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THE BASIC PRINCIPLES OF FORECASTING OF AVIATION SYSTEMS OPTIMAL DEVELOPMENT

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Abstract. *The article deals with the general mathematical approaches and basic information about the forecasting of modern aircraft systems development. In particular, it considers the dynamic model of determining the optimal characteristics of the aviation system, which are intended for evaluation of reliability and development of the forecasting technology of the aircraft modernization possibility. Dynamic models of optimal development of aviation systems forecasting also allows to evaluate the conditions of these aviation systems' competitiveness and to be defined with the program for their implementation in the transportation system of a particular state.*

Keywords: aviation system; forecasting; mathematical model; transport system.

1. Introduction

Dynamic models are divided into variational and gaming, direct and inverse ones [Smith 1991]. In its turn, these models are divided into a number of variations. Variant of the model is determined by the view of the forecasting Aviation Systems (AS) and the aims of their development. In the general case a few variants for setting the variational problems of the AS forecasting can be considered. First of all the variational direct tasks of the AS development forecasting will be considered.

General formulation of the task. Suppose in some moment of time $t = 0$, when it is necessary to make the forecast of the AS development on the interval of the future time, the following is known:

1. The purpose and objectives of the AS, the main $\boldsymbol{E_0}^{\mathbf{c}}$ and support E_v^c, $v = \overline{1, N}$, effectiveness criteria and their scalar composition $\boldsymbol{E}^{\mathbf{c}}$.

2. Tactical-technical and economic structure of the existing and prospective AS$_i$ (Π_i, E_i, C_i, C_i^e – respectively characteristics, the criterion of combat effectiveness, the industrial value of the creation and the annual cost of operation (the intensity of the operational costs)), of which on the interval $0 \le t \le T$ the AS can be formed. The criterion of effectiveness E_i of each aircraft (AS), as the criterion of the aviation system effectiveness $\boldsymbol{E}^{\mathbf{c}}$, can be a scalar convolution of private criteria.

3. The number of the serial AS

$$x^0 = \{x_i^0\},\ i = \overline{1, n^0} \text{ at } t_0 = 0.$$

The number of new AS$_i$ $\left(i = \overline{n^0 + 1, n}\right)$, which can be implemented in the transport system,

$$x^0 = \{x_i^0\} = 0 \text{ at } t_0 = t_{1_i},$$

where t_{1_i} – the time of the development of the i-th AS$_i$.

The definition of t_1 of the promising AS is an important task. The analytical formulae for calculating t_1 in functions of the AS parameters, like the formulae determining the cost of the future AS according to its parameters, were developed. It should be emphasized that the development of new AS has been growing continuously.

The main reason of this growth is due to the complexity of the radio-electronic equipment and aircraft engines development.

4. Integral assignations for the creation and operation of AS:

$C^c(t) = C_1^c(t),\ 0 \le t \le T.$

In static models specified assignations for the development of AS only at the point T are known: $C_1^c(t)$.

5. Tactical-technical characteristics of AS$_j$, $j = \overline{1, m}$, and the development trajectory of $y(t) = \{y_j(t)\}$ AS$_Y$ competitive side «Y», opposed to the predicted AS$_X$ = AS of side «X». It is required to determine the optimal trajectory of AS$_X$ development

$$x_{\text{opt}} = \left\{x_{i_{\text{opt}}}(t)\right\},\ i = \overline{1, n},$$

on the interval $0 \le t \le T$.

The second task to be solved in forecasting of the AS development is the formulation of variational direct tasks of the AS development. There can be two models.

2. Problem solving

The first model. The purpose of this model is to get AS with maximum efficiency at the period $t = T$ – the beginning of the AS application. Functionality is the criterion of the AS efficiency at the point $t = T$: $E^c(T)$.

Formulation of the task. To define the optimal trajectory (forecast) of the AS development

$$x_{opt}(t) = \{x_{i_{opt}}(t)\},\ 0 \le t \le T,$$

ensuring the maximum value of the system effectiveness criteria when $t = T$:

$$E^c_{\max} = \max_{x \in X} E^c(x(T), y(T))$$

on condition that the quantity vector of AS_i $x(t) = \{x_i(t)\}$ is subordinate to the restrictions (the determination of the variable $x(t)$) X. In this case, the differential equations of the AS development is

$$\frac{dx}{dy} = \dot{x} = Q(x, u),$$

where $u(t) = \{u_i(t)\}$ is the vector of development control of the AS; $x(t) \ge 0$; $g(x(t)) \le 0$.

