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MATHEMATICAL MODELING OF DYNAMICS OF CURVED PIPES CONVEYING INTERNAL MASSES OF NONHOMOGENEOUS LIQUID

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Abstract. *The article considers computer simulation of dynamics of elastic tube serpentines with internal flows of boiling liquids. We propose the motion model of non-homogeneous boiling liquid clots. The technique of numerical solution of the constructed equations is based on the transfer matrix method.*

Keywords: cylindrical spirals; dynamics; internal flows; non-homogeneous liquid; numerical method; velocity; vibration.

1. Introduction

Tube rods in the shape of screw cylindrical spirals (Fig.1) interacting with internal movable (liquid) medium have gained wide application in technology as manifolds of heat exchangers in nuclear and heat power stations, in hydraulic systems of air- and spacecrafts, in pump units, etc.

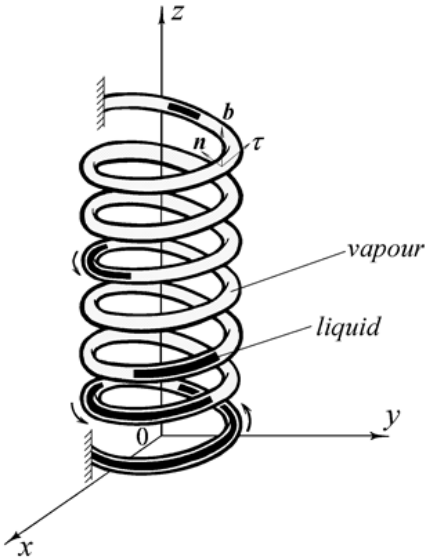

Fig. 1. Design scheme of a tube spiral with moving evaporating clots of liquid

The spiral shape of the tubes allows both the enlargement of the heat acceptance surface and the intensification of the heat acceptance, to compensate to essential temperature deformations of the structure. Liquid inside a tube, on being heated, begins to boil and transforms into water-vapour mixture. In response to the interaction between the internal flow of boiling liquid and the curvilinear tube, complicated static and dynamic effects are generated, which appear under the influence of forces acting on the tube from the flow side and which are accompanied by exchange of the potential and kinetic energies, as well as by static or dynamic loss of stability.

The forces initiating the effects incorporate the tangential forces of viscous friction dependent on the liquid viscosity and its velocity, as well as the centrifugal inertia forces normal to the rod axial line. The intensity of the latter is proportional to the moving liquid element mass, square of its velocity and curvature of the tube segment. Moreover, the Coriolis inertia forces are generated as a consequence of interaction of rotational and linear motions. On exposure to these forces, the tube structures begin to be involved into dynamic processes, analogous to the phenomena proceeding in elongated structures subject to action of moving loads and masses. The peculiarities of the dynamic behavior of this type of structure are connected with the effects, that in these cases the elements of moving masses take part in several types of motion simultaneously and are exposed to the action of inertia forces which depend upon the element position, and gyroscopic inertia forces conditioned by interplay of rotation and linear components of motion. As this takes place, the modes of the elastic system vibrations become more complicated, inasmuch as the phases of vibrations of its elements diversify, modes of its periodical motions cease to be steady, the node points begin to move and the vibration mode assumes the shape of a running wave, following motion of movable masses. Note also, that permanent varying of the considered elastic system mass geometry occurs at the flowing of the boiling fluid inside its channel, which is accompanied by change of its frequency spectrum. For this reason, in this case two sources of vibration generation come into being. The first one is connected with parametric generation, provoked by periodic variation of the system parameters (its mass geometry). The second source

is excitation of purely forced vibrations, induced by action of the inertia forces of movable fluid masses, which play the role of active forces in this case. By virtue of the fact that, owing to the absence of the spectrum of natural frequencies, the elastic system loses the mode of natural vibrations and the possibility to study it by the methods of spectral analysis is excluded. The most suitable methods for its investigation turn out to be the numerical methods of its immediate computer simulation.

The questions of analysis of dynamic behavior of rectilinear tubes with internal continuous flows were studied in [1-4]. Influence of elastic foundation on the tube flutter was investigated in [5]. Below the analysis of tube serpentine vibrations excited by nonstationary discontinuous internal flows, simulating boiling fluid is performed.

2. Differential equations of the spiral tube motion

To describe the dynamics of the tubular serpentine, it is convenient to use jointly internal and external geometries, applying the first to individualize the points of the curvilinear tubular rod and moving liquid clots, and the second to describe its geometry in the deformed state.

The internal geometry of the rod is specified by the coordinate, measured as the length of the axial line from the initial to the current point, and a moving right-handed coordinate system (u,v,w), the orientation of which at every point of the tube axial line is rigidly connected with the examined cross-section. The origin of this system lies at the center of gravity of the cross-section area, the u and v axes are directed along the principal central axes of inertia of the cross-section area, and the w axis is directed along the tangent to the elastic line. In this case the coordinate s is a concomitant one. The external geometry of the rod determinates the location of each of its points and the entire elastic line in the fixed inertial coordinate system $Oxyz$.

The Frenet natural trihedron of the elastic line of the rod with unit vectors of the principal normal $\mathbf{n}$, binormal $\mathbf{b}$ and tangent $\boldsymbol{\tau}$ is introduced.

The equations of bending an elastic tubular rod with distributed forces f and moments m are written in the form of the system of equilibrium equations [6].

$$\begin{aligned} &\frac{\tilde{d}F}{ds}+\boldsymbol{\omega}_{\chi}\times\mathbf{F}+f=0; \\ &\qquad\qquad\qquad\qquad\qquad\qquad ; \\ &\frac{\tilde{d}\mathbf{M}}{ds}+\boldsymbol{\omega}_{\chi}\times\mathbf{M}+\boldsymbol{\tau}\times\mathbf{F}+m=0 \end{aligned} \tag{1}$$

equations of elasticity

$$\begin{aligned} &\mathbf{M_u}=A(p-p_0),\ \mathbf{M_v}=B(q-q_0), \\ &\mathbf{M_w}=C(r-r_0), \\ &A=EI_u,\ B=EI_v,\ C=GI_w, \end{aligned} \tag{2}$$

and equations of kinematics

$$\begin{aligned} &\frac{d\boldsymbol{\tau}}{ds}=\frac{1}{R}\mathbf{n};\ \frac{d\mathbf{n}}{ds}=-\frac{1}{R}\boldsymbol{\tau}+\frac{1}{T}\mathbf{b}; \\ &\frac{db}{d\mathbf{s}}=-\frac{1}{T}\mathbf{n};\ \frac{d\rho}{d\mathbf{s}}=\boldsymbol{\tau}, \end{aligned} \tag{3}$$

where $\mathbf{F}$, $\mathbf{M}$ are the vectors of the internal forces and moments with components F_u, F_v, F_w and M_u, M_v, M_w, respectively; R is the radius of curvature; T is the torsion radius; $\boldsymbol{\rho}=x\mathbf{i}+y\mathbf{j}+z\mathbf{k}$ is the radius vector of the points of the axial line; A, B, C are the parameters of the flexural and torsional stiffness; p, q, r are the curvatures and torsion of the axial line in a deformed state; p_0, q_0, r_0 are the similar values of the undeformed spiral; E is the elasticity module of the rod material; G is the shear module; I_u, I_v are the inertial moments of the rod cross-section; I_w is the polar inertia moment; $\boldsymbol{\omega}_{\chi}$ is the Darboux vector which equals

$$\boldsymbol{\omega}_{\chi}=\frac{1}{R}\mathbf{b}+\left(\frac{1}{T}+\frac{d\chi}{ds}\right)\boldsymbol{\tau}.$$

