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ALGORITHM OF GUARANTEED-AND-ADAPTIVE CONTROL OF AEROSTATIC VEHICLE UNDER UNDETERMINED EXTERNAL TURBULENCES

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Abstract. *The guaranteed-and-adaptive algorithm of motion control of aerostatic vehicle are synthesized. It's based on differential transformation of initial mathematic model of differential game of airship motion and is more complicated in comparison with other algorithms. The obtained algorithm has the adaptation properties to turbulences and provides a guarantee of aerostatic vehicle translation to the given states during the worst combination of turbulence factors. The point of given algorithm is the achievement of lowest terminal errors during the process of airship motion control at the action of turbulences.*

Keywords: aerostatic vehicles; control algorithm; differential game; differential transformation.

1. Introduction

The motion of aerostatic vehicles, such as airships, is multimode and is characterized by different engine operating conditions, aerodynamic control movements, practically by sudden change of vehicle mass at the moment of ballast drop (if it is necessary) and potential engine failures [4]. Motion optimization of multimode aerostatic vehicle allows realizing of maximum possible flight characteristics, and improves reliability due to increased flight stability of control algorithms to external turbulences. For this reason, the airship control system should be build with the fullest accounting of all operation features.

Control algorithm synthesis of aerostatic vehicle under turbulences is a complex problem.

The high order of nonlinear differential equations of airship motion complicates the problem solution.

At that, there is no a priori information regarding components of external turbulences.

At the same time, the requirements for terminal parameters during takeoff and landing require consideration of the impact of unpredictable turbulences to achievement the control aims.

One way for evaluation of indeterminate form associated with the unpredictable influence of external turbulences, is the concept of guaranteed-and-adaptive approach to the control algorithms synthesis of trajectory airship motion.

This concept uses the principle of maximum guaranteed result as the control process is seen in the most adverse conditions that may occur when exposed to turbulences.

The problem of guaranteed-adaptive control synthesis under uncertain impact of turbulences requires transformation from optimization tasks to tasks of bidirectional optimization, as discussed in the theory of differential game [5].

In such circumstances, to consider the terminal control task is appropriate in the form of mathematical model of differential game of two players, whose research is based on the principle of maximum guaranteed result [2, 6].

The first player forms the vehicle control and the vector of turbulence is formed specifically by second player.

The control aims of first and second players are opposed.

The task of the first player is the vehicle transfer from the initial state to a given final in which the quality control criterion is minimized by its maximizing by the other player.

The game approach ensures the terminal conditions achievement at any allowable realizations of vector of turbulences as well as the terminal control algorithms synthesis focused on the most adverse conditions of influence of turbulence.

The paper in [1] presents the method of terminal control algorithm synthesys of dynamic objects under influence of turbulents on the basis of differential transformations of the model of differential game.

The approach does not require for its realization of numerical integration of differential equations of dynamic object motion, using the mathematical apparatus of differential transformations of functions and equations [6].

At that, the synthesis task of optimal adaptive algorithm reduces to the solution of nonlinear equations for its free parameters.

The paper [3] this method has been modernized for solving of the synthesis task of guaranteed-and-adaptive algorithms for motion control of multimode dynamic objects under influence of turbulence on the basis of differential transformation of mathematical model of differential game.

The objective of this paper is a guaranteed-and-adaptive control algorithm synthesis of multimode aerostatic vehicle using the above modernized method.

2. Synthesis of guaranteed-and-adaptive control algorithm – statement of problem

All controllable aerostatic vehicle motion is conditionally divided into r given time frame, inside which the parameters of the vehicle have no sudden changes and control switching.

All changes in the form of given springs happen at boundaries of selected intervals

$$T_i = t_i - t_{i-1},\ i = \overline{1,r},\ \sum_{i=1}^{r} T_i = T,$$

where T – a duration of controllable motion of aerostatic vehicle.

The deferential game model of multimode aerostatic vehicle motion at each motion segment at undefined turbulence we shall present as the vector differential equation:

$$\frac{dx_i}{dt} = f_i(t, x_i, u_i, v_i),\ x_i(t_{i-1}) = x_i^0,\ i = \overline{1,r}, \qquad (1)$$

where $x_i = x_i(t)$ – n-measurement of state vector;

$u_i = u_i(t)$ – m-measurement of control vector;

$v_i = v_i(t)$ – ℓ-measurement vector of turbulence;

f_i – continuous and continuously differentiable on plurality variable t, x_i, u_i, ν_i the vector function of generalized force;

$t \in [t_{i-1}, t_i]$.

