UDC 621.435

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AIRCRAFT MAXIMAL DISTANCE HORIZONTAL FLIGHTS IN THE CONCEPTUAL FRAMEWORK OF SUBJECTIVE ANALYSIS

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Abstract. *This paper reflects the aspects of optimal choice for an aircraft's maximal distance horizontal flights on the basis of multi-alternativeness of the flights operational modes control and at the possibility of the subjectively preferred controls conflicts within the conceptual framework of subjective analysis (preferences theory) in application to the decision making process under the uncertainty.*

Keywords: decision making; extremal; horizontal flight; individual's preferences; maximal distance; modes control; optimal choice; subjective entropy; variational problem.

1. Introduction

Horizontal flights of maximal distance and duration are important operational parameters of any kind of aircraft equipped with any type of the aircraft powerplant [Kroes, Wild 1994]. It is generally undoubtedly that the maximal distance and duration of the horizontal flight exist objectively.

2. Importance of the researches

Urgency of the researches in the sphere of aircraft operational modes optimal control is dictated by the importance of the safety and fuel savings issues.

Also, the problem is important since it relates the optimal choice behavior.

3. Analysis of the latest researches

Analyzing the sources of information [Kroes, Wild 1994; Kosmodemianskii 1966; Kasianov 2004; Kasianov, Goncharenko 2013; Kasianov 2007; Kasianov 2003; Kasianov, Goncharenko 2012; Goncharenko 2012] we see the necessity of further researches for the scientific explanations of some certain operational mode controls, or specific combinations of the controlling modes, in the direction of the evaluation of individuals' preferences of alternatives.

The interdisciplinary investigations [Kasianov, Goncharenko 2013; Kasianov 2007; Kasianov 2003; Kasianov, Goncharenko 2012; Goncharenko 2012] must go on.

4. The task setting

For this paper, we shall find the optimal combination of possible alternative modes of control for aircraft maximal flight distance.

Most of the intermediate mathematical expressions and their derivations have been dropped for the purposes of this paper should be abridged.

The usage of the idea of the individual's subjective preferences entropy extremization principle allows solving a special case of the generalized simplest problem of the calculus of variations.

5. The problem formulation

On the basis of the theoretical results achieved by our predecessors [Kosmodemianskii 1966], for the horizontal flight with the maximal distance for an aircraft equipped with the sky rocket engine [Kosmodemianskii 1966, § 5, p. 212]:

$$f = \left(\frac{A}{B}\right)^{\frac{1}{n}} v^2 \left[\frac{v + V_r}{(2n-1)V_r + (n-1)v}\right]^{\frac{1}{n}}, \qquad (1)$$

where f – function, which is being variated, or the free/loosened function, – the law of the airplane mass change [Kosmodemianskii 1966, § 5, p. 198];

A and B – constants, being determined by the expressions [Kosmodemianskii 1966, § 5, p. 202]:

$$A = \frac{C_{x_0}\rho S}{2M_0}, \qquad B = \frac{bg^n (2M_0)^{n-1}}{(\rho S)^{n-1}}, \qquad (2)$$

C_{x_0} – value of the head resistance force coefficient at the value of the lifting force when it is equal to zero;

ρ – density of the air at the given altitude;

S – character square-area of the flying object [Kosmodemianskii 1966, § 5, p. 199];

M_0 – mass of the flying apparatus at the initial moment in time (at the point of the airplane coming up to the straight line horizontal trajectory) [Kosmodemianskii 1966, § 5, p. 201, 202];

b – some stable value which is being determined within the given diapason of speeds from the blowings in wind tunnels (aerodynamic tubes);

g – acceleration, stipulated by the gravitational force, which is considered being constant and equaled to $g = 9.81$ m/s^2;

n – certain constant, being determined in an analogous way likewise C_{x_0} and b [Kosmodemianskii 1966, § 5, p. 199];

v – speed of the flying object center of masses;

V_r – effective relative speed of the burning products flowing out from the nozzle of the reactive (jet) engine, being $V_r = \text{const}$ [Kosmodemianskii 1966, § 5, p. 199].