It should be noted that, if the scalar composition of the AS efficiency criteria $E^c\left(E^c_0, E^c_v\right)$, is not optimized, and the main E^c_0 characterizing the effectiveness of the implementation system of the main tasks set before it, then the set of restrictions $g(x(t))$ includes, in particular, restrictions on auxiliary efficiency criteria.

$$E^c_v(t) \ge E^c_{1v}(t),\ v = \overline{1, N}.$$

The maximum value of the criterion $E^c_{\max}(x_{opt}(t))$ is reached at the point $t = T$.

In the second model the aim of development is to get the AS with the necessary effectiveness for the minimum time. The functional is the time T of the system output of the given efficiency level of E^c_1.

Formulation of the task. To define the optimal trajectory (forecast) of the AS development $x_{opt}(t)$, $0 \le t \le T$, ensuring minimal time of output of the system at a given efficiency level:

$$T_{\min} = \min_{x \in X} T(x(t), y(t))$$

in the domain of the variable $x – X$:

$$E^c(T_{\min}) = E^c_1;\ \dot{x} = Q(x, u);$$

$$x(t) = 0;\ g(x(t)) \le 0.$$

Besides the direct variational problems one should also consider the inverse variational problems of AS forecasting development.

In this case, the aim is to get the AS with a certain efficiency

$$E^c_1(t) = E^c(t),\ 0 \le t \le T,$$

and minimum assignations for its creation and operation. Functional is the total cost of the system $C^c(T)$ to the moment of time when $t = T$.

Formulation of the task. To determine the optimal trajectory of the AS development,

$$x_{opt}(t) = \{x_{i_{opt}}(t)\},\ 0 \le t \le T,$$

providing minimum system costs with a given efficiency

$$C^c_{\min} = \min_{x \in X} \int_0^T \frac{dC^c(x(t))}{dt} dt$$

in the domain of the variable $x – X$:

$$E^c(x(t), y(t)) = E^c_1(t),\ 0 \le t \le T;$$

$$\dot{x} = Q(x, u_0);\ x(t) \ge 0;\ g(x(t)) \le 0,$$

where u_0 – the vector of AS development control in the inverse task of forecasting.

The inverse and direct tasks of forecasting of AS development are equivalent: their solving gives the same answer about the optimal trajectory of the AS. However, with limited conditions of the assignation for the AS development in more natural way is a direct problem of forecasting. Moreover, its solution is more simple and elegant.

Another problem that arises in the forecasting of the AS development is the variational direct task of forecasting of the AS development on the time interval $0 \le t \le T$.

The purpose of this task is the creation of the system, capable to give the greatest impact not in the long term at the time T, as it was considered in the previous models, but for the entire interval of time $0 \le t \le T$. Functionality is the total result of the AS functioning on the interval $0 \le t \le T$:

$$E^c = \int_0^T f_0(x(t)) dt,$$

where $f_0(x(t))$ – the result of the AS functioning in a unit of time (the intensity of the functioning).

Formulation of the task. To determine the optimal trajectory of the AS development,

$$x_{opt}(t) = \{x_{i_{opt}}(t)\},\ 0 \le t \le T,$$

for the maximum result of the AS functioning to $0 \le t \le T$:

$$E^c_{\max} = \max_{x \in X} \int_0^T f_0(x(t))dt$$

in the domain of the variable $x - X$:

$$\dot{x} = Q(x,u);\ x(t) \ge 0;\ g(x(t)) \le 0.$$

Let's also consider the gaming model of forecasting of the AS development of the side «X» AS_X and the side «Y» AS_Y (differential games).

In strict competition the aim is to create AS_X, able to achieve the best results in the future moment of time $t = T$ at the worst development of the AS_Y. Functional is the difference between the criteria of effective systems.

Formulation of the task. To determine the optimal development trajectory of the AS_X

$$x_{\text{opt}}(t) = \{x_{i_{\text{opt}}}(t)\},\ i = \overline{1,n}$$

and AS_Y,

$$y_{\text{opt}}(t) = \{y_{j_{\text{opt}}}(t)\},\ j = \overline{1,m},$$

which ensure the maximin result of functioning systems at the moment of time $t = T$:

$$\max_{x \in X} \min_{y \in Y}\left(E^c_X(x(T), y(T)) - E^c_Y(y(T), x(T))\right)$$

in the domain of the variable $x - X$:

$$\dot{x} = Q(x,u),\ x(t) \ge 0,\ g(x(t)) \le 0;$$

in the domain of the variable $y - Y$:

$$\dot{y} = Q_Y(y,v),\ y(t) \ge 0,\ g_Y(y(t)) \le 0.$$

In non-strict competition the aim is to create AS_X and AS_Y, able to achieve maximum results in the future point in time $t = T$. The functionals E^c_X and E^c_Y are criteria for combat effectiveness of AS_X and AS_Y measuring the outcome of their rivalry.