The conjugation boundary and starting conditions of segments of the process of control motion are set in the form of given border of requirements [7]:

$$\varphi_i\left[x_i(T_i), x_{i+1}^0; u_i(T_i), u_{i+1}^0; T_i\right] = 0,\ i = \overline{1,r}. \qquad (2)$$

The task of first player consists in the vehicle translation from the given initial state $x_1(t_0)$ to final (terminal) state $x_r(T)$, which is determined in the point of time $t = T$ by q- measurement ($q \le n$) vector equation [8]:

$$S[x_r(T), T] = 0. \qquad (3)$$

The control aim consists in such control selection $u_i(t)$, which in the process of vehicle motion ensures minimization of the functional:

$$I = G[x_r(T), T] + \sum_{i=1}^{r} \int_{t_0}^{T} \Phi_i(t, x_i, u_i, \nu_i)\,dt, \qquad (4)$$

$$i = 1,2,3,...r$$

upon condition of its maximization during vector of turbulence selection $\nu_i(t)$ by second player.

Assume, that given functions G and Φ_i have continuous partial derivatives on x_i, u_i, ν_i.

Functions $u_i(t)$ and $\nu_i(t)$ are termed as program strategies of players.

Restriction on player strategies is taken into account during the selection of the functional type (4).

Pair of player strategies u_i^0 and ν_i^0 is termed as optimal, if there is the ratio:

$$I(u_i^0, \nu_i) \le I(u_i^o, \nu_i^o) \le I(u_i, \nu_i^0). \qquad (5)$$

The necessary criterions of strategies optimality u_i^0 and ν_i^0 are [4]:

$$\frac{\partial I}{\partial u_i} = 0, \quad \frac{\partial I}{\partial \nu_i} = 0, \qquad (6)$$

$$\frac{\partial^2 I}{\partial u_i^2} \ge 0, \quad \frac{\partial^2 I}{\partial \nu_i^2} \le 0, \qquad (7)$$

and sufficient criterions are the ratio (6) and criterion (7), which has the strict inequality.

Player strategies u_i^0 and ν_i^0, that satisfy the sufficient criterions, ensuring the existence of saddle point (5) of differential game (1)-(4).

The control process we shall consider within the frame of such mathematical models of differential games, which satisfy conditions (6) та (7).

From ratio (5) follows, that random law of vector of turbulence variation, other than optimal ν_i^0, doesn't impair the quality of object process control, which is achieved under the optimal control u_i^0.

Therefore, the control u_i^0 is guaranteed the quality of control process no worse of definition (u_i^o, v_i^0) at conditions of restricted random turbulences.

Taking into account, that control u_i^0 ensures obtaining of guaranteed assessment of control quality and adaptability to the specific type of turbulences action, we will call the control u_i^0 as guaranteed-and-adaptive control [2, 6].

Simulation of multimode dynamic object motion control in the form of differential game removes the uncertainty caused by the influence of turbulences.

Evaluation of indeterminate forms comes at the cost of loss of simplicity of the mathematical model and simulation process, as a result, in addition to optimal control u_i^0, it is necessary to determine the law of vector of turbulence ν_i^0 variation that describes the maximum opposition of purposes of terminal control.

3. Method of control algorithm synthesis

For control algorithm synthesis of multimode aerostatic vehicle motion under undetermined conditions of turbulences influence we will use the mathematical apparatus of differential transformations [6].

The differential transformations allow to replace the function $x(t)$ continuous argument t by their spectral models in the form of discrete functions $X(k)$ of integer argument $k = 0, 1, 2, \ldots$.

The differential transformations are the functional transformations of the type:

$$\underline{x(t)} = X(k) = \frac{h^k}{k!}\left[\frac{d^k x(t)}{dt^k}\right]_{t=t_0}, \tag{8}$$

where $x(t)$ – the original, which represents the continuous and bounded together with all its derivatives function of real argument t;

$X(k)$ – the discrete function of integer argument k, which is termed as a differential spectra of function $x(t)$ in the point $t = t_0$;

h – the scale stationary value having dimensionality of argumant t.

The line below is the transformation character.

The mathematical models, which are obtained from initial mathematical model by application of differential transformations (8) to them, are termed as spectral models.

Later on, we assume that the time functions, describing the control processes in task (1) - (4) within each motion segment are analytic.

Synthesis of guaranteed-and-adaptive control algorithms we will realize in two stages.

At the first stage will perform a synthesis of optimal gaming algorithms of program control $u_i^0(t)$ and maximum opposing turbulence $\nu_i^0(t)$ which satisfy the conditions (6) and (7), in the middle of each control segment in the class of analytic functions $u_i(\tau, A_i)$ and $v_i(\tau, B_i)$, where $A_i = (a_{i1}, a_{i2}, .. a_{iN})$ and $B_i = (b_{i1}, b_{i2}, .. b_{iM})$ are vectors of free parameters, τ is a local time argument.