It is assumed, that when the fuel is being burnt, the center of masses of the aircraft has no displacement relatively to its fuselage hull, hence, the vector differential equation of the center of masses motion will not be different from the equation of the material point with the changeable mass motion, that is from the equation by I.V. Meschersky (1893-1897) [Kosmodemianskii 1966, p. 7, § 5, p. 199], T. Levi-Civita (1928) [Kosmodemianskii 1966, p. 9, 11, 12, § 1, p. 19].

The function (1) is the extremal of the corresponding functional [Kosmodemianskii 1966, § 5, p. 202]:

$$L = \int_{v_E}^{v_0} \frac{(f + f' V_r) v^{2n-1}}{A v^{2n} + B f^n} dv, \quad (3)$$

where L – the distance of the flight;

v_0 – value of the initial speed of the flying object horizontal flight [Kosmodemianskii 1966, § 5, p. 202, 208, 210];

v_E – speed of the airplane flight at the end of the active segment of the horizontal flight, that is at the end of the engine run [Kosmodemianskii 1966, § 5, p. 202];

f' – derivative of the flying object mass change function with respect to the speed of the horizontal flight, that is $f' = \frac{df}{dv}$ [Kosmodemianskii 1966, § 5, p. 201].

It is written in the view of the integral (3) on the basis of [Kosmodemianskii 1966, § 5, p. 202]:

$$dL = v dt = -\frac{(f + f' V_r) v^{2n-1}}{A v^{2n} + B f^n} dv, \quad (4)$$

where t – time;

We elaborate methodic [Kasianov, Goncharenko 2013; Kasianov 2007; Kasianov 2003; Kasianov, Goncharenko 2012; Goncharenko 2012] for estimation the system's active element controlling influence upon the optimal operational mode.

Already developed elements of the general methodic imply the compilations of more general operational control functionals of the types of [Kasianov 2007, p. 119]:

$$\Phi_\pi = -\sum_{i=1}^{N} \pi_i \ln \pi_i - \beta \sum_{i=1}^{N} \pi_i F_i + \gamma \left[\sum_{i=1}^{N} \pi_i - 1 \right], \quad (5)$$

where π_i – the function of the individual's subjective preferences of the i-th achievable alternative;

N – number of the achievable alternatives;

β – structural parameter;

F_i – function, related to the i-th achievable alternative;

γ – structural parameter.

The structural parameters β and γ can be considered in different situations as Lagrange coefficients, weight coefficients or endogenous parameters that represent some certain properties of the individual's psych.

The other types of the functionals are [Kasianov, Goncharenko 2013, p. 42], [Kasianov, Goncharenko 2012, p. 57]:

$$\Phi_\pi = \int_{t_0}^{t_1} \left\{ -\sum_{i=1}^{N} \pi_i(t) \ln \pi_i(t) + \beta \sum_{i=1}^{N} \pi_i(t) F_i + \gamma \left[\sum_{i=1}^{N} \pi_i(t) - 1 \right] \right\} dt, \quad (6)$$

or [Kasianov, Goncharenko 2012, p. 57]:

$$\Phi_\pi = \int_{t_0}^{t_1} \left(-\sum_{i=1}^{N=4} \pi_i(t) \ln \pi_i(t) + \beta [\pi_1(t) x(t) + \alpha_2 \pi_2(t) \dot{x}(t) + \alpha_3 \pi_3(t) x(t) \dot{x}(t) + \alpha_4 \pi_4(t) \dot{x}(t) / x(t)] + \gamma \left[\sum_{i=1}^{N=4} \pi_i(t) - 1 \right] \right\} dt, \quad (7)$$

where $x(t)$, $\dot{x}(t)$, $x(t)\dot{x}(t)$, and $\dot{x}(t)/x(t)$ – in the simplest problem setting we consider as the subjective effectiveness functions of F_i for the four achievable alternatives with the corresponding preferences of $\pi_i(t)$;

α_i – coefficients that consider the differences in the measurement units for the effectiveness functions.