Formulation of the task. To determine the optimal trajectory of the AS_X development

$$x_{\text{opt}}(t) = \{x_{i_{\text{opt}}}(t)\}$$

and AS_Y

$$y_{\text{opt}}(t) = \{y_{j_{\text{opt}}}(t)\}$$

which provide maximum results of the system functioning at the point of time $t = T$:

$$E^c_{X\max} = \max_{x \in X} E^c_X(x(T), y(T));$$

$$E^c_{Y\max} = \max_{y \in Y} E^c_Y(y(T), x(T))$$

both in the same domains X and Y determining variables $x(t)$ and $y(t)$, and that the strict competition.

Solution of the task. The dynamic models of forecasting the AS development are based on the differential equations of AS development:

$$\dot{x} = Q(x,u).$$

We get them for the direct problem of forecasting.

We assume known:

– assignation for the system development

$C^c(t)$, $Q \le t \le T$;

– the cost of AS_i creation

$$C_i = C_i^{R\&D} + C_{\text{п}_i},\ i = \overline{1,n};$$

– the intensity of operating costs (annual operating cost) AS_i C_i^{e}, C_i and C_i^{e} generally depend on the number of AS_i x_i and future time t;

– the intensity of waste AS_i from the system due to natural losses of $\omega_i^g(t)$;

– the intensity of waste AS_i from the system due to their sales abroad ω_i^p.

Let's introduce several assumptions:

– suppose the basic assignations ΔC^c, allocated for small time Δt for the creation and operation of AS, can be implemented at this time;

– the function $x(t) = \{x_i(t)\}$ is continuous;

– natural losses and sale AS_i abroad is an ongoing process.

Assumptions allow to obtain the equations of development in the differential form [Boyce 2009].

The change in the AS amount in the transport system is determined by the rate (intensity) of AS_i supply in this transport system from the $\dot{x}_i^{pos}$ production and the rate of AS_i loss from it at the expense of their natural losses $\dot{x}_i^g$:

$$\frac{dx_i}{dt} = \frac{dx_i^{pos}}{dt} - \frac{dx_i^g}{dt}.$$

But as far as

$$\frac{dx_i^{pos}}{dt} = \frac{dx_i^{pr}}{dt} - \frac{dx_i^p}{dt},$$

where $\frac{dx_i^{pr}}{dt}$ and $\frac{dx_i^p}{dt}$ respectively the rate of AS production and the rate of AS sale abroad, then

$$\frac{dx_i}{dt} = \frac{dx_i^{pr}}{dt} - \frac{dx_i^p}{dt} - \frac{dx_i^g}{dt}$$

or

$$\frac{dx_i}{dt}=\frac{dx_i^{pr}}{dt}-\frac{dx_i^{ot}}{dt},\ i=\overline{1,n},$$

where

$$\frac{dx_i^{ot}}{dt}=\frac{dx_i^{p}}{dt}+\frac{dx_i^{g}}{dt}$$

is the total rate of AS loss on the account of the AS sale abroad and natural losses.

We assume that the rate of AS$_i$ loss on account of their sales and natural losses are proportional to their amount in the system:

$$\frac{dx_i^{p}}{dt}=\omega_i^p(t)x_i(t);\ \frac{dx_i^{g}}{dt}=\omega_i^g(t)x_i(t).$$

Then

$$\frac{dx_i^{ot}}{dt}=\omega_i(t)x_i(t),$$

where $\omega_i(t)=\omega_i^p(t)+\omega_i^g(t)$ – the summery intensity of AS$_i$ loss.