Differential transformations (8) of functions $u_i(\tau, A_i)$ and $v_i(\tau, B_i)$ are determined at $h = T_i$ and assume $\tau = 0$ its differential spectra as:

$$\underline{u_i}(\tau, A_i) = U_i(k, A_i) = \frac{T_i^k}{k!}\left[\frac{d^k u_i(t_{i-1} + \tau, A_i)}{dt^k}\right]_{\tau=0}. \tag{9}$$

$$\underline{v_i}(\tau, A_i) = V_i(k, B_i) = \frac{T_i^k}{k!}\left[\frac{d^k \nu_i(t_{i-1} + \tau, B_i)}{dt^k}\right]_{\tau=0}. \tag{10}$$

Diffenetial equation (1) in the image field on the basis of transformations (8) is written as the following spectral model:

$$\begin{aligned} &X_i(k+1, A_i, B_i, X_i^0) = \\ &= \frac{T_i}{k+1}\underline{f_i}\left[T_i, X_i(k, A_i, B_i, X_i^0), U_i(k, A_i), V_i(k, B_i)\right]; \\ &X_i(0) = X_i^0(A_{i-1}, A_{i-2}, \ldots A_1, B_{i-1}, B_{i-2}, \ldots, B_1); \\ &X_1(0) = X_1^0 = x_0; \; i = \overline{1, r}. \end{aligned} \tag{11}$$

Recursion expression (11) allows to find the differential spectra $X_i(k, A_i, B_i, X_i^0)$ of state vector $x_i(t)$ in the differential spectra (9) and (10).

Let's take advantage of the property of the differential transformations, according to which the algebraic total of all discretes of differential spectra of any analytical function in point $t = t_\nu$, is equal to zero doscrete of a differential spectrum of function in point $t_{\nu+1} = t_\nu + h$ or value of the original of function in the same point [6]:

$$\sum_{k=0}^{\infty} X_\nu(k) = X_{\nu+1}(0) = x(t_\nu + h). \tag{12}$$

From the obtained relation (12) at $t_\nu = t_{i-1}$ and $h = T_i$ we determine a state vector at the end of each control segment:

$$x_i(T_i, A_i, B_i, x_i^0) = \sum_{k=0}^{\infty} X_i(k, A_i, B_i, X_i^0), \quad i = \overline{1, r}. \qquad (13)$$

Than the equation of the final state (3) in view of the expression for conjugation of boundary and initial segments (2), and also the expressions for a state vector at the end of each segment (13) is conversed as followed [3]:

$$S[A_1, A_2, ..., A_r, B_1, B_2, ..., B_r] = 0. \qquad (14)$$

The given terminal condition in the implicit shape define q-components of vectors of free parameters A_i and B_i, $i = \overline{1, r}$ as functions T_i and x_i^0.

Remaining M+N-q components of vectors of free parameters are determined from the stationary conditions (6) of the functional (4).

The differential transformations (8) of functional (4) allow presenting functional (4) as the function of vectors of undetermined parameters A_i та B_i:

$$I(A_1, A_2, ...A_r, B_1, B_2, ..., B_r) = G[A_1, A_2, ...A_r, B_1, B_2, ..., B_r] + \sum_{i=1}^{r} T_i \times \sum_{k=0}^{\infty} \frac{\Phi_i[T_i, X_i(k, A_i, B_i, X_i^0, U_i(k, A_i), V_i(k, B_i)]}{k+1}. \qquad (15)$$

The stationary conditions (6) of function (15) enable to receive combined equations for determining remaining M+N-q of unknown components of vectors of free parameters $A_1, A_2, ..., A_r$ and $B_1, B_2, ..., B_r$:

$$\frac{\partial I(A_1, A_2, ...A_r, B_1, B_2, ..., B_r)}{\partial a_{ij}} = 0, \ q+1 \le i \le N, \qquad (16)$$

$$\frac{\partial I(A_1, A_2, ...A_r, B_1, B_2, ..., B_r)}{\partial b_{ij}} = 0, 1 \le j \le M. \qquad (17)$$

Solving the systems of nonlinear algebraic equations (14), (16) and (17), in the case of their consistency allows to find the components of vectors of free parameters $A = (A_1, A_2, ..., A_r)$ and $B = (B_1, B_2, ..., B_r)$ of program strategies of both players as functions from a vector of an arbitrary initial state $x_0 = x_1(t_0)$.

Then can be verified sufficient conditions (6), (7) of player strategies optimality at strict inequality in the expression (7).

When system of equations (14), (16) and (17) is inconsistency, the differential game (1)–(7) has no solution in selected function types $u(t, A)$ and $v(t, B)$ and than, the type of functions with free parameters should be changed or expand the dimension of vectors of free parameters [1].

