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## DESIGN OF ROBUST TWO-AXIS SYSTEMS FOR STABILIZATION AND TRACKING OF INFORMATION-MEASURING DEVICES OPERATED ON GROUND VEHICLES

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**Abstract.** Design features of the information-measuring robust stabilization and tracking systems operated on the ground vehicles are considered. The mathematical description of the control object mounted in the two-axis gimbals is obtained. Design features of the two-axis robust stabilization and tracking systems based on the structural  $H_{\infty}$ -synthesis are researched. The simulation results are given.

**Keywords:**  $H_{\infty}$ -synthesis; ground vehicles; information-measuring devices; robust two-axis stabilization and tracking systems.

### 1. Introduction

Now the stabilization and tracking systems are widely used for control by the position of the information-measuring devices line of sight. It is very difficult problem to keep the stable position of the line of sight in the space if the informationmeasuring device is mounted on the vehicle. Control by the line of sight orientation using the stabilization and tracking systems based on the gyro stabilization principles allows to solve this problem. Such systems combine functions of the gyro stabilization and servo systems. Solution of the stabilization and tracking tasks for the information-measuring devices designed for operation on the ground vehicles is important for many applications. As a rule, for such systems the stabilization and tracking processes are implemented in the horizontal and vertical planes that requires to use the two-axis stabilization and tracking systems. Operation of the informationmeasuring devices on the ground vehicles is characterized by such feature as the necessity to provide the high precision requirements in conditions of the sufficient parametric and intensive external disturbances. The effective implementation of stabilization and tracking in such conditions is possible by means of the robust control.

Unconditionally, the basic motivation for improvement of performances of the stabilization and tracking systems operated on the vehicles is the real necessity in such measures. It is known that the optical sensors precision performances and their resolution have been drastically increased last years [1]. These best trends may not be realized without appropriate progress in the stabilization and tracking means. Such situation causes the necessity to improve the precision and dynamic performances of

the stabilization and tracking system in the difficult conditions of the real operation. One of the most important problems is providing of such systems stability to the internal parametric and external coordinate disturbances. The modern approaches to design of the studied systems lie in use of the robust control. This requires adaptation of the robust synthesis known methods taking in account features of the control objects of the studied class.

#### 2. Analysis of the last researches and publications

Many papers and textbooks, for example [5, 10], deal with the design of the robust systems of the wide class. Design features of the information-measuring devices stabilization and tracking systems operated on the ground vehicles are represented in the papers [7, 8]. But in these papers the systems realizing stabilization in the horizontal and vertical planes separately are represented only.

The goal of the represented paper is research of design features of the robust stabilization and tracking systems, which provide control by the processes of stabilization and tracking relative two axes. In such system two one-axis systems with the same principle of operation are combined in the single system [4]. The two-axis system platform has two degrees of freedom relative to the vehicle (in the horizontal and vertical planes). Rotation of the platform relative to the third axis is implemented together with the vehicle.

Design of the robust systems may be based on the robust parametric optimization or the robust structural synthesis. For the parametric optimization the controller structure is believed to be known based on experience of development of the previous systems of the studied class [2, 7].

It should be noted that present controllers structures for the systems of the studied type are sufficiently complex. Moreover, the control system for one channel has very high order that complicates creation of the mathematical description for the system consisting of the plant and controller taking into consideration interconnections between the horizontal and vertical channels. The robust parametric optimization is sufficiently complicated due to such conditions. In the case of the robust structural synthesis the control object model taking into consideration interconnections between the horizontal and vertical channels although is complex but nevertheless allows to avoid calculating difficulties during structural synthesis procedure execution.

Algorithms for the structural synthesis of the systems for stabilization and tracking of the information-measuring devices operated on the ground vehicles are represented in the papers [6, 8] with sufficient degree of completeness. Therefore in the presented paper only new problems reflected the specific character taking into account interaction between the horizontal and vertical channels of the system are considered. First of all, this covers the creation of the mathematical description of the control object mounted in the two-axis gimbals. Some features of augmented object forming take place too.

# 3. The mathematical description of the stabilization and tracking object mounted in the two-axis gimbals

The studied stabilization and tracking system provides control by the space orientation of the measurement and observation equipment mounted in the two-axis gimbals. The gimbals consist of the platform representing the internal gimbal and the external gimbal suspended in the ball-bearings. The information-measuring equipment and gyro sensors are mounted on the platform. These sensors measure the absolute angular rate of the information-measuring device lines if sight.