Now let's find the intensity of production AS$_i$ – $\frac{dx_i^{pr}}{dt}$. For the elementary time Δt on the development of AS is allocated ΔC^{c} assignations and ΔC_p^{c} from the sale of AS abroad, so that the total basic expenditures on the AS development are

$$\Delta C_\Sigma^{c}=\Delta C^{c}+\Delta C_p^{c}.$$

Part of these costs ΔC_{pr}^{c} is used for the AS production, the other part ΔC_e^{c} – in the operation. Therefore

$$\Delta C_\Sigma^{c}=\Delta C_{pr}^{c}+\Delta C_e^{c}.$$

The quantity of the producing AS depends on ΔC_{pr}^{c}. Therefore, we find

$$\Delta C_{pr}^{c}=\Delta C_\Sigma^{c}-\Delta C_e^{c}=\Delta C^{c}+\Delta C_p^{c}-\Delta C_{\mathrm{e}}^{c}.$$

To determine ΔC^{c} we use information about integral assignations on development of AS $C^{c}(t)$. Approximately:

$$\Delta C^{c}=\mathrm{tg}\,\beta\Delta t=f(t)\Delta t,$$

where

$$f(t)=\mathrm{tg}\,\beta=\frac{dC^{c}(t)}{dt}$$

is the intensity of assignations on the AS development.

The income from the AS sale abroad at the time Δt will be:

$$\Delta C_{p\Sigma}^{c}=\sum_{i=1}^{n}C_i^p\Delta x_i^p,$$

where C_i^p – the cost of the sales of i-th AS$_i$;

Δx_i^p – the amount of sold ones for Δt_i AS$_i$. But of this income on the AS development is allocated only a certain amount $0 \le \alpha \le 1$. Therefore

$$\Delta C_p^{c}=\alpha\Delta C_{p\Sigma}^{c}=\alpha\sum_{i=1}^{n}C_i^p\Delta x_i^p.$$

or

$$\Delta C_p^{c}=\alpha\sum_{i=1}^{n}C_i^p\omega_i(t)x_i(t)\Delta t,$$

where $x_i(t)$ – the amount of AS in operation in the moment of time t.

The operating costs for AS maintenance during the time Δt depend on the annual costs (overspending per unit of time) on the operation of each AS and their amount and are as follows:

$$\Delta C_e^{c}=\sum_{i=1}^{n}C_i^e x_i(t)\Delta t.$$

Having made the corresponding substitution we'll find the assignations for the AS production in the time interval Δt:

$$\Delta C_{pr}^{c}=\Delta t\left[f(t)-\sum_{i=1}^{n}\left(C_i^e-\alpha C_i^p\omega_i^p\right)x_i\right].$$

Of this sum on the AS$_i$ production $\Delta C_{pr}(t)$ is spent. Therefore:

$$\Delta C_{pr}^{c}=\sum_{i=1}^{n}\Delta C_{pr_i}(t).$$

We divide the left and the right part of equation into $\Delta C_{pr}^{c}(t)$. Then

$$1=\sum_{i=1}^{n}u_i(t),$$

where $u_i(t)=\frac{\Delta C_{pr_i}(t)}{\Delta C_{pr}^{c}(t)}$ – the share of funds allocated in the moment of time t in the production of the i-th part of AS$_i$, it is limited on the left and on the right:

$0 \le u_i(t) \le 1$.

The function of the $u_i(t)$ is logically called the controlling one (control), as it characterizes the distribution of resources (in this case financial). It should be noted that the management of any process always comes down to the choice of a share of the resources available. The actual meaning of resources can be different. For example, the resources for transport aircraft, in particular, are the limiting value of load carrying capacity at different flight range, and its control is to change the loading of the aircraft within these limits. As the resources can also be considered the limited angles of the stabilizer rotation. And their control is to change the angle of the stabilizer deflection in the limits of the available resources.

Now we find the amount of AS_i produced within Δt:

$$\Delta x_i^{pr} = \frac{\Delta C_{pr_i}}{C_i} = \frac{\Delta C_{pr}^{c}}{C_i} u_i ,$$

where C_i – the cost of i-th AS_i creation.

Using the above mantioned, we get:

$$\frac{\Delta x_i^{pr}}{\Delta t} = \\ = \left(f(t) - \sum_{i=1}^{n} \left(C_i^e (x_i, t) - \alpha(t) C_i^p (t) \omega_i^p (t) \right) x_i(t) \right) \times \\ \times \frac{u_i(t)}{C_i(x_i, t)}.$$

Turning $\Delta t \to 0$, we find $\frac{dx_i^{pr}}{dt} = q_i u_i(t)$, where

$$q_i = \frac{f(t) - \sum_{i=1}^{n} C_{0_i}^{e}(x_i, t) x_i(t)}{C_i(x_i, t)}$$

maximum intensity of the i-th AS_i revenue into the transport system ($u_i(t) = 1$);

$C_{0_i}^e = C_i^e - \alpha C_i^p \omega_i^p$ – the annual value of the costs of AS_i operation.