As a result of execution of the first stage of synthesis of the control algorithms in the implicit form, the nonlinear communication of program strategies of both players $u(t, A)$ and $v(t, B)$ with a vector of the initial state $x_0 = x_1(t_0)$ is established.

These strategies can be utilized only in initial instant t_0 and do not account changes of state during motion.

To take into account the current state of the control process is necessary to synthesize control algorithms and maximum counteract turbulences in the form of positional strategies of the players

$$u = u(x, t), v = v(x, t).$$

At the second stage of synthesis we shall make the following assumption.

We will consider only such models of control process in which there are player strategies and allow to associate an arbitrary initial condition within a given region of state space with given terminal conditions (3).

The strategies synthesis beyond a given region of state space is not considered.

The solution of combined equations (14), (16) and (17) for current instant t for each current state of game $x(t)$ sets pair of player strategies $u^0[t, A(T, x)]$ and $v^0[t, B(T, x)]$, linking current state of game with terminal requirements (3).

If organize a time continuous process of calculating parameters A and B of players strategies, then on the set of solutions can be formulated player strategies on each motion segment as $u[t, A(T, x)]$ and $v[t, B(T, x)]$.

The first player who realizes a potential strategy $u[t, A(T, x)]$, which continuously determined from combined equations (14), (16) and (17), ensures the aerostatic vehicle control with achievement of the given terminal conditions (3) at the maximum counteract of turbulence which action is modeled by a strategy of second player $v[t, B(T, x)]$.

If necessary to find the optimal trajectory $x(t, A, B)$, its components can be identified as a Taylor series or using inverse differential transformation in the form of polynomials of Legendre, Chebyshev, Fourier series [6].

At that, free parameters of approximating functions are determined from comparison of the differential spectra of the state vector components and differential spectra of approximating functions.

The constraint equation between free parameters of some approximating functions and discretes of differential spectrum of an unknown time function is given in [6].

4. Synthesis of guaranteed-and-adaptive algorithm

As an aerostatic vehicle we shall consider the airship.

Synthesis of guaranteed-and-adaptive control can be executed on mathematical model of differential game which contains a description of the trajectory airship motion with taking into account the effects of turbulences.

The corresponding spectral model of airship motion is given in the paper [8].

A scheduling of the airship thrust vector angle sets as a sum of two components:

$$\varphi = u + \sigma, \tag{18}$$

where u – motion control in the absence of the influence of turbulences;

σ – increasing of thrust vector angle required for counteracting of the influence of external turbulences.

Component σ is a combined characteristic of the effects of various turbulences in motion airship control.

In contrast to stochastic models, description of the trajectory airship motion control in the form of differential game does not require a priori information about stochastic models of external turbulence and ensures the achievement of given terminal conditions in the worst case of turbulence factors combination.

Assume, that a control component u_i has first player, and additional control component σ (18) is determined by effects of turbulence factors to be considered from the standpoint of second player who has the opposite control aims.

Program strategies syntheses of both players perform in the class of analytic functions [1]:

$$u_i = e_0 + e_1 t, \tag{19}$$

$$\sigma_i = b_0 + b_1 t, \tag{20}$$

where e_0, e_1, b_0, b_1 - parameters to be determined.

Then the scheduling of thrust vector angle is written as following

$$\varphi = a_0 + a_1 t, \tag{21}$$

where

$$a_0 = e_0 + b_0, \; a_1 = e_1 + b_1.$$

The task of first player consists in airship translation from given initial state:

$$V_X(0) = V_{X_0}, V_Y(0) = V_{Y_0}, H(0) = H_0$$

to given terminal:

$$H(T_i) = H_{T_i}, \; V_Y(T_i) = V_{Y_{T_i}}, \; V_X(T_i) = V_{X_{T_i}} \tag{22}$$

with a minimum value of the quality functional, provided to maximize its second player that mimics the effect of the most unfavorable combination of turbulence factors. Introduced symbols correspond to the symbols in the spectral model of trajectory airship motion [8].

We will consider the most important airship motion stages - takeoff and landing.

Let us introduce for consideration the functional in the form:

$$I_i = \frac{\lambda_1}{2}\left[H_{T_i} - H(T_i)\right]^2 + \frac{\lambda_2}{2}\left[V_{Y_{T_i}} - V_Y(T_i)\right]^2 + + \frac{\lambda_3}{2}\int_0^T \left(u_i^2 - \nu\sigma_i^2\right)dt, \tag{23}$$

where $\lambda_1, \lambda_2, \lambda_3$ and ν – additional weight coefficients.

The first two expressions (23) are characterizing the terminal errors in altitude and vertical speed component.

Integral component of the functional (23) limits the airship control selection and influence of turbulence factors in trajectory motion control.