In deduction of equations (1) it is taken into account that they are written out in the (u,v,w) coordinate system, which changes from a point to point, so the total derivatives $\mathrm{d}\mathbf{F}/\mathrm{d}s$ and $\mathrm{d}\mathbf{M}/\mathrm{d}s$ are calculated through the use of the equalities

$$\frac{d\mathbf{F}}{ds}=\frac{\tilde{d}\mathbf{F}}{ds}+\boldsymbol{\omega}_{\chi}\times\mathbf{F},\ \frac{d\mathbf{M}}{ds}=\frac{\tilde{d}\mathbf{M}}{ds}+\boldsymbol{\omega}_{\chi}\times\mathbf{M},$$

which stem from the Euler's equalities known in classical mechanics. Here $\mathrm{d}\tilde{\mathbf{F}}/\mathrm{d}s$ and $\mathrm{d}\tilde{\mathbf{M}}/\mathrm{d}s$ are the local derivatives. So the vectors $\mathbf{F}$, $\mathbf{M}$, $\mathrm{d}\tilde{\mathbf{F}}/\mathrm{d}s$, $\mathrm{d}\tilde{\mathbf{M}}/\mathrm{d}s$ and $\boldsymbol{\omega}_{\chi}$ have the components F_u, F_v, F_w, M_u, M_v, M_w, $\mathrm{d}F_u/\mathrm{d}s$, $\mathrm{d}F_v/\mathrm{d}s$, $\mathrm{d}F_w/\mathrm{d}s$, $\mathrm{d}M_u/\mathrm{d}s$, $\mathrm{d}M_v/\mathrm{d}s$, $\mathrm{d}M_w/\mathrm{d}s$ and p, q, r correspondingly.

If the axial line of the rod is preset by the equalities

$$x=x(s),\ y=y(s),\ z=z(s) \tag{4}$$

its geometrical characteristics can be determined via the formulae

$$\frac{1}{R}=\sqrt{(x'')^2+(y'')^2+(z'')^2}\ ,\ \frac{1}{T}=R^2\begin{vmatrix}\mathbf{x}' & \mathbf{y}' & \mathbf{z}'\\ \mathbf{x}'' & \mathbf{y}'' & \mathbf{z}''\\ \mathbf{x}''' & \mathbf{y}''' & \mathbf{z}'''\end{vmatrix},$$

$$p=\frac{1}{R}\sin\chi\ ,\ q=\frac{1}{R}\cos\chi\ ,\ r=\frac{1}{T}+\frac{\mathrm{d}\chi}{\mathrm{d}s}. \qquad (5)$$

Here χ is the angle between the $\mathbf{n}$ unit vector and the u axis, the superindex prime denotes differentiation with respect to s.

It is useful also to remember, that the equations of kinematics are not independent, inasmuch as they have six first integrals

$$|\boldsymbol{\tau}|=1,\ |\mathbf{n}|=1,\ \boldsymbol{\tau}\cdot\mathbf{n}=0,\ \boldsymbol{\tau}\times\mathbf{n}=\mathbf{b} \qquad (6)$$

issuing from the condition of the Frenet basis orthonormality.

It is further assumed that with the selected system parameters, the tubular coil will accomplish small vibrations that can be described by linear differential equations. These equations can be relations (1) linearized in the vicinity of an initial undeformed state. One can write them in scalar form, having eliminated the vector $\mathbf{b}$ from them by means of first-integral formulas:

$$\partial\Delta F_u/\partial s=F_v\Delta r+\Delta F_v r_0-F_w\Delta q-\Delta F_w q_0-\Delta f_u,$$

$$\partial\Delta F_v/\partial s=-F_u\Delta r-\Delta F_u r_0+F_w\Delta p+\Delta F_w p_0-\Delta f_v,$$

$$\partial\Delta F_w/\partial s=F_u\Delta q+\Delta F_u q_0-F_v\Delta p-\Delta F_v p_0-\Delta f_w,$$

$$\partial\Delta p/\partial s=(\Delta F_v-Cq_0\Delta r+Br_0\Delta q)/A,$$

$$\partial\Delta q/\partial s=(-\Delta F_u-Ar_0\Delta p+Cp_0\Delta r)/B,$$

$$\partial\Delta r/\partial s=(-Bp_0\Delta q+Aq_0\Delta p)/C,$$

$$\partial\Delta\tau_x/\partial s=\Delta n_x\sqrt{p_0^2+q_0^2}+n_x(p_0\Delta p+q_0\Delta q)/\sqrt{p_0^2+q_0^2},$$

$$\partial\Delta\tau_y/\partial s=\Delta n_y\sqrt{p_0^2+q_0^2}+n_y(p_0\Delta p+q_0\Delta q)/\sqrt{p_0^2+q_0^2},$$

$$\partial\Delta\tau_z/\partial s=\Delta n_z\sqrt{p_0^2+q_0^2}+n_z(p_0\Delta p+q_0\Delta q)/\sqrt{p_0^2+q_0^2}, \qquad (7)$$

$$\partial\Delta n_x/\partial s=-\Delta\tau_x\sqrt{p_0^2+q_0^2}-\tau_x(p_0\Delta p+q_0\Delta q)/\sqrt{p_0^2+q_0^2}+(\Delta r-\partial\Delta\chi/\partial s)(\tau_y n_z-\tau_z n_y)+(r_0-\partial\chi/\partial s)\times(\Delta\tau_y n_z+\tau_y\Delta n_z-\Delta\tau_z n_y-\tau_z\Delta n_y),$$

$$\partial\Delta n_y/\partial s=-\Delta\tau_y\sqrt{p_0^2+q_0^2}-\tau_y(p_0\Delta p+q_0\Delta q)/\sqrt{p_0^2+q_0^2}+(\Delta r-\partial\Delta\chi/\partial s)(\tau_z n_x-\tau_x n_z)+(r_0-\partial\chi/\partial s)\times(\Delta\tau_z n_x+\tau_z\Delta n_x-\Delta\tau_x n_z-\tau_x\Delta n_z),$$

$$\partial\Delta n_z/\partial s=-\Delta\tau_z\sqrt{p_0^2+q_0^2}-\tau_z(p_0\Delta p+q_0\Delta q)/\sqrt{p_0^2+q_0^2}+(\Delta r-\partial\Delta\chi/\partial s)(\tau_x n_y-\tau_y n_x)+(r_0-\partial\chi/\partial s)(\Delta\tau_x n_y+\tau_x\Delta n_y-\Delta\tau_y n_x-\tau_y\Delta n_x),$$

$$\partial\Delta x/\partial s=\Delta\tau_x\ ,\partial\Delta y/\partial s=\Delta\tau_y\ ,\partial\Delta z/\partial s=\Delta\tau_z.$$

