}$, $\Delta t$, $\Delta C$ are increment of the respective parameters in the traffic congestion.

To obviate $\Delta\lambda_{jk}$ we need to execute a recalculation of the traffic light cycle considering the increased traffic intensity on the $k$-lane in $\Delta\lambda_{jk}$, i.e. $(\lambda_{\max jk} + \Delta\lambda_{jk})$.

Otherwise, if you submit equation (6) as follows $\lambda_{\max jk}C\tau_{jk} = (t_{gr}-t_r)3600$, when part of the vehicles have not managed to cross the stop line within allowing cycle duration, we should add the additional time, calling it congestion lapse in the allowing cycle, which corresponds to the intensity of the arrival of vehicles to brake-line:

$$(t_{gr}-t_r+t_{cgs})3600 = (\lambda_{\max jk}+\Delta\lambda_{jk})C\tau_{jk}, \qquad (6)$$

where $t_{cgs}$ – congestion lapse in the allowing cycle of the traffic light, sec.

Value (7) is the total time length of the queue formed at the generating intersection stop line whilst crossing the stop-line (right-hand part of equality), and the total allowing time interval (the total duration of the allowing cycles), that are required to pass the queue occurred within an hour of the traffic lights regulation (left-hand part). The congestion time interval is calculated [11, 45, 52]:

$$t_{gr} = (\lambda_{jk}+\Delta\lambda_{jk})C\tau_{jk} - (t_{gr} - t_r), \text{ sec/cycle}; \qquad (7)$$

$$t_{cgs} = \frac{3600}{C}(\lambda_{jk} + \Delta\lambda)C\tau_{jk} - (t_{gr} - t_r)\frac{3600}{C} = 3600(\lambda_{jkh} + \Delta\lambda_{jk})\tau_{jk} - \frac{3600(t_{gr} - t_r)}{C}, \text{ sec/hour}. \qquad (8)$$

Equations (8) and (9) represent the congestion time interval in one cycle and per hour activity of traffic lights. The proportion of the congestion time interval per traffic lights hour can be defined as follows:

$$t_{cgs}/3600 = (\lambda_{jk}+\Delta\lambda_{jk})\tau_{jk} - (t_{gr} - t_r)/C, \text{ sec/hour}.$$

## 6. Determination of the number of vehicles that are not passed within the time of the allowing cycle

To determine of the number of vehicles that are not passed within the time of the allowing cycle we need to divide the time interval should be devided between the front bumpers:

$$n_{unp} = t_{cgs}/\tau_{jk}, \qquad (9)$$

where $n_{unp}$ – the number of vehicles that are not passed within the time of the allowing cycle, veh/cycle.

We may also determine the number of vehicles that are not passed within the time of the allowing cycle otherwise. Hence, the number of vehicles that arrived at the stop-line during the cycle of traffic light regulation is $C\lambda_{jk}$, veh/sec, and the number of passed through the intersection within the allowing cycle is $(t_{gr}-t_r)/\tau_{jk}$, veh/cycle. Thus, the number of vehicles that are not passed on $k$-lane in $j$-direction at the adjustable intersection can be determined by the following expression [3]:

$$n_{unp} = C\lambda_{jk}-(t_{gr}-t_r)/\tau_{jk}. \qquad (10)$$

In $j$-direction, that involves $K$ traffic lanes:

$$n_{unp} = \sum_{k=1}^{K}\left[C\lambda_{jk} - (t_{gr} - t_r)/\tau_{jk}\right]. \qquad (11)$$

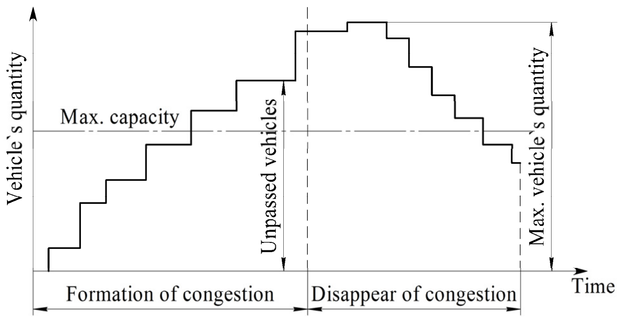


**Fig. 1.** Dynamics of formation and disappearance of queues of vehicles forming a queue of cars that has not passed through during one cycle of the traffic light

Fig. 1 demonstrates: the more $n_{unp}$ is, the more intensity of the congestion conditions formation will be and conversely, when the number of unpassed vehicles is less than the maximum one for this intersection the congestion decreases.