In the general case, the system dynamic may be described by the Euler equations [3]

$$\dot{\omega}_{x}J_{x} + \omega_{y}\omega_{z}(J_{z} - J_{y}) - (\omega_{y}^{2} - \omega_{z}^{2})J_{yz} -$$

$$-(\omega_{x}\omega_{y} + \dot{\omega}_{z})J_{xz} + (\omega_{x}\omega_{z} - \dot{\omega}_{y})J_{xy} = M_{x};$$

$$\dot{\omega}_{y}J_{y} + \omega_{x}\omega_{z}(J_{x} - J_{z}) - (\omega_{z}^{2} - \omega_{x}^{2})J_{xz} -$$

$$-(\omega_{z}\omega_{y} + \dot{\omega}_{x})J_{xy} + (\omega_{x}\omega_{y} - \dot{\omega}_{z})J_{yz} = M_{y};$$

$$\dot{\omega}_{z}J_{z} + \omega_{x}\omega_{y}(J_{y} - J_{x}) - (\omega_{x}^{2} - \omega_{y}^{2})J_{xy} -$$

$$-(\omega_{x}\omega_{z} + \dot{\omega}_{y})J_{yz} + (\omega_{x}\omega_{z} - \dot{\omega}_{x})J_{xz} = M_{z},$$

$$(1)$$

where  $\omega_x$ ,  $\omega_y$ ,  $\omega_z$  – projections of the platform angular rates onto its own axes;

 $J_x, J_y, J_z$  – the inertia moments of the platform with useful payload mounted on it relative to the gimbals axes;

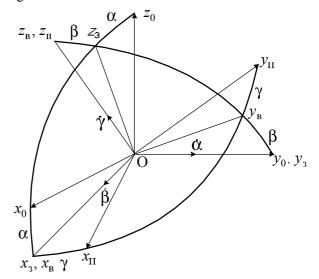
 $J_{yz}, J_{xz}, J_{xy}$  – the centrifugal inertia moments relative to the gimbals axes;

 $\dot{\omega}_x$ ,  $\dot{\omega}_y$ ,  $\dot{\omega}_z$  – projections of the platform angular accelerations onto its own axes;

 $M_{\scriptscriptstyle X}, M_{\scriptscriptstyle Y}, M_{\scriptscriptstyle Z}$  — the moments acting by the gimbals axes.

It should be noted that use of the two-axis gimbals is the most acceptable for many applications, as two mutually-orthogonal axes represent the minimum quantity necessary for determination of the direction in the three-dimensional space [1].

The mutual position of the coordinate axes necessary for representation of kinematic of the stabilization and tracking system is represented in Fig. 1.



**Fig. 1.** Mutual position of the coordinate system connected with the vehicle and the platform

The represented coordinate systems may be characterized in the following way. The coordinate system  $Ox_0y_0z_0$  is connected with the ground vehicle. The coordinate system  $Ox_1y_1z_1$  is connected with the external gimbal. The coordinate system  $Ox_2y_2z_2$  is connected with the internal gimbal (platform). The platform motion relative to the axis z is not stabilized and the platform is deviated from this axis together with the vehicle.

In accordance with the Fig. 1 the angular rate projections of the platform with mounted on it useful payload onto its own axes may be represented in the following form:

$$\omega_x = \dot{\alpha}\cos\beta; 
\omega_y = \dot{\beta}; 
\omega_z = \dot{\alpha}\sin\beta.$$
(2)

Based on the relationships (2) the Euler kinematic equations, which correspond to the sequence of angles represented in the Fig. 1, become:

$$\dot{\alpha} = \omega_x \cos \beta + \omega_z \sin \beta ;$$

$$\dot{\beta} = \omega_y . \tag{3}$$

The mathematical model of the control object of the system for stabilization and tracking of the information-measuring devices operated on vehicles depends essentially on the type of the vehicle. In the represented paper the mathematical model assigned for operation on the ground vehicles is considered.