6. The problem solution

For now, we combine the subjective entropy extremization principle (5)–(7) with the results obtained as the development of (1)–(4), in the view of

$$L=\int_{M_0}^{M_E}-\frac{2\eta Q\rho v^2 S}{C_{x_0}\left(\rho v^2 S\right)^2+b\left(2mg\right)^2}\,dm\,, \tag{8}$$

where M_E – mass of the flying apparatus at the end of the active segment of the horizontal flight, that is at the end of the engine run;

η – efficiency (coefficient of the useful action) of the propulsive complex;

Q – low calorific value of the fuel by its working mass;

m – mass of the flying apparatus;

$$v_L(m)=\sqrt[4]{4\frac{bm^2g^2}{C_{x_0}\rho^2S^2}}\,, \tag{9}$$

where $v_L(m)$ – the extremal (optimal speed) of the functional (8), as the function of the aircraft changeable mass found on conditions of the Euler's-Lagrange's equation compliance for (8).

Let us introduce the principle (5)–(7) and the functional (8) into the functionals of more general form, for example, for two reachable alternative speeds of the horizontal flight:

$$\Phi_\pi=\int_{M_0}^{M_E}\left\{H_\pi-\beta\left[\pi_1\frac{2\eta Q\rho v_{opt}^2 S}{C_{x_0}\left(\rho v_{opt}^2 S\right)^2+b\left(2mg\right)^2}+\right.\right.$$

$$\left.+\pi_2\frac{2\eta Q\rho v^2 S}{C_{x_0}\left(\rho v^2 S\right)^2+b\left(2mg\right)^2}\right]+$$

$$\left.+\gamma\left[\sum_{i=1}^{N=2}\pi_i-1\right]\right\}dm\,, \tag{10}$$

where H_π – is subjective entropy,

$$H_\pi=-\sum_{i=1}^{N=2}\pi_i\ln\pi_i\,, \tag{11}$$

v_{opt} – unknown optimal speed of the horizontal flight with regards to the distance (length) of the flight;

v – arbitrary chosen function of speed.

On conditions of the Euler's-Lagrange's equations system compliance for (10), we get canonical distributions for the functions of preferences π_i (the extremals) similar to [Kasianov 2007, P. 115-135], and for the optimal speed of the aircraft horizontal flight of the maximal distance v_{opt} (also the extremal), we find the expression which is identical to (9).

7. Practical application of the problem solution

For example, let us make an assumption that an aircraft has the supposed flight parameters of the sort of:

M_0 = 10,000 kg;

M_E = 8,000 kg;

η = 0.3;

Q = 42,700·10^3 J/kg;

ρ = 1 kg/m^3;

S = 50 m^2;

C_{x_0} = 0.02;

b = 0.045.

Imagine two programs of a flight with the only two different speed-on-mass dependencies for the given flight, none of the dependencies are the extremals of the kind of (9) of the functionals (8) and (10).

If the flight task is to cover the longest possible, in such a case, distance in the horizontal segment of the flight trajectory, controlling active element's logic strategy will challenge his intellectual skills.

This kind of variational problem could be solved numerically. This approximate solution will converge in the limit to the real optimal combination of the two modes, the smaller the dependencies segments variances, in the limit the biggest tends to zero; the greater the number of variants, in the limit it tends to infinity; the more accurate solution it will yield.

The precise solution, if it exists, can be obtained with the application either of the principle of maximum by L.S. Pontryagin or the principle of optimality by R. Bellman.

Although, whichever of those three methods cannot definitely be called "simple" or "easy".

The principle of the individual's subjective preferences entropy extremization is a "convenient tool" for solving such a problem.