## 7. The analysis of the congestion queue at the intersection

Considering the formation of congestion queue in the $j$-direction of the intersection, if in a certain time interval (for instance the morning rush hour) there is an increase of traffic that exceeds the maximum capacity of allowing cycle, hence the inequalities are executed (1–4). During the first cycle of traffic light regulations when the traffic intensity exceeds the maximum capacity, a queue of unpassed cars is formed $n_{unp1}$, within the second overloaded cycle a certain number of other vehicles is added – the queue $n_{unp2}$ etc. (fig. 2).

The queue increases accordingly to the sum of the increasing progression, the number of unpassed automobiles for each of the following cycles depends on the excess of the factual intensity of the arriving vehicles above the maximum capacity [3]. The congestion queue increases all the time long, when the intensity of vehicles arriving to the stop line in $j$-direction of $h$-intersection the maximum capacity. Thus the span during which the traffic intensity exceeds the maximum capacity of the intersection is the time interval of growth or formation of the congestion queue. All this interval long $n_{cgsi\text{-}1} < n_{cgsi}$.

In case in a certain cycle of the traffic lights regulation the traffic intensity is equal to the maximum capacity, then during the following period the congestion queue will not increase and even start to decrease: $n_{cgsi\text{-}1} = n_{cgsi}$.

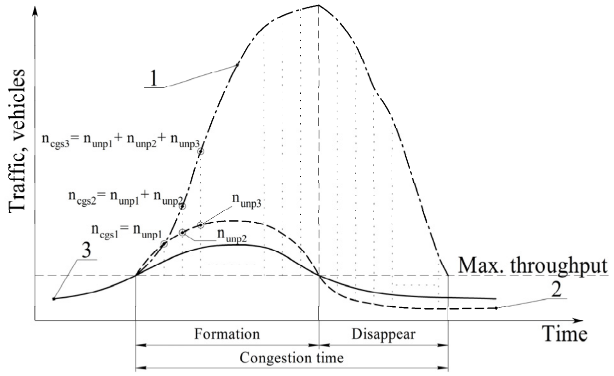


**Fig. 2.** Formation of the congestion queue $n_{cgs}$ on the intersection of the roadways:

1 – congestion queue; 2 – vehicles arrival rate; 3 – vehicles quantity per cycle

Reduction of the congestion queue is started at the moment when the intensity of arriving of vehicles to the following intersection gets less than its maximum capacity, hereof the number of passed vehicles during the time of allowing cycle will be more than the number of vehicles that are added to the end of the queue.

Therefore, the time interval, within which the intensity gets less than the capacity form the moment $n_{cgsi\text{-}1} > n_{cgsi}$ and up to $n_{cgs} = 0$, is the tome interval of the congestion queue reduction. In total both of the intervals represents the time of congestion at the intersection:

$$T_{cgs} = t_{frm} + t_{dis}, \tag{12}$$

where $T_{cgs}$ – the time of congestion at the intersection;

$t_{frm}$ – the formation time of congestion;

$t_{dis}$ – the disappearance time of congestion.

In general, the formation of a congestion queue can be represented by the equation:

$$n_{cgsi+1} = n_{cgsi} + n_{arr} - n_{pass}, \tag{13}$$

where $n_{cgsi+1}$ – congestion queue size in each subsequent cycle of the traffic light regulation, veh;

$n_{cgsi}$ – size of the already formed queue of the unpassed vehicles, veh;

$n_{arr}$ – the number of vehicles that arrived during this cycle, veh;

$n_{pass}$ – the number of vehicles that was passed during the allowing interval of the cycle, veh.