Based on the expressions (1)-(3)mathematical models of the one-axis system for stabilization and tracking of the information-measuring devices operated on the ground vehicles and represented in the paper [9], the mathematical model of the two-axis gyro stabilization and tracking system may be obtained. It should be noted that for creation of the system control object model it is expedient to accept some simplifications for the set of equations (1), namely, to neglect by the centrifugal inertia moments. Taking into account this simplification the mathematical model of the system providing stabilization and control by the lines of sight of the informationmeasuring equipment operated on the ground vehicles in the horizontal and vertical planes becomes:

$$\begin{split} \dot{\alpha} &= \omega_x \cos \beta + \omega_z \sin \beta \,; \\ \dot{\beta} &= \omega_y \,; \\ \dot{\alpha}_e &= \omega_{e\alpha} \,; \\ \dot{\beta}_e &= \omega_{e\beta} \,; \\ \dot{U}_{\omega\alpha} &= U_{\omega d\alpha} \,; \\ \dot{U}_{\omega\beta} &= U_{\omega d\beta} \,; \\ \dot{\omega}_x &= [-(J_z - J_y)\omega_y \omega_z - M_{fix} \mathrm{sign}\omega_x - \\ &- M_{unbx} \cos \alpha + c_r (\alpha_g - \alpha)/n_r]/J_x \,; \\ \dot{\omega}_y &= [-(J_y - J_x)\omega_x \omega_z - M_{fiy} \mathrm{sign}\omega_y - \\ &- M_{unby} \cos \beta + k_{spr} (A - \beta) + \frac{c_r (\beta_g - \beta)}{n_x}]/J_y \,; \end{split}$$

$$\begin{split} \dot{\omega}_{e\alpha} &= \left[ -M_{fre} \mathrm{sign} \omega_{e\alpha} + \frac{c_m}{R_w} U_\alpha + \frac{c_r (\alpha_g - \alpha)}{n_r} \right] / J_e \,; \\ \dot{\omega}_{e\beta} &= \left[ -M_{fre} \mathrm{sign} \omega_{e\beta} + \frac{c_m}{R_w} U_\beta + \frac{c_r (\beta_g - \beta)}{n_r} \right] / J_e \,; \\ \dot{U}_\alpha &= \left[ -U_\alpha + k_{PWM} U_{PWM\alpha} - c_{ed} \omega_{e\alpha} \right] / T_{arm} \,; \\ \dot{U}_\beta &= \left[ -U_\beta + k_{PWM} U_{PWM\beta} - c_{ed} \omega_{e\beta} \right] / T_{arm} \,; \\ \dot{U}_{\alpha d\alpha} &= \left[ -2 \nu T_0 U_{\omega e\alpha} - U_{\omega\alpha} + k_{ars} \omega_x \right] / T_0^2 \,; \\ \dot{U}_{\omega d\beta} &= \left[ -2 \nu T_0 U_{\omega e\beta} - U_{\omega\beta} + k_{ars} \omega_y \right] / T_0^2 \,, \end{split}$$

where  $\alpha$ ,  $\beta$  – the turn angles of the platform with the useful payload installed on it;

 $\omega_x$ ,  $\omega_y$  – the platform angular rates in the horizontal and vertical planes correspondingly;

 $\omega_{e\alpha}$ ,  $\omega_{e\beta}$  – the rates of the engines mounted by the axes x, y correspondingly;

 $\alpha_e$ ,  $\beta_e$  – the turn angles of the engines mounted by the axes x, y;

 $U_{\omega\alpha}$ ,  $U_{\omega\beta}$  — the output signals of the angular rate signals measuring the absolute angular rates of the platform with mounted on it useful payload by the axes x, y;

 $U_{\omega d\alpha}$ ,  $U_{\omega d\beta}$  – the derivatives of the sensor output signals;

 $J_x$ ,  $J_y$ ,  $J_z$  — the inertia moments of the platform with mounted on it useful payload relative its own axes x, y, z;

 $M_{\it fix}$ ,  $M_{\it fiy}$  – the nominal dry friction moments acting by the gimbals axes x, y;

 $M_{unbx}$ ,  $M_{unby}$  — the unbalanced moments by the axes x, y;

 $k_{spr}$  - the rigidity coefficient of the spring compensator;

A — the initial angle of spring resetting;

 $c_{r}$  – the reducer rigidity;

 $\alpha_g$ ,  $\beta_g$  – the turn angles of the platform taking into account presence of the drive gap;

 $M_{frex}$ ,  $M_{frey}$  – the nominal dry friction moments of engines installed at the gimbals axes x, y;

 $c_m$  - the constant of the load moment at the engine shaft;

 $R_{\rm w}$  – the resistance of the engine armature winding;

 $U_{\alpha}$ ,  $U_{\beta}$  – the armature voltages of engines mounted by the gimbals axes;

 $n_r$  – the reducer gear ratio;

 $T_{arm}$  — the time constant of the engine armature circuit:

 $k_{PWM}$  – the transfer constant of the linearized pulse width modulator;

 $U_{PWM}$  – the voltage at the pulse width modulator input;