For determination of guaranteed-and-adaptive control algorithm, calculated discretes of differential spectra of variable of airship trajectory motion [8] express as a function of the initial values of the variables of the mathematical model (1), free parameters a_0, a_1 of thrust vector angle control φ of propulsion system that is predicted, and the duration T_i i-segment of airship motion control.

Using the property of differential transformations (8), obtain an equation linking the control parameters (a_0, a_1) and trajectory airship motion parameters with given at the end of each selment altitude H_{T_i}, vertical speed $V_{Y_{T_i}}$ and flying speed $V_{X_{T_i}}$, respectively:

$$H(T_i)=H_0+T_iV_{Y_0}+\frac{T_i^2}{2}\left[\tilde{\Phi}_i+\tilde{P}_{\Sigma_i}\sin a_0\right]+$$
$$+T_i^3\begin{bmatrix}\frac{1}{6}(C_6+C_7)V_{Y_0}\bar{P}_{\Sigma_i}\cos a_0- \\ -\frac{1}{6}(C_6+C_7)V_{X_0}\tilde{P}_{\Sigma_i}\sin a_0- \\ -\frac{1}{6}(C_6+C_7)V_{X_0}\tilde{\Phi}_i- \\ -\frac{1}{6}(C_6+C_7)V_{Y_0}C_4V_0^2+ \\ +\frac{1}{6}\tilde{P}_{\Sigma_i}\cdot a_1\cdot\cos a_0\end{bmatrix}=H_{T_i}. \qquad (24)$$

$$V_Y(T_i)=V_{Y_0}+T_i\left[\tilde{\Phi}_i+\tilde{P}_{\Sigma_i}\sin a_0\right]+$$
$$+T_i^2\begin{Bmatrix}\frac{1}{2}(C_6+C_7)V_{Y_0}\bar{P}_{\Sigma_i}\cos a_0- \\ -\frac{1}{2}(C_6+C_7)V_{X_0}\tilde{P}_{\Sigma_i}\sin a_0- \\ -\frac{1}{2}(C_6+C_7)V_{X_0}\tilde{\Phi}_i- \\ -\frac{1}{2}(C_6+C_7)V_{Y_0}C_4V_0^2+ \\ +\frac{1}{2}\tilde{P}_{\Sigma_i}\cdot a_1\cdot\cos a_0\end{Bmatrix}=V_{Y_{T_i}}. \qquad (25)$$

$$V_X(T_i)=V_{X_0}+T_i\left[\bar{P}_{\Sigma_i}\cos a_0-C_4V_0^2\right]+$$
$$+T_i^2\begin{bmatrix}-\left(\frac{C_5}{2}V_{Y_0}+C_4V_{X_0}\right)\bar{P}_{\Sigma_i}\cos a_0+ \\ +\left(\frac{C_5}{2}V_{X_0}-C_4V_{Y_0}\right)\tilde{P}_{\Sigma_i}\sin a_0+ \\ +\left(\frac{C_5}{2}V_{X_0}-C_4V_{Y_0}\right)\tilde{\Phi}_i+ \\ +\left(\frac{C_5}{2}C_4V_{Y_0}+C_4^2V_{X_0}\right)V_0^2- \\ -\frac{1}{2}\bar{P}_{\Sigma_i}\cdot a_1\cdot\sin a_0\end{bmatrix}=V_{X_i}. \qquad (26)$$

Differential transformation of expression (21) gives the differential spectrum of thrust vector angle as:

$$\varphi(k)=(e_0+b_0)\mathrm{ъ}(k)+(e_1+b_1)T_i\mathrm{ъ}(k-1).$$

Substituting the differential spectrum of functions (19) and (20) into expression (23), which applied differential conversion, can provide the functional (23) in the form

$$I_i=\frac{\lambda_1}{2}\left[H_{T_i}-H(T_i)\right]^2+\frac{\lambda_2}{2}\left[V_{Y_{T_i}}-V_Y(T_i)\right]^2+$$
$$+\frac{\lambda_3T_i}{2}\begin{bmatrix}e_1^2-\upsilon b_0^2+T(e_0e_1-\upsilon b_0b_1)+ \\ +\frac{T_i^2}{3}(e_1^2-\upsilon b_1^2)\end{bmatrix}. \qquad (27)$$

