UDC 51:533.6

Mikhail Lopatjuk

SPLINE-FUNCTIONS IN THE TASK OF THE FLOW AIRFOIL PROFILE

Kyiv State Maritime Academy named after hetman Petro Konashevich-Sahaydachny
Frunze street 9, 04071, Kyiv, Ukraine
E-mails: academy@maritime.kiev.ua; Victoria@maritime.kiev.ua

Abstract. *The method and the algorithm of solving the problem of streamlining are presented. Neumann boundary problem is reduced to the solution of integral equations with given boundary conditions using the cubic spline-functions.*

Keywords: airfoil profile; spline-function; the potential flow; velocity at the profile contour.

1. Introduction

For the mathematical modeling of real aerodynamic processes numerical calculations are widely used. The method of spline functions has been proving itself for a long time as a flexible mathematical tool for solving systems of differential and integral equations. While studying aerodynamic surfaces we should use cubic spline functions [Zavialov et al. 1980; Lopatjuk 2011].

2. Analysis of studies and publications

A great deal of works is devoted to the task of the flow airfoil discrete profile, for example [Pavlovets 1971]. Among the existing methods for solving the problem the method of spline functions is the most appropriate. But there are many kinds of it: trigonometric splines, splines of different degrees and defects, splines "the load" and others [Zavialov et al. 1980]. Among them, only the cubic spline functions are the optimal order splines as they have minimum of intersection points, which at the same time provide the necessary second order smoothness of contours which are interpolated. Furthermore, their other positive qualities should be noted such as simplicity and uniformity of the input task. For the specifying any function only the value of the abscissa and ordinate at nodal points and boundary conditions is required. Inaccurate boundary conditions results only in local boundary curvature. Interpolation algorithm is easy to implement on a computer, ensuring high precision results. Matrix of spline function "moments" for a given discrete set is calculated once, stored and further it represents, in fact, the analytical form of the set, which allows to solve related problems, including the problem of numerical integration and differentiation, to obtain approximate solutions of boundary value problems. Requirements of systematic approach necessitate the single digital parametric mathematical construction of the object of model study.

The cubic spline-functions method provides the unity of the storing basic parameters and calculating the required information [Lopatjuk 2011].

The aim is to develop methods and algorithm of solving the problem of streamlining of airfoil with fluid flow.

3. Calculation of geometric and aerodynamic parameters of a discrete profile

Consider the problem of flow past smooth airfoil profile by the stream of ideal incompressible fluid. In addition to that this flow is considered to be established, vortex. Mathematically, this problem is a Neumann boundary problem, which turns to the solution of the Laplace equation:

$$\Delta\varphi = 0$$

under these boundary conditions:

– unleakage of fluid through the profile contour, streamlining continuity:

$$\frac{\partial\varphi}{\partial n}\Big|_L = 0;$$

– uniformity of fluid flow beyond the profile streamlined:

$$\bar{\nabla}\varphi = V_\infty, \quad r \to \infty.$$

In his work Pavlovets [1971] considers special cases.

In this paper for the solution of integral equations obtained in the study, the method of cubic spline-functions is used.

Presenting the potential flow as a superposition of unperturbed flow and stream made by specifics of vortices types, distributed over the surface of the body, we are looking for a solution as a sum of a unperturbed flow potential and potential of continuous vortex layer placed on the profile circuit with a linear density

$$W = V_\infty z - \frac{1}{2\pi i}\oint \gamma(\zeta)\ln(z-\zeta)dz,$$

where $\zeta = \xi + i\eta$ – the complex coordinate of a point on the profile;

$z = x + iy$ – complex coordinate of any point of the flow plane.

This potential and stream function as real and imaginary parts of the complex potential, are as follows:

$$\varphi = V_\infty x - \frac{1}{2\pi}\int_0^l \gamma(\tilde{s}) arctg \frac{y - \eta(\tilde{s})}{x - \xi(\tilde{s})} d\tilde{s},$$

$$\psi = y + \frac{1}{2\pi}\int_0^l \gamma(\tilde{s}) \ln\sqrt{(x - \xi(\tilde{s}))^2 + (y - \eta(\tilde{s}))^2} d\tilde{s},$$

where $\tilde{s}$ – curvilinear coordinate is measured along the contour profile;

$l = \tilde{s}_N$ – total length of the path profile.