Now on the left-hand sides of these equations, the derivatives with respect to s are partial, since the terms Δf_u, Δf_v, Δf_w contain derivatives with respect to time

3. Simulation of the inertia forces of the boiling liquid

To determine forces generated by a boiling liquid, it is necessary to elaborate a model of dynamic interaction of the spiral tube and liquid moving inside it. As experimental studies carried out in connection with analysis of boiling fluid motions in glass tubes heated on the outside testify, at some thermodynamical states and values of geometrical and mechanical parameters of the system there appear the cases of the so-called slug flows. They reside in the fact that in the tube heat-exchanging systems the regimes of fluid boiling are possible, when the generated vapour-water mixture is not homogeneous but consists of some fluid and vapour segments alternating and moving at high velocities. As the mixture flows, the process of boiling continues, thus the lengths of the tube segments filled with a fluid (called fluid clots) are decreasing and the lengths of cavities filled with a vapour (gas slugs) are increasing. In this case their velocities considerably increase.

The observations made on heated glass spiral tubes show that the lengths of fluid clots change from approximately 10 internal diameters of the pipe on their formation to a zero on a complete evaporation, and the volume of a fluid, as it evaporates, increases tenfold. On boiling, the volume of gas cavities can change from a zero to 50 diameters of the pipe and then, as a result of clot evaporation, they merge.

In studying the dynamical interaction between an elastic pipe and an inner flow, T. B. Benjamin [1] showed that viscous friction forces occurring during

flow appeared to be relatively small. As these forces are directed along the axis line of a pipe, they may be neglected in investigation of its transverse vibration. Thus, the fluid is assumed to be perfect and, while investigating its influence on the dynamics of the tube, only its inertial properties will be taken into consideration. In investigating the problem of vibrations of a pipe with an inner nonhomogeneous flow, the motion of a fluid element along a vibrating and dynamically bending pipe-line will be considered. Calculation will be made of its acceleration in the direction perpendicular to the pipe axis and determination of the inertial force acting on the fluid element and transferring to the pipe walls.

In the calculations, the distributed moments of external forces $\mathbf{m}$ are ignored. The role of the vector of external forces in this case is played by the summarized vector $\mathbf{f} = \mathbf{f}^{\mathbf{i}} + \mathbf{f}^{\mathbf{fr}}$ of the inertial-forces vector $\mathbf{f}^{i}$ and friction forces vector $\mathbf{f}^{fr}$. Since a fluid element accomplishes compound motion, its absolute acceleration $\mathbf{a}_{fl}$ is calculated by formula

$$\mathbf{a}_{fl} = \mathbf{a}^{e} + \mathbf{a}^{r} + \mathbf{a}^{c}. \tag{8}$$

Here, $\mathbf{a}^{e}$ is the vector of the reference-frame acceleration of the fluid in its movement with the tube. Therefore,

$$a_x^e = \ddot{x},\ a_y^e = \ddot{y},\ a_z^e = \ddot{z}. \tag{9}$$

The acceleration $\mathbf{a}^{r}$ of the fluid element caused by its motion in the curvilinear channel of the tube is relative. The vector $\mathbf{a}^{r}$ lies in a contiguous plane; therefore, it is conveniently represented in the axes of a moving trihedron

$$\mathbf{a}^{r} = \boldsymbol{\tau}\,\mathrm{d}v/\mathrm{d}t + \sqrt{p^2+q^2}\,v^2\mathbf{n}. \tag{10}$$

In the considered case $v \neq const$, but this term is not taken into account, as the inertia force connected with it is not applied to the tube walls.

The Coriolis acceleration $\mathbf{a}^{c}$ is due to interaction of the rotational motion of the tube when it vibrates and the relative motion of the liquid in it. It is calculated as

$$\mathbf{a}^{c} = 2\,\boldsymbol{\omega} \times \mathbf{v}.$$

The vector $\boldsymbol{\omega}$ determines the angular velocity of rotation of the trihedron $\mathbf{n}$, $\mathbf{b}$, $\boldsymbol{\tau}$ in the *Oxyz* system. It is expanded in the components of the unit vectors

$$\begin{aligned}
\omega_n &= \tau_x\,\mathrm{d}b_x/\mathrm{d}t + \tau_y\,\mathrm{d}b_y/\mathrm{d}t + \tau_z\,\mathrm{d}b_z/\mathrm{d}t,\\
\omega_b &= n_x\,\mathrm{d}\tau_x/\mathrm{d}t + n_y\,\mathrm{d}\tau_y/\mathrm{d}t + n_z\,\mathrm{d}\tau_z/\mathrm{d}t,\\
\omega_\tau &= b_x\,\mathrm{d}n_x/\mathrm{d}t + b_y\,\mathrm{d}n_y/\mathrm{d}t + b_z\,\mathrm{d}n_z/\mathrm{d}t.
\end{aligned} \tag{11}$$

Knowing the total acceleration $\mathbf{a}_{fl}$, one finds the inertial force acting on the fluid element

$$\mathbf{f}_{fl}^{i} = -\rho_{fl}\,\mathbf{a}_{fl}. \tag{12}$$

For a tube element, one has

$$\mathbf{f}_{t}^{i} = -\rho_t \mathbf{a}_t = -\rho_t(\ddot{x}\mathbf{i} + \ddot{y}\mathbf{j} + \ddot{z}\mathbf{k}). \tag{13}$$