Differentiating equation (27) for the free control parameters e_0, e_1, b_0, b_1, we find the derivatives that are required under the terms of the extremum of function (27) are equal to zero:

$$\frac{\partial I_i}{\partial e_0}=-\lambda_1\left[H_{T_i}-H(T_i)\right]\cdot\frac{\partial H(T_i)}{\partial e_0}-$$
$$-\lambda_2\left[V_{Y_{T_i}}-V_Y(T_i)\right]\cdot\frac{\partial V_Y(T_i)}{\partial e_0}+ \qquad (28)$$
$$+\frac{\lambda_3T_i^2}{2}(2e_0+e_1T_i),$$

$$\frac{\partial I_i}{\partial b_0}=-\lambda_1\left[H_{T_i}-H(T_i)\right]\cdot\frac{\partial H(T_i)}{\partial b_0}-$$
$$-\lambda_2\left[V_{Y_{T_i}}-V_Y(T_i)\right]\cdot\frac{\partial V_Y(T_i)}{\partial b_0}- \qquad (29)$$
$$-\frac{\lambda_3T_i^2}{2}(2b_0+b_1T_i),$$

$$\frac{\partial I_i}{\partial e_1}=-\lambda_1\left[H_{T_i}-H(T_i)\right]\cdot\frac{\partial H(T_i)}{\partial e_1}-$$
$$-\lambda_2\left[V_{Y_{T_i}}-V_Y(T_i)\right]\cdot\frac{\partial V_Y(T_i)}{\partial e_1}+ \qquad (30)$$
$$+\frac{\lambda_3T_i^2}{2}\left(e_0+\frac{2}{3}e_1T_i\right),$$

$$\frac{\partial I_i}{\partial b_1}=-\lambda_1\left[H_{T_i}-H(T_i)\right]\cdot\frac{\partial H(T_i)}{\partial b_1}-$$
$$-\lambda_2\left[V_{Y_{T_i}}-V_Y(T_i)\right]\cdot\frac{\partial V_Y(T_i)}{\partial b_1}- \qquad (31)$$
$$-\frac{\lambda_3T_i^2}{2}\left(b_0+\frac{2}{3}b_1T_i\right).$$

Analysis of the expression (24), (25) with taking into account the expression (21) suggests making conclusion:

$$\frac{\partial H(T_i)}{\partial e_0}=\frac{\partial H(T_i)}{\partial b_0};\ \frac{\partial V_Y(T_i)}{\partial e_0}=\frac{\partial V_Y(T_i)}{\partial b_0}; \qquad (32)$$

$$\frac{\partial H(T_i)}{\partial e_1}=\frac{\partial H(T_i)}{\partial b_1};\ \frac{\partial V_Y(T_i)}{\partial e_1}=\frac{\partial V_Y(T_i)}{\partial b_1}. \qquad (33)$$

The difference of expressions (28) and (29) at the performance of ratios (32) gives the equation for the determination of free parameters:

$$2(e_0+\nu_ib_0)+T_i(e_1+\nu_ib_1)=0. \qquad (34)$$

Subtracting (31) from expression (30) and taking into account the relation (33), obtain the second equation:

$$\left(e_0+\nu_i b_0\right)+\frac{2}{3}T_i\left(e_1+\nu_i b_1\right)=0. \qquad (35)$$

Solving the equation system (34) and (35) to set the ratio between the optimal values of the control parameters and turbulences [5]:

$$e_0=-\nu_i b_0, \qquad (36)$$

$$e_1=-\nu_i b_1. \qquad (37)$$

From an analysis of expressions (28) - (31) follows that the second derivatives of the function (27) with respect to parameters e_0 and e_1 are positive, and with respect to parameters b_0 and b_1 are negative at zero terminal errors on altitude and vertical speed.

This confirms the existence of the saddle point of the function (27) with the exact airship translation to the given terminal conditions.

Expression (36) and (37) between the control parameters and turbulences allows to reduce the problem of control synthesis to determination parameters e_0, e_1, and the duration T_i of *i*-segment of airship takeoff / landing, which can be found from equation (26) obtained from the boundary condition (22) taking into account the expression (21).

Thus, for determination of three unknowns, e_0, e_1 and the duration T_i of *i*-segment of airship takeoff / landing there are three equations (28) and (30) and (26).

From these equations exclude on the basis of expressions (36) and (37) turbulence parameters b_0 and b_1.

From equation (30) explicitly define the control parameter e_1

$$e_1=-\frac{1}{T_i}\cdot\frac{\lambda_1 N_3 N_4+\lambda_2 M_3 M_4+\dfrac{\lambda_3}{2}\varphi_i}{rT_i\left(\lambda_1 N_4^2+\lambda_2 M_4^2\right)+\dfrac{\lambda_3}{3}},$$

where

$$N_4=\frac{T_i}{6}\cdot\tilde{P}_{\Sigma_i}\cos r_i\varphi_i;$$

$$M_4=\frac{1}{2}\cdot\tilde{P}_{\Sigma_i}\cos r_i\varphi_i;$$

$$N_3=H_0-H_{T_i}+T_iV_{Y_0}+\frac{T_i^2}{2}\left[\tilde{\Phi}_i+\tilde{P}_{\Sigma_i}\sin r_i\varphi_i\right]+$$