Preher proved the equivalence of conditions for unleakage through the contour and vortex density layer of liquid velocity outside the circuit:

$$\gamma(s) = V^+(s) = (\frac{\partial \varphi}{\partial s})^+ .$$

Based on this and on the properties of the derivative of the potential φ, to determine the speed $V(s)$ we get the integral Fredholm equation of the second kind:

$$\frac{1}{2}V(s) + \frac{1}{2\pi}\int_0^l V(\tilde{s}) \frac{\partial}{\partial s} arctg \frac{y - \eta}{x - \xi} d\tilde{s} = -V_\infty \frac{dx}{ds}.$$

Determining the derivative under the integral sign and going to a given rate on the profile contour that is streamlined

$$v = \frac{V(s)}{V_\infty}\sqrt{(\frac{dx}{ds})^2 + (\frac{dy}{ds})^2},$$

we obtain the following equation

$$v - \frac{1}{\pi}\int_0^{s_N} v(\tilde{s}) \frac{\frac{dy}{ds}(x - \xi) - \frac{dx}{ds}(y - \eta)}{(x - \xi)^2 + (y - \eta)^2} ds =$$

$$= -2(\frac{dx}{ds}\cos\alpha + \frac{dy}{ds}\sin\alpha),$$

where α – angle of attack.

Note that when the profile points (ξ, η) and (x, y) coincide, the kernel of the integral equation is uncertain species $\frac{0}{0}$. But since the profile interpolates with the help of spline - functions, while ensuring contour smoothness of the second order, then it is going to the boundary, we obtain:

$$\lim_{(\xi,\eta)\to(x,y)} \frac{\frac{dy}{ds}(x - \xi) - \frac{dx}{ds}(y - \eta)}{(x - \xi)^2 + (y - \eta)^2} = \frac{\frac{dx}{ds}\frac{d^2y}{ds^2} - \frac{dx^2}{ds^2}\frac{dy}{ds}}{2((\frac{dx}{ds})^2 + (\frac{dy}{ds})^2)}.$$

We assume that the density of the vortex layer is distributed along the profile contour according to the law of cubic spline-functions. Then, turning to a uniform grid $H_i = s_{i+1} - s_i = \text{const}$, we get

$$v(s) = \frac{6}{H}[vM_{j-1}(s_j - s)^3 + vM_j(s - s_{j-1})^3 +$$
$$+ (6v_{j-1} - vM_j H^2)(s_j - s) + (6v_j - vM_j H^2)(s - s_{j-1})],$$
$$j = 2, \ldots, N. \quad (1)$$

Performing identical transformation and introducing data replacement, we obtain the corresponding system:

$$\sum_{j=1}^{N} a_{ij} K_{ij} v_j = c_i, \quad i = 1, \ldots, N,$$

where

$$a_{ij} = \begin{cases} 1{,}5 \quad i \neq j, j = 1, N; \\ 1{,}5 - \frac{3\pi N}{s_N} \quad i = j, j = 1, N; \\ 4 \quad i \neq j, j = 2n, n = 1, \ldots, \frac{N-1}{2}; \\ 4 - \frac{3\pi N}{s_N} \quad i = j, j = 2n, n = 1, \ldots, \frac{N-1}{2}; \\ 3 \quad i \neq j, j = 2n+1, n = 1, \ldots, \frac{N-3}{2}; \\ 3 - \frac{3\pi N}{s_N} \quad i = j, j = 2n+1, n = 1, \ldots, \frac{N-3}{2}; \end{cases}$$

$$K_{ij} = \begin{cases} \dfrac{\frac{dy}{ds}\Big|_{s=s_i}(x_i - x_j) - \frac{dx}{ds}\Big|_{s=s_i}(y_i - y_j)}{(x_i - x_j)^2 + (y_i - y_j)^2}, \quad i \neq j, \\ \dfrac{\frac{dx}{ds}\Big|_{s=s_i} My_i - \frac{dy}{ds}\Big|_{s=s_i} Mx_i}{2[(\frac{dx}{ds}\Big|_{s=s_i})^2 + (\frac{dy}{ds}\Big|_{s=s_i})^2]}, \quad i = j, \end{cases}$$

$$c_i = \frac{6\pi N}{s_N}(\frac{dx}{ds}\Big|_{s=s_i}\cos\alpha + \frac{dy}{ds}\Big|_{s=s_i}\sin\alpha), \; i = 1, \ldots, N.$$

Using this system as basis and methods for solving linear systems with a large number of unknowns, we can find value of v_i and vM_i in accordance with the system (1).