We emphasize that, in general C_i^e and C_i are functions of the AS_i x_i amount and the time t.

Knowing that $\frac{dx_i^{pr}}{dt}$ and $\frac{dx_i^{pr}}{dt}$, it is easy to write the system of differential equations of AS development:

$$\frac{dx_i}{dt} = q_i(x, t) u_i(t) - \omega_i(t) x_i(t) = Q_i(x, u_i), \; i = \overline{1, n} ,$$

with restrictions on the control functions

$$0 \le u_i(t) \le 1, \; \sum_{i=1}^{n} u_i(t) = 1$$

and coordinate variables

$$x_i(t) \ge 0, \; g(x(t)) \le 0$$

and with initial conditions:

$$x_i = x_i^0(0), \; i = \overline{1, n^0} \text{ , at } t = 0$$

and

$$x_i = x_i(t_{1_i}) = 0, \; i = \overline{n^0, n} \text{ at } t = t_{1_i} .$$

If the functions $u_i(t)$, $i = \overline{1, n}$, are selected, then the integration allows you to find the trajectory of AC $x_i(t)$ development.

Optimal control

$$u_{\text{opt}}(t) = \{ u_{i_{\text{opt}}}(t) \}$$

gives the optimal trajectory of the AS development

$$x_{\text{opt}}(t) = \{ x_{i_{\text{opt}}}(t) \} ,$$

$u_{\text{opt}}(t)$ and $x_{\text{opt}}(t)$ are determined by solving the variational task.

3. Conclusions

Thus, it should be noted that in the inverse task of forecasting the equation of AS development has a different view. As a control, you can select the intensity of AS_i production

$$u_{0_i}(t) = \frac{dx_i^{pr}(t)}{dt} .$$

To achieve the given system effectiveness $E^{c}(x(t)) = E_1^{c}(t)$, it provides the necessary rate of AS_i revenue into transport system $\frac{dx_i}{dt}$, and also compensation of AS_i waste, at the expense of natural losses $\frac{dx_i^g}{dt} = \omega_i^g x_i$ and sales abroad $\frac{dx_i^p}{dt} = \omega_i^p x_i(t)$:

$$\frac{dx_i^{pr}}{dt} = u_{0_i} = \frac{dx_i}{dt} + \frac{dx_i^g}{dt} + \frac{dx_i^p}{dt} .$$

Thus, the equation of the AS_i development has the form

$$\frac{dx_i}{dt} = u_{0_i} - \omega_i x_i = Q_i(u_{0_i}, x_i), \; i = \overline{1, n} ,$$

where $\omega_i = \omega_i^g + \omega_i^p$ – the total intensity of the aircraft waste, at the expense of their destruction ω_i^p and sale ω_i^p .

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І.І. Ліннік[1], М.В. Олег[2], К.В. Богайськая[3]. Базові принципи прогнозування оптимального розвитку авіаційних систем
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Розглянуто загальні математичні підходи та основні відомості про прогнозування розвитку сучасних авіаційних систем. Зокрема, наведено динамічні моделі визначення оптимального вигляду авіаційної системи, які призначені для оцінки достовірності та відпрацювання технології прогнозування можливості модернізації літального апарата. Показано, що динамічні моделі прогнозування оптимального розвитку авіаційних систем дозволяють оцінити умови конкурентоспроможності цих авіаційних систем та визначитися з програмою впровадження їх у транспортну систему конкретної держави.

Ключові слова: авіаційна система; математична модель; прогнозування; транспортна система.

И.И. Линник[1], М.В. Олег[2], Е.В. Богайская[3]. Базовые принципы прогнозирования оптимального развития авиационных систем
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Рассмотрены общие математические подходы и основные сведения о прогнозировании развития современных авиационных систем. В частности, приведены динамические модели определения оптимального облика авиационной системы, предназначенные для оценки достоверности и отработки технологии прогнозирования возможности модернизации летательных аппаратов. Показано, что динамические модели прогнозирования оптимального развития авиационных систем позволяют оценить условия конкурентоспособности этих авиационных систем и определиться с программой внедрения их в транспортную систему конкретного государства.

Ключевые слова: авиационная система; математическая модель; прогнозирование; транспортная система.

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