The sum of $\mathbf{f}_{t}^{i}$ and $\mathbf{f}_{fl}^{i}$ gives the total inertial force acting on a coil element with liquid

$$\mathbf{f}^{i} = \mathbf{f}_{t} + \mathbf{f}_{fl}. \tag{14}$$

After corresponding transformations the $\mathbf{f}^{i}$ vector projections on the axes of the movable coordinate system *Oxyz* can be represented as follows:

$$\begin{aligned}
f_x^i &= -(\rho_t + \rho_{fl})\,\ddot{x} - 2\rho_{fl}V_{fl}[\dot{\tau}_x(b_x^2 + n_x^2) + \dot{\tau}_y(b_x b_y + \\
&+ n_x n_y) + \dot{\tau}_z(b_x b_z + n_x n_z)] - \rho_{fl}V_{fl}^2\sqrt{p^2+q^2}\,n_x,\\
f_y^i &= -(\rho_t + \rho_{fl})\,\ddot{y} - 2\rho_{fl}V_{fl}[\dot{\tau}_x(b_x b_y + n_x n_y) + \dot{\tau}_y \times \\
&\times (b_y^2 + n_y^2) + \dot{\tau}_z(b_y b_z + n_y n_z)] - \rho_{fl}V_{fl}^2\sqrt{p^2+q^2}\,n_y,\\
f_z^i &= -(\rho_t + \rho_{fl})\,\ddot{z} - 2\rho_{fl}V_{fl}[\dot{\tau}_x(b_x b_z + n_x n_z) + \\
&+ \dot{\tau}_y(b_y b_z + n_y n_z) + \dot{\tau}_z(b_z^2 + n_z^2)] - \\
&- \rho_{fl}V_{fl}^2\sqrt{p^2+q^2}\,n_z.
\end{aligned} \tag{15}$$

Note, that here ρ_{fl} denotes the mass of the unit length of the flow. Depending on the type of the medium fraction, which is located at the considered point of the tube channel, it can be associated either with the liquid density or with the vapour density.

If the external friction forces $\mathbf{f}^{fr}$ are taken into account their components are represented in the form

$$f_x^{fr} = -\eta\,\dot{x},\quad f_y^{fr} = -\eta\,\dot{y},\quad f_z^{fr} = -\eta\,\dot{z},$$

where η is the friction coefficient.

In constitutive equations (7) there are the linearized components Δf_u, Δf_v, Δf_w of the total external forces. For their construction it is also necessary to linearize f_x, f_y, f_z in the vicinity of the equilibrium state. Then,

$$\begin{aligned}
\Delta f_x &= -(\rho_t + \rho_{fl})\Delta\ddot{x} - 2\rho_{fl}v_{fl}[\Delta\dot{\tau}_x(b_x^2 + n_x^2) + \\
&+ \Delta\dot{\tau}_y(b_x b_y + n_x n_y) + \Delta\dot{\tau}_z(b_x b_z + n_x n_z)] - \rho_{fl}v_{fl}^2 \times \\
&\times \begin{bmatrix} n_x(p_0\Delta p + q_0\Delta q)/ \\ /\sqrt{p_0^2+q_0^2} + \sqrt{p_0^2+q_0^2}\,\Delta n_x + \sqrt{p_0^2+q_0^2}\,n_x \end{bmatrix} - \eta\Delta\dot{x},
\end{aligned}$$

$$\Delta f_y = -(\rho_t + \rho_{fl})\Delta\ddot{y} - 2\rho_{fl} v_{fl}[\Delta\dot{\tau}_x(b_x b_y + n_x n_y) + \\ + \Delta\dot{\tau}_y(b_y^2 + n_y^2) + \Delta\dot{\tau}_z(b_y b_z + n_y n_z)] - \rho_{fl} v_{fl}^2 \times \\ \times \begin{bmatrix} n_y(p_0\Delta p + q_0\Delta q)/ \\ /\sqrt{p_0^2+q_0^2} + \sqrt{p_0^2+q_0^2}\Delta n_y + \sqrt{p_0^2+q_0^2}\, n_y \end{bmatrix} - \eta\Delta\dot{y}, \quad (16)$$

$$\Delta f_z = -(\rho_t + \rho_{fl})\Delta\ddot{z} - 2\rho_{fl} v_{fl}[\Delta\dot{\tau}_x(b_x b_z + n_x n_z) + \\ + \Delta\dot{\tau}_y(b_y b_z + n_y n_z) + \Delta\dot{\tau}_z(b_z^2 + n_z^2)] - \rho_{fl} v_{fl}^2 \times \\ \times \begin{bmatrix} n_z(p_0\Delta p + q_0\Delta q)/ \\ /\sqrt{p_0^2+q_0^2} + \sqrt{p_0^2+q_0^2}\Delta n_z + \sqrt{p_0^2+q_0^2}\, n_z \end{bmatrix} - \eta\Delta\dot{z}.$$

In that the problem solving the constitutive equations (7) are expressed through the components along the axes u,**v**,w, then using correlations (16) and the formulae of transition [1], one can transfer from the components $\Delta f_x, \Delta f_y, \Delta f_z$ to the projections of the $\Delta\mathbf{f}$ vector on the axes u,v,w. To do this an angle $\chi = arctg\dfrac{p}{q}$ is introduced between the unit vector **n** and axis u. After straightforward transformations [6] one can write down

$$\Delta f_u = \left(\Delta f_x n_x + \Delta f_y n_y + \Delta f_z n_z\right)\cos\chi + \\ + \left(\Delta f_x b_x + \Delta f_y b_y + \Delta f_z b_z\right)\sin\chi, \\ \Delta f_v = -\left(\Delta f_x n_x + \Delta f_y n_y + \Delta f_z n_z\right)\sin\chi + \\ + \left(\Delta f_x b_x + \Delta f_y b_y + \Delta f_z b_z\right)\cos\chi, \quad (17) \\ \Delta f_w = \Delta f_x \tau_x + \Delta f_y \tau_y + \Delta f_z \tau_z.$$

To calculate the forces Δf_u, Δf_v, Δf_w, one must take into account discontinuities in parameters of density and inner flow velocities of the liquid-vapour mixture, and assign the law of the fluid clot flow and the vapou-filled cavities motion in its channel proceeding from the condition of preserving the overall vapour-water mixture flow mass rate at the inlet and outlet. The model for changing the flow parameters of motion is formed assuming that the clots of length a_0 enter the channel with a velocity of V_0. At the inlet, a gap between two neighbouring clots is equal to zero. During the motion caused by boiling, the length of a clot varies as $a_1 = a_0 e^{-kt}$ and decreases at the rate of $\dot{a} = d\,a_1/d\,t = -k\,a_0 e^{-k\,t}$. As a result, the lengths of the spaces (cavities) between clots increase at the rate of $\dot{b} = d\,b_1/d\,t = c\,k\,a_0 e^{-k\,t}$. The volume of vapour in a space is considered to be c times as much as that of a fluid from which it was formed, therefore the relation $\rho_{fl} = c\rho_v$ is performed between the densities of the fluid and the vapour.