4. Conclusions

Thus, we have a set of coefficients speed “moments”

$$vM_{ki}, \quad k = 1,\ldots,m; \quad i = 1,\ldots,N,$$

where m – number of airfoils along the wingspan.

In turn it makes possible to find the coefficients of “moments” of pressure

$$pM_{ki}, \quad k = 1,\ldots,m; \quad i = 1,\ldots,N$$

for each profile. Together the “moments” of geometric spline functions, velocity spline functions and pressure spline functions can be used as a general mathematical model of a wing surface of streamlining parameters [Lopatjuk 2011].

Methodology and algorithm allow to calculate the geometric and aerodynamic profile parameters during the designing of new types of aircraft.

References

Lopatjuk, M.M. 2011. *Spline - functions in hydroaeromechanics problems*. Kyiv, Water Transport. KDAVT scientific papers collection. N 2 (15): 147–150 (in Ukrainian).

Lopatjuk, M.M. 2011. *The mathematics model of the limited span wing using the spline-functions*. Materials X International Scientific and Technical Conference “AVIA-2011”. Kyiv, National Aviation University. Vol. 1: 6.34–6.37 (in Ukrainian).

Pavlovets, G.A. 1971. *Methods of calculation of wing sections streamlining of perfect incompressible flow*. Moscow, Proceedings of the TsAGI. Vol. 1344 (in Russian).

Zavialov, J.S.; Kvasov, B.I.; Miroshnichenko, V.L. 1980. *Methods of spline - functions*. Moscow, Nauka. 352 p. (in Russian).

Received 3 October 2013.

М.М. Лопатюк. Сплайн-функції в задачі обтікання крилового профілю

Київська державна академія водного транспорту ім. гетьмана Петра Конашевича-Сагайдачного
вул. Фрунзе, 9, Київ, Україна, 04071
E-mails: academy@maritime.kiev.ua; Victoria@maritime.kiev.ua

Розглянуто задачу обтікання гладкого крилового профілю потоком ідеальної нестисливої рідини. Потенціальний потік подано у вигляді суперпозиції незбуреного потоку і потоку, утвореного особливостями типу вихрів, розподілених по поверхні тіла. Описано розв’язання задачі у вигляді суми потенціалу незбуреного потоку і потенціалу неперервного вихрового шару, що розміщений на контурі профілю з лінійною густиною. Показано, що густина вихрового шару розподілена по контуру профілю за законом кубічних сплайн-функцій. Після тотожних перетворень задачу зведено до розв’язування лінійної системи з великою кількістю невідомих. Розроблено методику і алгоритм, що дозволяє розрахувати геометричні та аеродинамічні параметри профілів у процесі проектування нових типів авіаційної техніки.

Ключові слова: аеродинамічний профіль; момент; потенціал потоку; сплайн-функція; швидкість на контурі профілю.

М.М. Лопатюк. Сплайн-функции в задаче обтекания крылового профиля

Киевская государственная академия водного транспорта им. гетмана Петра Конашевича-Сагайдачного
ул. Фрунзе, 9, Киев, Украина, 04071
E-mails: academy@maritime.kiev.ua; Victoria@maritime.kiev.ua

Рассмотрена задача обтекания гладкого крылового профиля потоком идеальной несжимаемой жидкости. Потенциальный поток представлен в виде суперпозиции невозмущенного потока и потока, созданного особенностями типа вихрей, распределенных по поверхности тела. Описано решение задачи в виде суммы потенциала невозмущенного потока и потенциала непрерывного вихревого слоя, размещенного на контуре профиля с линейной плотностью. Показано, что плотность вихревого слоя распределена по контуру профиля по закону кубических сплайн-функций. После тождественных преобразований задача сведена к решению линейной системы с большим количеством неизвестных. Разработаны методика и алгоритм, позволяющие рассчитывать геометрические и аэродинамические параметры в процессе проектирования новых типов авиационной техники.

Ключевые слова: аэродинамический профиль; момент; потенциал потока; скорость на контуре профиля; сплайн-функция.

Lopatjuk Mikhail. Candidate of Engineering. Associate Professor.
Department of Mathematics, Kyiv State Maritime Academy named after hetman Petro Konashevich-Sahaydachny, Kyiv, Ukraine.
Education: Faculty of Mechanics and Mathematicity, Kyiv National University named after Taras Shevchenko (1974).
Research areas: applied mathematics, aerodynamics and gas dynamics of aircraft.
Publications: 16. E-mails: academy@maritime.kiev.ua; Victoria@maritime.kiev.ua