 $c_{ed}$  – the coefficient of proportionality between the engine angular rate and the electromotive force;

v – the relative damping coefficient;

 $T_0$  – the time constant of the angular rate sensor;

 $k_{ars}$  — the transfer constant of the angular rate sensor

In the represented non-linear equations (4) the angles  $\alpha_g$ ,  $\beta_g$  may be defined in accordance with the expressions

$$\alpha_g = \alpha_e / n_p$$
, if  $|\alpha_e / n_p - \alpha| \ge 0.5\Delta$ ;  
 $\alpha_g = \alpha$ , if  $|\alpha_e / n_p - \alpha| < 0.5\Delta$ ;

$$\beta_g = \beta_e / n_p, \text{ if } |\beta_e / n_p - \beta| \ge 0.5\Delta;$$
  
$$\beta_g = \beta, \text{ if } |\beta_e / n_p - \beta| < 0.5\Delta,$$

where  $\Delta$  – the value of the experimentally determined system drive gap.

For further researches it is necessary to implement linearization of the equations (4) relative to the nominal values of the phase coordinates. Such linearization must include the following stages:

- 1) linearization of the expressions for the friction and unbalanced moments of the engine and stabilization object;
- 2) neglect by the drive gap and the friction moments at the bearings of the gimbals and at the engine shaft;
  - 3) neglect by the error of the angular rate sensor;
- 4) assumption of smallness of the platform turn angles for linearization of the trigonometric functions.

After these actions the set of equations (4) may be transformed to the linearized form and represented in the space of states for the vectors of states by four matrices: **A**, **B**, **C**, **D**.

here  $f_{\alpha}$ ,  $f_{\beta}$  are the linearized coefficients of the friction moments.

# 4. The algorithm of the structural synthesis for the robust stabilization and tracking system

One of the modern approaches to the structural synthesis of the robust stabilization and tracking system is the  $H_{\infty}$ -synthesis. Basic stages of such synthesis implementation are represented in many papers and textbooks [5, 10].

The robust structural synthesis is based on solutions of two Riccati equations, check of some conditions and minimization of the mixed sensitivity function  $H_{\infty}$ -norm for the system including the control object and controller and represented by the vector of outputs characterized the system quality, vector of inputs and vector of controls and measured signals used for feedback implementation [5, 10].

The modern approach to the structural  $H_{\infty}$ -synthesis problem solution is based on forming of the desired frequency characteristics of the system. This is implemented by means of forming of the augmented object due to introduction of the weighting transfer functions. Such approach is called loop-shaping [5, 10]. In this case, the optimization criterion represents the  $H_{\infty}$ -norm of the mixed sensitivity function of the augmented system

$$J_{H_{\infty} s} = \begin{bmatrix} W_1 S \\ W_2 R \\ W_3 T \end{bmatrix}, \tag{6}$$

where  $W_1, W_2, W_3$  – the weighting transfer functions; S, R, T – the sensitivity functions by the command signal, control and the complementary sensitivity function.

It is known that the sensitivity function is a measure of the system accuracy [5, 10]. The complementary function is a measure of the system

robustness. The sensitivity function by control allows to estimate expenses by control. So, introduction of these functions in the optimization criterion (6) allows to solve the multi-objective design problem.

In the practical situations during design of systems for stabilization and tracking of the information-measuring devices it is expedient to carry out the research of the suboptimal robust controller, for which the  $H_{\infty}$ -norm of the mixed sensitivity function of the system including the augmented object must not exceed some minimally possible number  $\gamma$  [5, 10].

$$J_{H_{\infty}} = \begin{bmatrix} W_1 S \\ W_2 R \\ W_3 T \end{bmatrix}_{\infty} < \gamma.$$

The robust structural synthesis is implemented with use of the control object mathematical model in the space of states, described by the four matrices (5).

Check of the synthesis results has been carried out by means of the non-linear model (4). The simulation results taking into account the system parametric disturbances are represented in the Fig. 2.