The decisive role of the congestion queue formation is determined by the difference of the number of vehicles that arrived and the passed vehicles: in case during the cycle of traffic lights regulation the number of vehicles is more, than the one passing during the allowing cycle, then the congestion queue increases, and conversely, the excess of passing vehicles above the arriving ones, means reduction of the congestion queue. In general the dynamics of the congestion queue is calculated by the system (see also fig. 3) [3]:

$$\begin{cases} n_{cgsi+1} > n_{cgsi}, \text{ if } n_{arr} > n_{pass}; \\ n_{cgsi+1} = n_{cgsi}, \text{ if } n_{arr} = n_{pass}; \\ n_{cgsi+1} < n_{cgsi}, \text{ if } n_{arr} < n_{pass}. \end{cases} \tag{14}$$

Equation (14) can also be represented as:

$$n_{cgsi+1} = n_{cgsi} + C\lambda_{jk} - (t_{gr} - t_r)/\tau_{jk}, \text{ veh.} \tag{15}$$

Then the system (15), which determines the dynamics of the formation the congestion queue is as follows:

$$\begin{cases} n_{cgsi+1} > n_{cgsi}, \text{ if } C\lambda_{jk} > (t_{gr} - t_r)/\tau_{jk}; \\ n_{cgsi+1} = n_{cgsi}, \text{ if } C\lambda_{jk} = (t_{gr} - t_r)/\tau_{jk}; \\ n_{cgsi+1} < n_{cgsi}, \text{ if } C\lambda_{jk} < (t_{gr} - t_r)/\tau_{jk}. \end{cases} \tag{16}$$

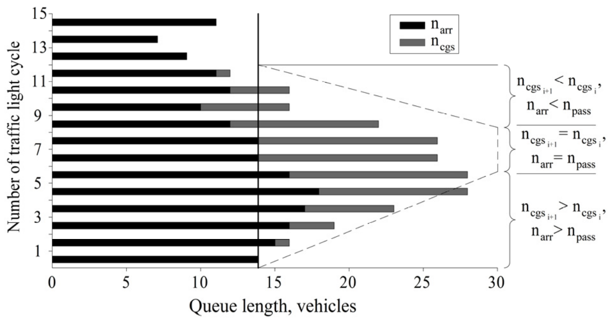


**Fig. 3**. Dynamics of the congestion queue formation at the intersection of the roadways

## 8. Determination of the congestion queue length

In case the intensity of the vehicles arriving to the stop line within an hour of the traffic light activity is equal, than during the number of 3600/C cycles the length of the congestion queue is determined by the number of all the unpassed vehicles during this period of time [3]:

$$n_{cgs} = 3600(C\lambda_{jkh} - (t_{gr} - t_r)/\tau_{jkh})/C, \text{ veh/hour}, \tag{17}$$

or $n_{cgs} = 3600\lambda_{jkh} - 3600(t_{gr} - t_r)/C\tau_{jkh}$, veh/hour, (18)

where $\lambda_{jkh}$ – arriving intensity of vehicles to the stop-line of the $j$-direction in the $k$-lane at $h$-intersection, veh/sec;

$\tau_{jkh}$ – the time interval between the front bumpers of vehicles of the *j*-direction in the *k*-lane at *h*-intersection, sec/veh.

Formulas (18) and (19) determines the length of the congestion queue during 3600/*C* cycles, that is the intensity of the congestion formation in the traffic during an hour of traffic lights activity.

In case the intensity of vehicles arriving to the stop-line is changes during a certain number of cycles according to the definite law or discretely than the length of the congestion queue is determined as a total of all the unpassed vehicles during $N$ cycles of the traffic light regulation. (check fig. 4, *b*).

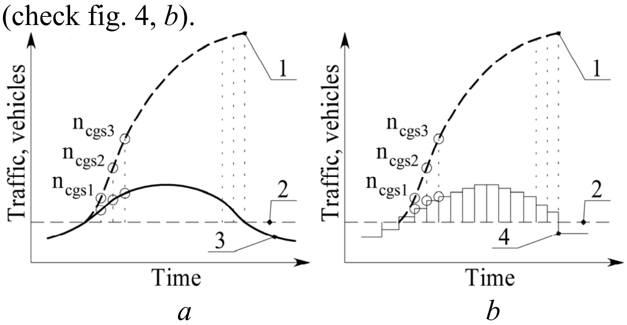


**Fig. 4.** Determination of the congestion queue length: *a* – in case the traffic intensity is changed by the function $\lambda(t)$; *b* – in case of a discrete change $\lambda$:
1 – congestion queue;
2 – max. roadway capacity;
3 – traffic intensity is defined by the function;
4 – traffic intensity is defined discretely.