As the volume of the space of a cavity increases, the velocity V_{i+1} of the $i+1$-th clot increases relative to the previous one as $V_{i+1} = V_i(c-1)\dot{a}$. The velocity of vapour in the cavity between clots is assumed to be distributed linearly.

The system of equations (7) and (17), along with the corresponding boundary and initial conditions, determines the dynamics of a curvilinear tube with internal fluid flow. Underline its total order with respect to variable s equals 15. But inasmuch as it has three first integrals, only 12 boundary conditions should be formulated at the edges $s = 0$, $s = S$, as additional 3 boundary conditions issue from the first integrals.

4. The technique of solution

With the aim of reducing the system of equations (7), (17) with partial derivatives relative to the independent variables s, t to the system of ordinary differential equations relative to the variable s, use an implicit time finite difference scheme (the Houbolt method), according to which the derivatives relative to time at the t_{n+1} time moments are substituted by their four-step finite-difference analogs [6]

$$\dot{X}(s,t_{n+1}) = \dot{X}_{n+1}(s) = \frac{1}{6\Delta t}\times \\ \times\left[11X_{n+1}(s) - 18X_n(s) + 9X_{n-1}(s) - 2X_{n-2}(s)\right], \\ \ddot{X}(s,t_{n+1}) = \ddot{X}_{n+1}(s) = \frac{1}{(\Delta t)^2}\times \quad (18) \\ \times\left[2X_{n+1}(s) - 5X_n(s) + 4X_{n-1}(s) - X_{n-2}(s)\right],$$

where Δt is the time increment. Its value is predetermined by the condition of the calculation convergence.

Assume, that at the time instants t_{n-2}, t_{n-1}, t_n the deformed states of the tube system are known. Then substituting the derivatives by t in (7), (17) by finite-differences (18), one gains the system of ordinary differential equations of the 15th order at the time instant t_{n+2}. This system is rewritten in the general form

$$d\vec{y}/dx = \mathbf{A}(x)\vec{y} + \vec{f}(x). \quad (19)$$

Here $\vec{y} = \vec{y}(s)$ is the 15-dimensional vector of the unknown functions; x the independent redenoted variable s changing within the limits of $0 \le x \le S$; S the spiral length; $\mathrm{A}(x)$ the known discontinuous

matrix-function of the independent variable x; $\vec{f}(x)$ the preset vector of right members determined by the known solution functions at previous steps in time.

It should be noted that the deformed state of the tube system at the t_n time instant in (18) is determined through application of equation (19) at $t = t_n$ using the deformed states at the previous time instants t_{n-3}, t_{n-2}, t_{n-1} and analogously is done for the states at $t = t_{n-1}$ and $t = t_{n-2}$ in (18). The tube system states at $t_0 = 0$, $t_1 = \Delta t$ and $t_2 = 2\Delta t$ are found via the use of appropriate initial conditions at $t = 0$.

The solution to (19) must be subjected to boundary conditions at the interval bounds, which are predetermined at the beginning $x = 0$ and at the end $x = S$ of the integration interval.

They are represented in the general form as

$$\mathbf{B}\vec{y}(0) = 0,\ \mathbf{D}\vec{y}(L) = 0, \tag{20}$$

where matrices **B** and **D** measure (6×15).

Notice, that the number 12 of boundary conditions (20)is not equal to the system (19) which is of order 15. This is associated with the availability of the systems first three integrals which complement the number of boundary equations making a total of 15.

For constructing the solution $\vec{y}(x)$, 6 components $y_j(x)$ are chosen from the $y_i(x)$ $\left(i = \overline{1,15}\right)$ components, any values $y_j(0)$ of which don't violate the first equation (20) and the three first integrals at zero values of the other components. After renumbering the unknown values $y_i(x)$ $\left(i = \overline{1,15}\right)$ in such a way that the index j could take on the values $j = \overline{1,6}$, the solution to problem (19), (20) can be given as [6]

$$\vec{y}(x) = \mathbf{Y}(x)\vec{C} + \vec{y}_0,$$

where $\vec{y}_0$ is the solution to the Cauchy problem for system (19) at zero initial conditions, $\mathbf{Y}(x)$ is a (15×9) matrix of particular solutions y_{ij} to the homogeneous matrix differential equation

$$\mathrm{d}\mathbf{Y}/\mathrm{d}x = \mathbf{A}(x)\mathbf{Y} \tag{21}$$

with initial conditions $y_{ij}(0) = \delta_i^j$ $\left(i = \overline{1,15},\ j = \overline{1,6}\right)$ for independently modified variables, and with initial conditions chosen from the first equation of system (20) and three first integrals for the other variables $y_{ij}(0)$ $\left(j = \overline{7,15}\right)$.

Here δ_i^j is the Kronecker symbol.

The vector of the constants $\vec{C} = \left(\overline{C_1, C_6}\right)^T$ is chosen so that the equality

$$\mathbf{D}\,\mathrm{Y}(L)\vec{C} + \mathbf{D}\,\vec{y}_0(L) = 0$$

following from the second conditions of system (20) could be satisfied.

The construction of the matrix-function $\mathbf{Y}(x)$ and the vector-function $\vec{y}_0(x)$ is made by integrating equations (19) and (21) by the fourth order Runge-Kutta method. The peculiarity of using such an approach is that due to the presence of large factors in the coefficients of system (7), it is rigid and there are rapidly growing functions among its particular solutions. Therefore in constructing the matrix of its fundamental solutions, the method of discrete orthogonalization by Godunov [6] is additionally used which makes it possible to obtain a stable computational process by orthogonalizing the vector-solutions to the Cauchy problems in the finite number of argument change interval points. Its essence is in the fact that the integration interval is divided into sections, and the numerical integration of the initial differential equation is carried out on each of these sections in the same way as in using the method of transfer matrix. The lengths of the sections are such that the particular solutions to a homogeneous equation within the limits of one section could remain linearly independent. When passing from one section to another, the matrix of the solutions is subject to linear transformation so that the vectors of particular solutions of the homogeneous and nonhomogeneous equations become orthogonal. Thus it is possible to preserve the linear independence of the equation solutions in the whole interval of integration. To avoid excessive increase of the numerical values of the nonhomogeneous equation solutions, the normalization factor is introduced at the section boundaries.

5. Research results

The procedure for solution of a system of equations (7), (17) with partial derivatives employs the Hubolt implicit difference scheme, which is distinguished by enhanced accuracy for its integration with respect to time [6]. It is used to construct a step-by-step

process in each step of which a two-point boundary-value problem is solved for the 15th-order equations with independent variable s that have three first integrals. Since some of the coefficients of this system have small divisors equal to the squares of the steps of integration with respect to time, this system is rigid and rapidly increasing functions are among its partial solutions. It is therefore solved by the joint application of the transfer matrix method, the discrete-orthogonalization method [7] and the Runge-Kutta method.