One of the robust structural synthesis important stages is choice of the weighting transfer functions for the control object augmentation. Such choice is implemented on the basis of the heuristic approaches. For the system of the studied type the expressions for the weighting transfer functions may be represented in the following form:

$$\mathbf{W}_{1} = \begin{bmatrix} \frac{1}{s^{2}} & 0\\ 0 & \frac{1}{s^{2}} \end{bmatrix}; \mathbf{W}_{2} = \begin{bmatrix} 0.04 & 0\\ 0 & 0.04 \end{bmatrix};$$
$$\mathbf{W}_{3} = \begin{bmatrix} 10 \frac{s}{s+50} & 0\\ 0 & 20 \frac{s}{s+200} \end{bmatrix}.$$

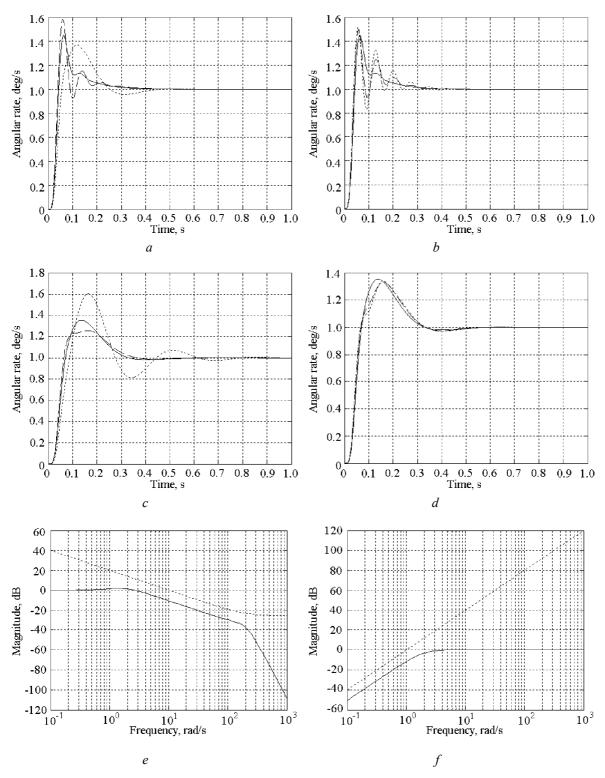


Fig. 2. The simulation results: a – angular rates of the horizontal channel with the changed inertia moment; b – angular rates of the horizontal channel with the changed coefficient of rigidity between the actuator and plant; c – angular rates of the vertical channel with the changed inertia moment; d – angular rates of the vertical channel with the changed coefficient of rigidity between the actuator and plant; d – angular rates of the vertical channel with the changed coefficient of rigidity between the actuator and plant;

e – the complementary sensitivity function (solid) bounded by the weighting transfer function  $1/W_3$  (dashed);

f – the sensitivity function (solid) bounded by the weighting transfer function  $1/W_1$  (dashed);

a, b, c, d: solid, dashed and dotted lines are used for the nominal, increased and decreased parameters

For approbation of the suggested approaches to design of the robust system such parametric disturbances as the plant inertia moment and the rigidity between the actuator and the moving base of the plant were chosen. These parameters change during operation and may achieve  $\pm 50\%$ .

Analysis of the simulation results proves the acceptable accuracy of the tracking processes in conditions of the considerable parametric changes corresponding to the difficult conditions of the real operation.

#### 5. Conclusions

The basic approaches to the robust structural synthesis of the two-axis system for stabilization and tracking of the information-measuring devices operated on the ground vehicles are represented. The mathematical description of the plant mounted in the two-axis gimbals is obtained.

The weighting transfer functions providing plant augmentation are chosen. The robust structural synthesis of the system of the studied type was carried out.

The efficiency of the suggested approaches is proved by the simulation results.

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## О.А. Сущенко. Проектування робастних двовісних систем стабілізації та наведення інформаційновимірювальних пристроїв, експлуатованих на наземних рухомих об'єктах

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Наведено математичний опис об'єкта управління, встановлюваного в двовісному кардановому підвісі. Визначено підходи до проектування систем досліджуваного типу на підставі структурного  $H_{\infty}$ -синтезу. Описано вирази для вагових передавальних функцій та результати моделювання синтезованої системи.

**Ключові слова:** інформаційно-вимірювальні пристрої; наземні рухомі об'єкти; робастні двовісні системи стабілізації та наведення;  $H_{\infty}$ -синтез.

### О.А. Сущенко. Проектирование робастных двухосных систем стабилизации и наведения информационноизмерительных устройств, эксплуатируемых на наземных подвижных объектах

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Приведено математическое описание объекта управления, установленного в двухосном кардановом подвесе. Определены подходы к проектированию систем исследуемого типа на основании структурного  $H_{\infty}$ -синтеза, представлены выражения для весовых передаточных функций и результаты моделирования синтезированной системы. **Ключевые слова:** информационно-измерительные устройства; наземные подвижные объекты; робастные двухосные системы стабилизации и наведения;  $H_{\infty}$ -синтез.

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