Mathematical modeling in the framework of the subjective analysis paradigm yields the sought result.

Calculation experiments for the presumed data are illustrated in Fig. 1.

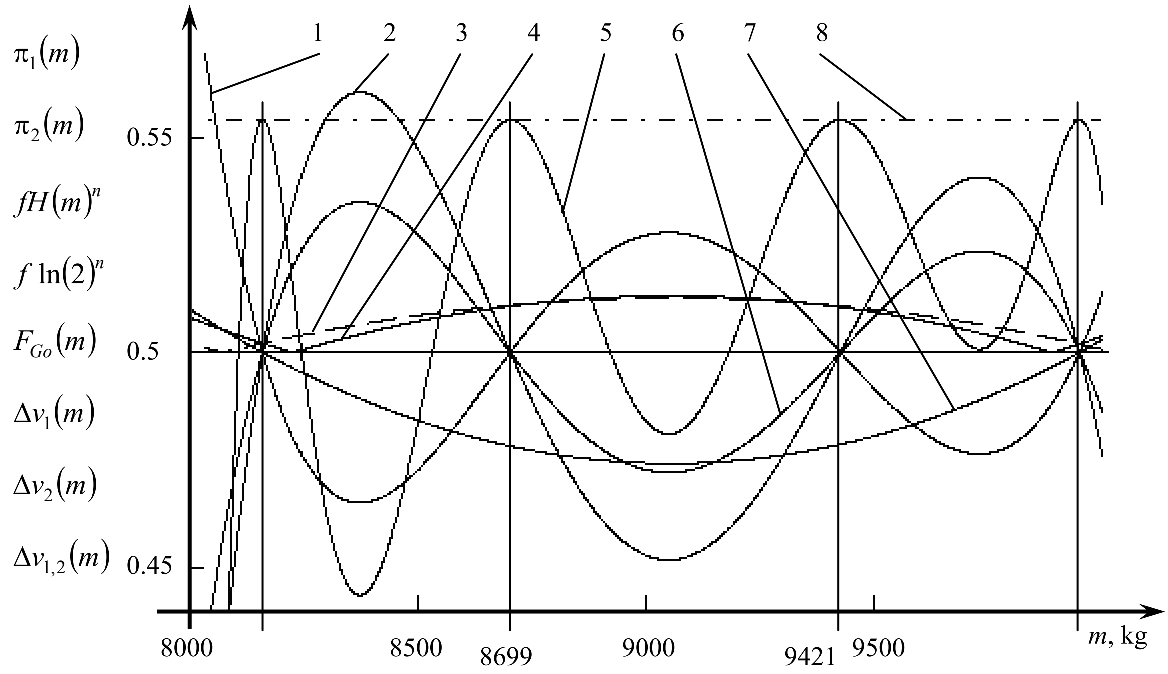

Fig. 1. Controlling operational modes preferences and subjective entropy formed by the effectiveness functions:
1 – $\pi_2(m)$ – function of the preferences of the second alternative;
2 – $F_{Go}(m)$ – function related to the effectiveness functions difference;
3 – $\Delta v_2(m)$ – function related to the differences between the extremal speed and the second alternative speed;
4 – $\Delta v_1(m)$ – function related to the differences between the extremal speed and the fist alternative speed;
5 – $fH(m)^n$ – subjective entropy $H(m)$ multiplied by the scale factor f and raised to the scale power n;
6 – $\pi_1(m)$ – function of the preferences of the first alternative;
7 – $\Delta v_{1,2}(m)$ – function related to the differences between the fist and second alternative speeds themselves;
8 – $f\ln(2)^n$ – subjective entropy $H(m)$ maximal value $\ln(2)$ multiplied by the scale factor f and raised to the scale power n

In Fig. 1, it is noticeable four maxima of entropy $H(m)$ at the values of knots of

$$F_{Go}(m) = \frac{1}{2} + Go(m)(F_1 - F_2) = \frac{1}{2}, \quad (12)$$

where $Go(m)$ – special scale function for the effectiveness functions difference; and intersections of $p_1(m)$ and $p_2(m)$.