$$+T_i^3\begin{bmatrix}\frac{1}{6}(C_6+C_7)V_{Y_0}\bar{P}_{\Sigma_i}\cos r_i\varphi_i-\\ -\frac{1}{6}(C_6+C_7)V_{X_0}\tilde{P}_{\Sigma_i}\sin r_i\varphi_i-\\ -\frac{1}{6}(C_6+C_7)V_{X_0}\tilde{\Phi}_i-\frac{1}{6}(C_6+C_7)V_{Y_0}C_4V_0^2\end{bmatrix},$$

$$M_3=V_{Y_0}-V_{Y_{T_i}}+T_i\left[\tilde{\Phi}_i+\tilde{P}_{\Sigma_i}\sin r_i\varphi_i\right]+$$

$$+T_i^2\begin{bmatrix}\frac{1}{2}(C_6+C_7)V_{Y_0}\bar{P}_{\Sigma_i}\cos r_i\varphi_i-\\ -\frac{1}{2}(C_6+C_7)V_{X_0}\tilde{P}_{\Sigma_i}\sin r_i\varphi_i-\\ -\frac{1}{2}(C_6+C_7)V_{X_0}\tilde{\Phi}_i-\frac{1}{2}(C_6+C_7)V_{Y_0}C_4V_0^2\end{bmatrix},$$

$$r_i=\frac{\upsilon_i-1}{\upsilon_i}.$$

Eliminating the parameter e_1 from expressions (28) and (26) and substituting arbitrary initial conditions by current values of variables of trajectory motion and control parameter e_0 by current value of thrust vector angle φ_i, receive the guaranteed-and- adaptive algorithm of airship motion control in the following form:

$$\varphi_i=\frac{T_i^2}{\lambda_3}\begin{bmatrix}\dfrac{\lambda_1\delta_1\left(H_{T_i}-H_0\right)}{T_i^2}+\dfrac{\lambda_2\delta_2V_{Y_{T_i}}}{T_i^2}-\\ -\dfrac{V_{Y_0}}{T_i}\left(\lambda_1\delta_1+\dfrac{\lambda_2\delta_2}{T_i}\right)-A_i\left(\dfrac{\lambda_1\delta_1}{2}+\dfrac{\lambda_2\delta_2}{T_i}\right)+\\ +\tilde{\Phi}_i\begin{pmatrix}\dfrac{\lambda_1\delta_1}{2}-\dfrac{\lambda_2\delta_2}{T_i}-\\ -\dfrac{B_i}{2}\left[\lambda_1\delta_1\dfrac{T_i}{3}+\lambda_2\delta_2\right]\end{pmatrix}-\\ -d_1\dfrac{T_i}{2}\left(\dfrac{\lambda_1\delta_1}{3}+\dfrac{\lambda_2\delta_2}{T_i}\right)\end{bmatrix}-\frac{T_i}{2}e_1, \qquad (38)$$

where

$$A_i=\tilde{P}_{\Sigma_i}\sin r\varphi_i;$$

$$B_i=\left(C_6+C_7\right)V_{X_0};$$

$$d_1 = (C_6 + C_7)V_{X_0}\tilde{P}_{\Sigma_i}\sin r\varphi_i -$$
$$-\left[(C_6 + C_7)V_{Y_0}\overline{P}_{\Sigma_i} + r\cdot a_1\cdot\tilde{P}_{\Sigma_i}\right]\cdot\cos r\varphi_i +$$
$$+(C_6 + C_7)V_{Y_0}C_4V_0^2;$$

$$d_2 = -(C_6 + C_7)V_{X_0}\tilde{P}_{\Sigma_i}\cos r\varphi_i -$$
$$-\left[(C_6 + C_7)V_{Y_0}\overline{P}_{\Sigma_i} + r\cdot a_1\cdot\tilde{P}_{\Sigma_i}\right]\cdot\sin r\varphi_i;$$

$$\delta_1 = \frac{T_i}{2}\left(\tilde{P}_{\Sigma_i}\cos r\varphi_i + \frac{T_i}{3}d_2\right);$$

$$\delta_2 = \tilde{P}_{\Sigma_i}\cos r\varphi_i + \frac{T_i}{2}d_2.$$

Duration T_i of i-segment of airship takeoff / landing with taking into account introduced symbols is determined from equation (26):

$$T_i = \frac{1}{2d_3}\left[-D_i + \sqrt{D_i^2 + 4\left(V_{X_{T_i}} - V_{X_0}\right)d_3}\right],$$

where

$$D_i = \overline{P}_{\Sigma_i}\cos r_i\varphi_i - C_4V_0^2\ ;$$

$$d_3 = -\left(\frac{C_5}{2}V_{Y_0} + C_4V_{X_0}\right)\overline{P}_{\Sigma_i}\cos r_i\varphi_i +$$
$$+\left(\frac{C_5}{2}V_{X_0} - C_4V_{Y_0}\right)\tilde{P}_{\Sigma_i}\sin r_i\varphi_i +$$
$$+\left(\frac{C_5}{2}V_{X_0} - C_4V_{Y_0}\right)\tilde{\Phi}_i +$$
$$+\left(\frac{C_5}{2}C_4V_{Y_0} + C_4^2V_{X_0}\right)V_0^2 - \frac{1}{2}\overline{P}_{\Sigma_i}\cdot r\cdot a_1\cdot\sin r_i\varphi_i$$