In the initial undeformed state, the axial line of the tubular coil is determined by the equations

$$x = R\cos\left(\frac{\cos\alpha}{R}s\right),\ y = R\sin\left(\frac{\cos\alpha}{R}s\right),\quad z = s\sin\alpha, \tag{22}$$

where R is the radius of the cylindrical surface of the coil and α is the angle of ascent of the coil.

They are used to calculate the components of the unit vectors of the moving trihedron

$$n_x = -\cos\left(\frac{\cos\alpha}{R}s\right),\ n_y = -\sin\left(\frac{\cos\alpha}{R}s\right),\ n_z = 0,$$

$$\tau_x = -\cos\alpha\sin\left(\frac{\cos\alpha}{R}s\right),\quad \tau_y = \cos\alpha\cos\left(\frac{\cos\alpha}{R}s\right),\ \tau_z = \sin\alpha, \tag{23}$$

$$b_x = \tau_y n_z - \tau_z n_y,\ b_y = \tau_z n_x - \tau_x n_z,\quad b_z = \tau_x n_y - \tau_y n_x,$$

and the parameters of curvature and torsion

$$p_0 = 0,\ q_0 = \sqrt{(x'')^2 + (y'')^2 + (z'')^2} = \frac{\cos^2\alpha}{R},$$

$$r_0 = \frac{1}{q_0^2}\begin{vmatrix} \mathbf{x}' & \mathbf{y}' & \mathbf{z}' \\ \mathbf{x}'' & \mathbf{y}'' & \mathbf{z}'' \\ \mathbf{x}''' & \mathbf{y}''' & \mathbf{z}''' \end{vmatrix} = \frac{\sin\alpha\cos\alpha}{R} \tag{24}$$

Relations (23) and (24) are used to calculate the coefficients of equations (7).

The above-described procedure was employed to study the vibrations of two types of steel tubular spirals. The first type tubes have the following characteristics : number of coils $N = 5$; $R = 0{,}5\ m$; $\alpha = 0{,}07214\ rad$; the curvature and torsion parameters $p_0 = 0$, $q_0 = 1{,}99\ m^{-1}$, $r_0 = 0{,}14\ m^{-1}$. For the tube of the second type these parameters comprise : $N = 10$; $R = 0{,}1\ m$; $p_0 = 0$; $q_0 = 9{,}95\ m^{-1}$; $r_0 = 7{,}19\ m^{-1}$. For both tube serpentines, flexural stiffnesses $A = B = 1253\ N\,m^2$; torsional stiffness $C = 955\ N\,m^2$; outside diameter of circular section of tube $d = 0{,}02\ m$; wall thickness of tube $h = 0{,}003\ m$; mass per unit length of flowing liquid (water) $\rho_{lq} = 1{,}54\cdot 10^{-1}\ kg/m$; mass per unit length of tube $\rho_t = 1{,}24\ kg/m$

Two cases of the problem were considered. In the first case it was assumed, that the external friction may be neglected and $\eta = 0$. For the second one η was taken to be 1 $N s/m^2$.

It is impossible to determine beforehand the period in which the tubular coil will respond to the inertial forces of the internal flow. The nature of the dynamic response of the coil is established after analysis of the calculation results.

Eight problems were solved in each case for the selected values of the parameters, which were different by the lengths a_0 of the water clots at entrance and the k parameter, which determined the rate of the clot evaporating.

The tube dynamics over a time interval equal to $9 \div 10\ s$, sufficient for establishment of general regularities of the dynamic process, was studied for each problem at a fixed clot velocity V_0. Then, to find the resonance modes of motion, V_0 was changed and the motion modelling was repeated for the new V_0 value. The smallest V_0 value at which the vibration amplitude began to increase without limit was considered to be critical. The step ΔV_0 of V_0 variation was $\Delta V_0 = 1\ m/\sec$. In the vicinity of the critical state, this value was $\Delta V_0 = 0{,}1\ m/\sec$. Table 1 corresponds to absence of friction forces ($\eta = 0$).

Table 1. The values of critical velocities of liquid clots entering into tube spiral and the periods of their forced vibrations at critical states ($\eta = 0$)

Case number	a_0	k, s^{-1}	$V_{0,cr}$, m/s	First type of the spiral ($R = 0{,}5\ m$; $N = 5$)			Case number	$V_{0,cr}$, m/s	Second type of the spiral ($R = 0{,}1\ m$; $N = 10$)		
				T, s	T_x, s	T_y, s			T, s	T_x, s	T_y, s
1	$S/8$	0,5	3,8	0,518	0,284	0,518	5	17,6	0,0447	0,0224	0,046
			4,5	0,437	0,219	0,324		33,5	0,0235	0,0276	–
2	$S/8$	1	2,7	0,729	0,364	0,361	6	16,9...17	0,0466	0,0252	0,0466
			4,9	0,402	0,326	0,135		29,7	0,0265	0,018	-
3	$S/4$	0,5	5,4...7	0,729	0,146	0,212	7	17,7	0,0889	0,0327	0,036
			7.2	0,547	0,274	0,616		34	0,0463	0,0339	0,308
4	$S/4$	1	5,1...5,8	0,772	0,386	0,771	8	16,9...17,1	0,0931	0,031	0,0465
			6,3	0,751	0,25	0,824		27,2	0,0579	0,0289	0,115

It was established in consequence of the result analysis that critical values $V_{0,cr}$ of the velocity of a water clot entering into the tube could be achieved when the amplitude of the spiral chatter began to enlarge indefinitely. In doing so, as the clot motions are not absolutely periodic, conventional periods T of conventional resonances of the tube vibrations can be established for every element of the spiral. Usually these values are different for the directions Ox and Oy. In Table 1 the values T_x and T_y are listed for the middle element ($s = S/2$) of the tube. It can be seen from it that there can be several critical values $V_{0,cr}$ or even unstable segments for the velocity V_0 and that $V_{0,cr}$ enlarges when a_0 diminishes. The critical velocity $V_{0,cr}$ also diminishes with reduction of k.

The modes of helix motion have high complexity with respect to both the space (s) and time (t) coordinates. Figures 2 a-h are graphs of the vibrations of the point $s = S/2$ on the axial line of the tube along the Ox and Oy axes for the precritical (fig. 2, *a*, *b*, $V_0 = 16{,}8\ m/s$), critical (fig. 2, *c*, *d*, $V_{0,cr} = 16{,}9\ m/s$), intercritical (aig. 2, *e*, *f*, $V_{0,cr} = 17\ m/s$) and second critical (fig. 2, *g*, *h*, $V_{0,cr} = 27{,}1\ m/s$) clot velocities for Case 8 in Table 1. It can be seen that at a low velocity $V_0 = 16{,}8\ m/s$ (fig. 2, *a*, *b*), the vibrations are in the form of beats, while in the critical case $V_{0,cr} = 16{,}9\ m/s$ the amplitude rises, but not linearly, as in the case of ordinary resonance vibrations (fig. 2, *c, d*). This type of dynamic stability loss is typical for parametric resonances of systems whose properties periodically change.