Also, in Fig. 1, there are functions:

$$\Delta v_1(m) = \frac{1}{2} + \frac{|v_{\max} - v_1|}{k}, \quad (13)$$

$$\Delta v_2(m) = \frac{1}{2} + \frac{|v_{\max} - v_2|}{k}, \quad (14)$$

$$\Delta v_{1,2}(m) = \frac{1}{2} + \frac{v_1 - v_2}{k}, \quad (15)$$

where $v_{\max}$ – extremal speed of the flight, unreachable for this problem setting;

v_1 and v_2 – alternative speeds of the flight correspondingly;

k – special scale coefficient.

The values of the scales f, n, $Go(m)$ and k are chosen for the expositional conveniences.

In accordance with the preferences, for the supposed data, diagrams plotted in Fig. 2 substantiate expediency of the operational mode change.

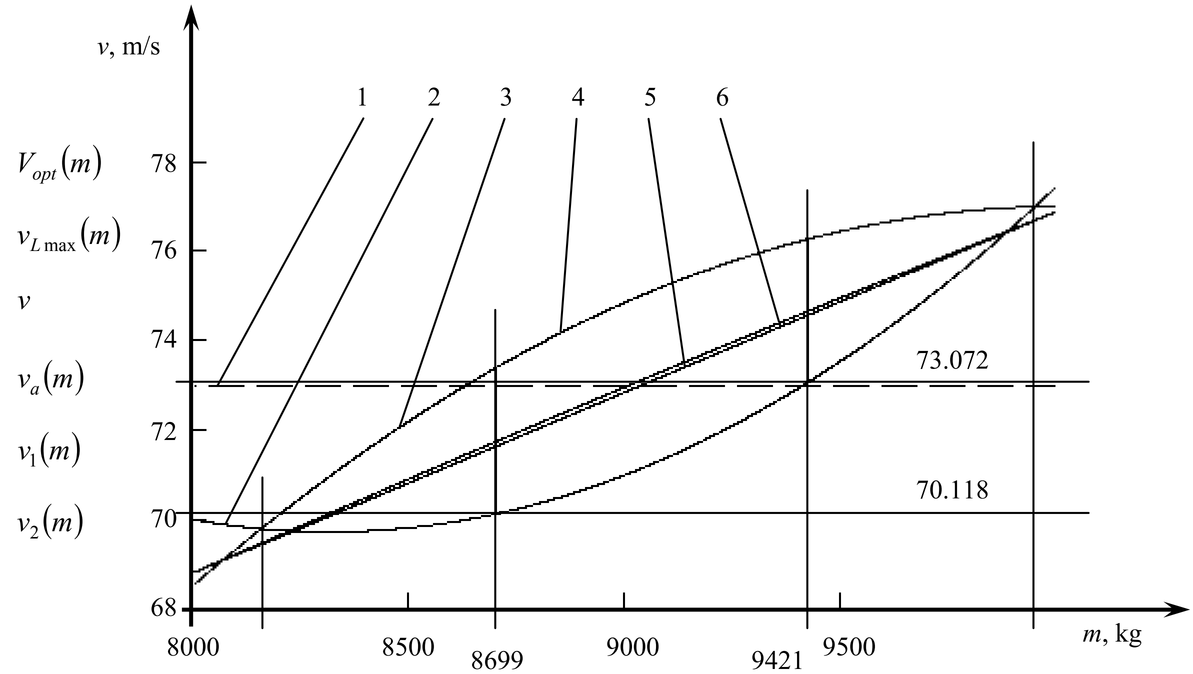

Fig. 2. Variants of the horizontal flight speeds:
1 – v – constant speed of the flight;
2 – $v_1(m)$ – fist alternative speed;
3 – $v_2(m)$ – second alternative speed;
4 – $V_{opt}(m)$ – optimal compositional speed of the flight;
5 – $v_a(m)$ – closest to the extremal, approximated speed, not optimal compared to the extremal one, although;
6 – $v_{L\max}(m)$ – extremal speed of the flight

In Fig. 2 we can observe shifts in operational modes at the knots only if $\Delta v_{1,2}(m) \neq 0.5$ but $F_{Go}(m) = 0.5$.