In case of a table (discrete) determination:

$$n_{cgs} = \sum_{n=1}^{N}\left(C\lambda_{jkhn} - (t_{gr} - t_r)/\tau_{jkh}\right) = C\sum_{n=1}^{N}\lambda_{jkhn} - N(t_{gr} - t_r)/\tau_{jkh}, \text{ veh,} \quad (19)$$

where $\lambda_{jkh}$ – arriving intensity of vehicles to the stop-line of the *j*-direction in the *k*-lane at *h*-intersection, veh/sec; *N* – the number of cycles chosen.

In case the arriving intensity of vehicles to the stop-line can be determined as a time function, then the length of the congestion queue is determined as difference of all the vehicles that arrived to the stop line during the time chosen by the researcher and the capacity of the intersection for the same period (check fig. 4, *a*):

$$n_{cgs} = \int_{t=0}^{T}\left[\lambda_{jkh}(t) - \frac{t_{gr} - t_r}{C\tau_{jkh}}\right]dt = \int_{t=0}^{T}\lambda_{jkh}(t)dt - \frac{T(t_{gr} - t_r)}{C\tau_{jkh}}, \text{ veh,} \quad (20)$$

where $t = 0$ corresponds the beginning of the congestion conditions formation $\lambda_{jkh}(t) > \lambda_{\max jkh}$;

*T* – time interval chosen by the researcher or the time of congestion queue formation in general;

$\lambda_{jkh}(t)$ – arriving intensity of vehicles to the stop-line of the *j*-direction in the *k*-lane at *h*-intersection as a time function *t*, veh/sec.

As a commentary to Figure 4 we should note that the functional intensity determination gives a more accurate result, but at the same time choosing a pace from tables is simpler and more reliable in terms of a particular experiment.

## 9. Conclusions

The paper shows that the behavior of congestion can be really learn via observation of queue parameters. Should be pointed out that properly configured length of traffic light allowing cycle can prevent the formation of traffic jams. These aspects can be useful for a more detailed research behaviors congestion areas and roads.

## References

[1] *Klinkovshteyn G. I.* Traffic organization / G. I. Klinkovshteyn, M. B.Afanasev. – M. : Transport, 1997–230 p. (in Russian).

[2] *Daganzo C. F.* Remarks on Traffic Flow Modeling and its Applications // Traffic and Mobility, Proc. Traffic and Mobility, Simulation, Economics and Environment Conference (Brilon, Huber, Schreckenberg and Wallentowitz, eds.), pp. 105–115, Aachen, Germany, Springer-Verlag, New York, N.Y., 1999.

[3] *Zhivoglyadov V. G.* Theory crossing traffic and pedestrian flows: monograph / V. G. Zhivoglyadov. – Rostov n/D: Rostov State University, 2003. – 411 p. (in Russian).

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**В. М. Першаков[1], Р. В. Кротов[2]. Теоретичні принципи дослідження виникнення заторових станів**
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В останні роки кількість транспортних засобів на автомобільних дорогах постійно зростає. У статті розглянуто сприятливі фактори заторів. Наведено приклад математичної моделі заторової ситуації на регульованому перехресті. Показано динаміку заторової черги на перехрестях, а також її довжину.
**Ключові слова:** дозволяючий сигнал; заторова черга; перехрестя; транспортний засіб; транспортний потік.

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В последние годы количество транспортных средств на автомобильных дорогах постоянно растет. В статье рассмотрены благоприятные факторы заторов. Представлен пример математической модели заторовой очереди на регулируемом перекрестке. Показана динамика заторовой очереди на перекрестках, а также ее длина.
**Ключевые слова:** заторовая очередь; перекресток; разрешающий сигнал; транспортный поток; транспортное средство.

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