This feature is inherent in the considered system, because its inertia (mass) characteristics change periodically with the clots movement.

It is necessary also to note that an increase in clot velocity increases not only the frequency of clot action on the structure but also the intensity of the inertial forces, which is proportional to the square of the velocity. Unlike in ordinary vibrational systems, therefore, the spiral vibrations can again be unstable in supercritical states, when V_0 is larger than the first critical value $V_{0,cr} = 16{,}9\ m/s$ (fig. 2, *g*, *h*).

It is not simple to separate a 3D mode of forced vibrations of the tube as the dynamic processes are not steady, so the deformed states of its centerline were analysed for different time instants. In Fig. 3 the outlines of the spiral states are shown for Case 8 in Table 1. They have different geometrics and it is rather difficult to distinguish any regularity in the spiral motion.

With the aim of investigating how the external friction influences the stability of the spiral vibrations, the 8 cases considered in Table 1 were recalculated with allowance made for the friction forces in (16) and (17) with the coefficient $\eta = 1\ Ns/m^2$. The calculation results (Table 2) testify that the friction forces do not practically influence on the modes and periods of the system vibrations, but their action brings slight enlargement of the critical values of the flow velocities and appearance of unstable segment at case 1.

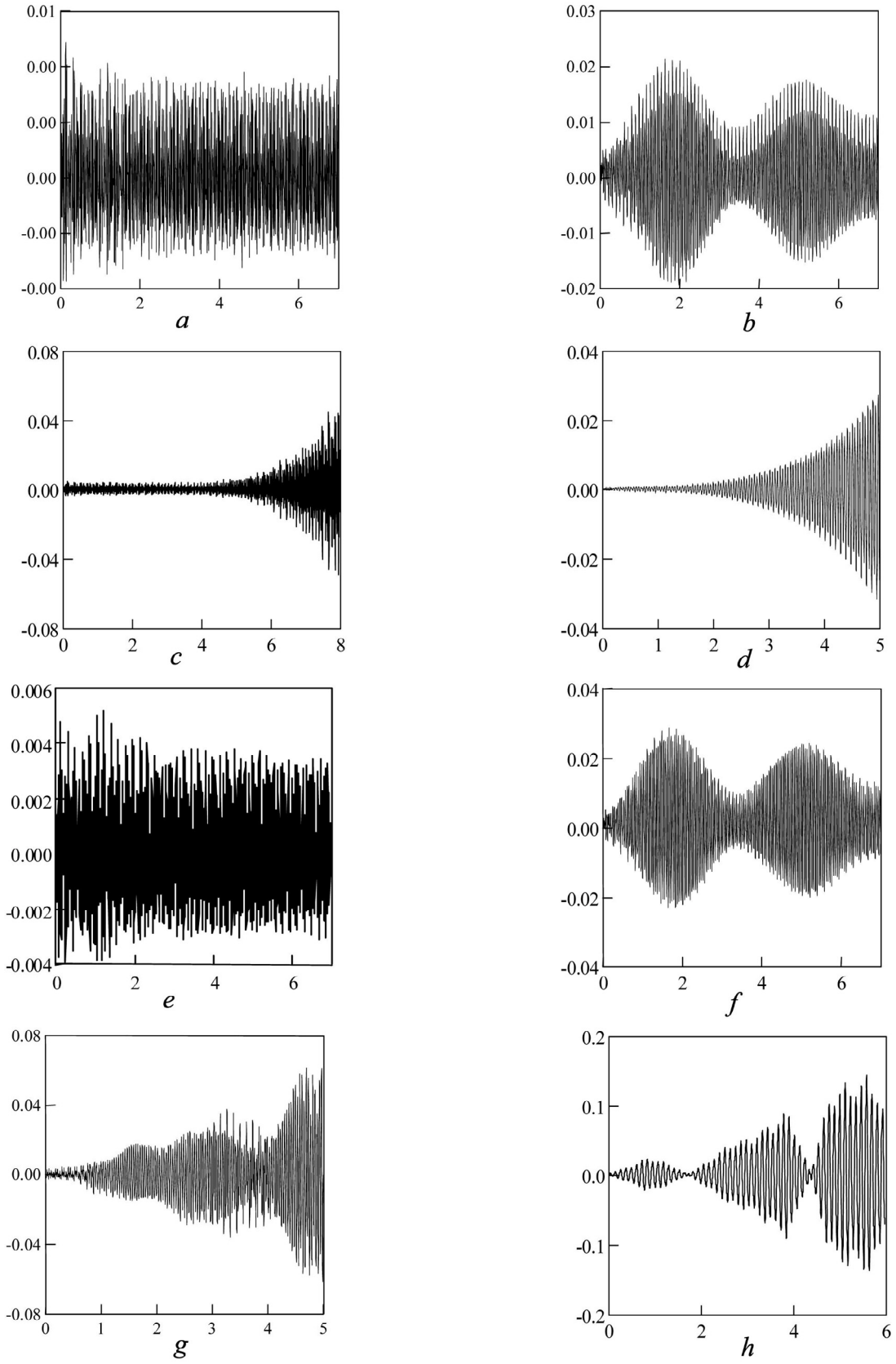

Fig. 2. Modes of vibration in time of the serpentine middle cross-section for different values of initial velocities of the entering fluid