The corner points of the flight speed changes are at the $\Delta v_{1,2}(m) = 0.5$ and $F_{Go}(m) = 0.5$.

The mere fact that the dependence of (12) has the very similar shape of the graph plotted in the Fig. 1 as that one of the $\pi_1(m)$ – function of the preferences of the first alternative precisely says about the very close connection between the effectiveness functions difference $F_1 - F_2$ and the subjectively preferred perception of that kind of effectiveness, it is like stimulus and perception.

For example, for the given two alternatives we might build an integral objective functional like (6), (7), (10):

$$\Phi_F = \int_{M_0}^{M_E} \left\{ -\sum_{i=1}^{N=2} \tilde{F}_i(m) \ln \tilde{F}_i(m) - \beta \sum_{i=1}^{N=2} \pi_i(m) \tilde{F}_i(m) + \gamma_F \left[\sum_{i=1}^{N=2} \tilde{F}_i(m) - 1 \right] \right\} dm, \quad (16)$$

where $\tilde{F}_i(m)$ – special effectiveness function as a stimulus function related with the psychophysical properties of the operators, being normalized, it reflects their cognitive estimation of the effectiveness functions of F_i *(cognitiveness*) of the reachable alternatives, and it is a free variated function to be sought;

γ_F – Lagrange uncertain multiplier.

From the necessary conditions for the functional (16) extremum we find

$$\frac{\partial R}{\partial \tilde{F}_i} = -\ln \tilde{F}_i - 1 - \beta \pi_i + \gamma_F = 0, \quad (17)$$

where R – integrand of the functional (16),

$$\tilde{F}_i = \exp[\gamma_F - 1] \exp[-\beta \pi_i], \quad (18)$$

for any *i*-th function.

Using the common member of $\gamma_F - 1$ from equation (17) or $e^{\gamma_F - 1}$ from equations (18) we come to

$$-\ln \tilde{F}_1 - \beta \pi_1 = -\ln \tilde{F}_2 - \beta \pi_2, \quad (19)$$

or

$$\frac{\widetilde{F}_1}{\exp[-\beta\pi_1]} = \frac{\widetilde{F}_2}{\exp[-\beta\pi_2]}, \qquad (20)$$

which allows us to note from equations (19) and (20)

$$\ln\frac{\widetilde{F}_1}{\widetilde{F}_2} = -\beta(\pi_1 - \pi_2). \qquad (21)$$

Equation (21) is the principal law of psychophysics.

From equations (17) and (18) with the use of the normalizing condition for the stimuli functions we get

$$\widetilde{F}_i = \frac{\exp[-\beta\pi_i]}{\sum\limits_{j=1}^{2}\exp[-\beta\pi_j]}. \qquad (22)$$

Equations (21) and (22) with respect to the normalizing condition for the preferences yield

$$\pi_1 = \frac{1}{2}\left[1 - \frac{1}{\beta}\ln\frac{\widetilde{F}_1}{\widetilde{F}_2}\right]. \qquad (23)$$

Equation (23) is identical to the canonical

$$\pi_1[F_1,\, F_2] = \frac{\exp[-\beta F_1]}{\exp[-\beta F_1] + \exp[-\beta F_2]}. \qquad (24)$$

From equations (23) and (24) it yields

$$\widetilde{F}_1[F_1,\, F_2] = \frac{\exp\{\beta[1 - 2\pi_1[F_1,\, F_2]]\}}{1 + \exp\{\beta[1 - 2\pi_1[F_1,\, F_2]]\}}. \qquad (25)$$

The entropy paradigm having found its applications in different spheres of science brings new results in the researches beginning from the economical [Kasianov, Goncharenko 2013] and sociological [Kasianov, Goncharenko 2013; Kasianov 2007; Kasianov 2003] and ending in the engineering [Kasianov 2007; Kasianov 2003; Kasianov, Goncharenko 2012; Goncharenko 2012].