Weighting factors $\lambda_1, \lambda_2, \lambda_3$ selection in expression (38) is determined by the requirements to terminal errors in the airship motion control on the stages of takeoff and landing, and also sufficient conditions for an extremum of function (27).

The guaranteed-and-adaptive algorithm (38), synthesized on the differential-gaming model of control process is more complex compared to other algorithms [8].

At the same time the complicated algorithm (38) provides a guarantee of airship translation to the given terminal conditions when an action of turbulences is bounded by functional component (23).

From equations (36), (37) follows that for rejection of worst combination of turbulence factors and airship motion control the level of parameter components for thrust vector angle has to be in ν – time bigger than the impact of turbulence parameters.

Therefore, the guarantee of airship motion control at the worst combination of turbulence factors can be realized only in the presence of control resource, sufficient for rejection of the turbulence action and airship translation to the given terminal conditions.

The feature of synthesized guaranteed-and adaptive algorithm compared to other algorithms is the achievement the smallest terminal errors at action of turbulences on the airship motion control.

Existence of saddle point of the functional (23) reduces terminal errors if in the real process of airship motion control on takeoff and landing stages a combination of turbulence factors are not the worst.

Gains of feedback on disagreement on altitude and vertical speed of the algorithm (38) are variable, depending on the trajectory parameters of the airship motion control on the takeoff and landing stages, airship characteristics, and also from parameter ν_i of limiting the influence of turbulences in the functional component (23).

5. Conclusions

The guaranteed-and-adaptive algorithm of airship thrust vector angle based on differential transformation of mathematical model of differential game is synthesized.

The algorithm not only has the properties of adaptation to the action of turbulences, but also ensures realization of airship translation to the given terminal conditions on takeoff and landing stages at effect of limited turbulences.

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А.В. Гусинін[1], О.М. Тачиніна[2]. Алгоритм гарантовано-адаптивного керування аеростатичним літальним апаратом в умовах дії невизначених зовнішніх збурень
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Синтезовано гарантовано-адаптивний алгоритм керування рухом аеростатичним літальним апаратом на основі диференціальних перетворень математичної моделі диференціальної гри. Показано, що гарантовано-адаптивний алгоритм, синтезований за диференціально-ігровою моделлю процесу керування, є більш складним порівняно з іншими алгоритмами. Визначено, що ускладнений алгоритм забезпечує гарантію переведення дирижабля в задані термінальні умови у випадку дії збурень. Зазначено, що отриманий алгоритм керування володіє властивостями адаптації до дії збурень та забезпечує гарантію переведення аеростатичного літального апарату в задані термінальні умови при найгіршому сполученні дії факторів обмежених збурень. Розглянуто особливість синтезованого гарантовано-адаптивного алгоритму – досягнення найменших термінальних помилок при дії збурень на процес управління рухом дирижабля.

Ключові слова: аеростатичний літальний апарат; алгоритм керування; динаміка руху; диференціальна гра; диференціальні перетворення.

А.В. Гусынин[1], Е.Н. Тачинина[2]. Алгоритм гарантированно-адаптивного управления аэростатическим летательным аппаратом в условиях действия неопределенных внешних возмущений
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Синтезирован гарантировано-адаптивный алгоритм правления движением аэростатическим летательным аппаратом на основе дифференциальных преобразований математической модели дифференциальной игры. Показано, что гарантированно адаптивный алгоритм синтезирован на основе дифференциально-игровой модели процесса управления, является более сложным по сравнению с другими алгоритмами и обеспечивает гарантию перевода дирижабля в заданные терминальные условия в случае действия возмущений. Описано, что полученный алгоритм управления обладает свойствами адаптации к действию возмущений и обеспечивает гарантию перевода аэростатического летательного аппарата в заданные терминальные условия при наихудшем сочетании действия факторов ограниченных возмущений. Рассмотрена особенность синтезированного гарантированно-адаптивного алгоритма – достижение наименьших терминальных ошибок при действии возмущений на процесс управления движением дирижабля.

Ключевые слова: алгоритм управления; аэростатический летательный аппарат; динамика движения; дифференциальная игра; дифференциальные преобразования.

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