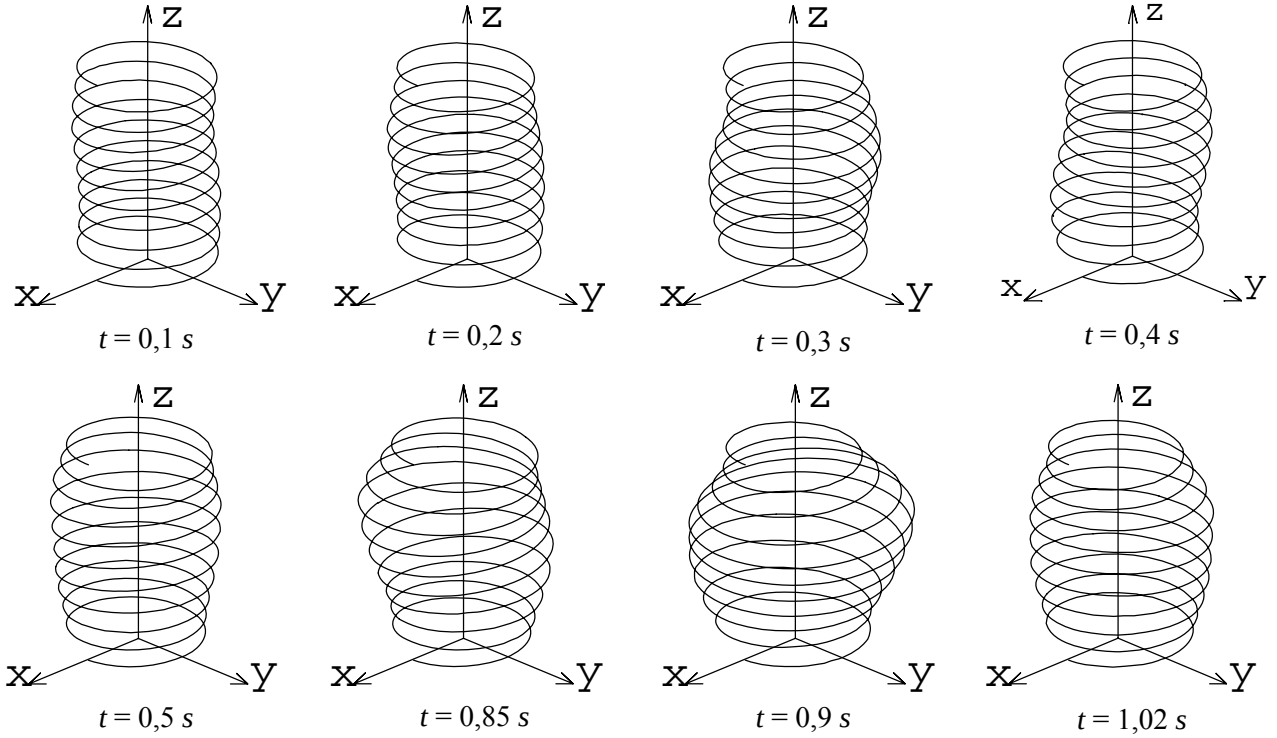

Fig. 3. Modes of the serpentine motions for case 8 in Table 1 ($V_0 = 27,2\ m/s$)

Table 2. The values of critical velocities of liquid clots entering into tube spiral and the periods of their forced vibrations at critical states ($\eta = 1\ \ Ns/m^2$)

Case number	a_0	k, s^{-1}	$V_{0,cr}$, m/s	First type of the spiral ($R = 0,5\ m$; $N = 5$)			Case number	$V_{0,cr}$, m/s	Second type of the spiral ($R = 0,1\ m$; $N = 10$)		
				T, s	T_x, s	T_y, s			T, s	T_x, s	T_y, s
1	$S/8$	0,5	3.9⋯4.2	0,518	0,285	0,519	5	17,9	0,0447	0,023	0,048
			4,8	0,437	0,22	0,322		33,9	0,0235	0,0281	–
2	$S/8$	1	3,0	0,729	0,366	0,360	6	17,1...17,4	0,0466	0,0261	0,047
			4,8	0,402	0,327	0,136		31,2	0,0265	0,019	–
3	$S/4$	0,5	5,7...7,3	0,729	0,147	0,212	7	18...19,1	0,0889	0,033	0,038
			7,5	0,547	0,277	0,617		35,7	0,0463	0,034	0,309
4	$S/4$	1	5,3...5,9	0,772	0,385	0,773	8	17,2...17,4	0,0931	0,033	0,0467
			6,7	0,751	0,253	0,826		28,3	0,0579	0,029	0,116

6. Conclusion

The problem of computer simulation of tube spiral vibrations under action of internal flows of boiling fluid is considered. A mathematic model of dynamics of the elastic serpentine is elaborated with allowance made for a discontinuous distribution of the parameters of the internal flow caused by the process of its heating and boiling. The action of inertial forces of positional and gyroscopical types is taken into account. The analysis of the results obtained for different values of the parameters of the flow nonhomogeneity and velocity makes it possible to make the following conclusions:

1. The nonhomogeneity of the inner fluid flow manifests itself both in the nonhomogeneity of centrifugal inertial forces acting on the pipe in the transverse direction and in the change with time of the system general mass geometry. In this connection purely dynamical and parametrical excitations of vibrations take place.

2. The possibility of establishment of stable and unstable regimes of motion is found out, which depend on the character of nonhomogeneity and velocity of the fluid clots and the rate of their evaporation.

3. The spatial modes of forced vibrations of the tube spiral are constructed. It can be noted that the centrifugal inertia forces normal to the elastic line of the curvilinear rod and the Coriolis inertia forces caused by slewing and rotation of the rod cross-sections lead to expansion and intricating of the vibration modes. Besides, generation of combined modes including longitudinal, bending and torsional modes followed by condensation and rarefaction of the spiral coils as well as by the enlargement and diminution of their diameters is peculiar to the studied regimes.

4. The influence of external friction forces on the tube forced vibrations is analysed. It is noted that these forces lead to displacement of critical values of the fluid velocities and to change of the vibration amplitudes.

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Є. Ю. Толбатов. Математичне моделювання динаміки вигнутих труб, які транспортують внутрішні маси неоднорідної рідини

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Запропоновано методику чисельного моделювання динаміки пружних серпантинних труб з внутрішніми потоками киплячої рідини. Отримано систему диференціальних рівнянь, яка містить розривні коефіцієнти у правій частині, котрі залежать від часу. Пропонується модель руху згустків неоднорідної киплячої рідини. Вона пов'язана з умовою збереження маси рідини. Методика чисельного рішення рівнянь побудована на основі методу трансфер-матриці. Виявлено можливість встановлення стійких та нестійких режимів руху, які залежать від характеру неоднорідності та швидкості руху згустків.

Ключові слова: внутрішні потоки; динаміка; коливання; неоднорідна рідина; циліндричні спіралі; чисельний метод; швидкість.

Е. Ю. Толбатов. Математическое моделирование динамики криволинейных труб, содержащих внутренние массы неоднородной жидкости

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Предложена методика численного моделирования динамики упругих спиральных труб с внутренними потоками кипящей жидкости. Получена система дифференциальных уравнений, содержащая разрывные коэффициенты в правой части, зависящие от времени. Предлагается модель движения сгустков неоднородной кипящей жидкости. Она связана с условием сохранения массы жидкости. Методика численного решения уравнений построена на основе метода трансфер-матрицы. Обнаружена возможность установления устойчивых и неустойчивых режимов движения, зависящих от характера неоднородности и скорости движения сгустков.

Ключевые слова: внутренние потоки; динамика; колебания; неоднородная жидкость; скорость; цилиндрические спирали; численный метод.

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