For the case of the horizontal flight for the maximal distance at two possible alternative operational modes control the subjective analysis paradigm yields the optimal combination of the two controls.

The diagrams plotted in Fig. 1 demonstrate positive conflictability of the “right” alternative, even if there is no extremal amongst them. The conflictability can be evaluated with the use of the hybrid model of the relative pseudo-entropy function researched in some detailed particular applications in the papers [Goncharenko 2012] for the reliability and safety issues. There is an opinion that conflicts might have positive as well as negative functions.

Herein, it is justified completely in the view of the conflict between preferences.

Interpreting the situations depicted in Figs 1, 2, we are able to see the fundamental importance of the preferences functions for any types of problem formulations (5)–(7), (10)–(25), and likewise.

For the practical application with two unextremal flight speeds, the functions of preferences distributions reflect optimal composition of the effectiveness functions.

8. Conclusions

The postulated in the subjective analysis principle of the individual’s subjective preferences entropy extremization allows, by itself, to find optimality in the control of alternative choice without any preconditions and even without knowing the extremal one.

Since the subjective entropy extremization principle allows; independently on the conditions of transversality, Weierstrass-Erdmann, principle of maximum by L.S. Pontryagin (USSR), as well as principle of optimality by R. Bellman (USA); finding the extremals, their optimal conjunctions of all kinds: either breaks with shifts, or both at smooth and corner points, for closed and restricted areas; stipulated by compliance with the only a priory condition of the Euler-Lagrange equations; it is suggested to call this principle by the name of its author, professor Vladimir Aleksandrovich Kasyanov, National Aviation University (Kyiv, Ukraine).

It is important to investigate other types of functionals of the kind of (5)–(8), (10), (16) as well as with the different sorts of functions of effectiveness, also research operational modes of optimal control for horizontal flights with segments of maximal distance and maximal duration.

The same to the similar problem formulations being guided by the principle possibility to optimize the horizontal flights of maximal distance and duration by means of the changeable angles of attack, as well as for the problem settings on the basis of the researches for optimization with regards to both the horizontal flight speeds and angles of attack.

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Received 3 October 2013.

А.В. Гончаренко. Горизонтальні польоти літака на максимальну дальність у концептуальних рамках суб'єктивного аналізу

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Розглянуто аспекти оптимального вибору для горизонтальних польотів літака на максимальну дальність на основі багатоальтернативності керування експлуатаційними режимами польотів та за можливості конфліктів суб'єктивно переважних керувань у концептуальних рамках суб'єктивного аналізу (теорії переваг) у застосуванні до процесу прийняття рішень в умовах невизначеності.

Ключові слова: варіаційна задача; горизонтальний політ; екстремаль; індивідуальні переваги; керування режимами; максимальна дальність; оптимальний вибір; прийняття рішень; суб'єктивна ентропія.

А.В. Гончаренко. Горизонтальные полеты самолета на максимальную дальность в концептуальных рамках субъективного анализа

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Рассмотрены аспекты оптимального выбора для горизонтальных полетов самолета на максимальную дальность на основе многоальтернативности управления эксплуатационными режимами полетов и при возможности конфликтов субъективно предпочитаемых управлений в концептуальных рамках субъективного анализа (теории предпочтений) в приложении к процессу принятия решений в условиях неопределенности.

Ключевые слова: вариационная задача; горизонтальный полет; индивидуальные предпочтения; максимальная дальность; оптимальный выбор; принятие решений; субъективная энтропия; управление режимами; экстремаль.

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