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NONLINEAR PERTURBATION METHOD FOR CALCULATING AXISYMMETRIC CAVITATIONAL FLOWS

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Abstract. *A mathematical model of a cavity under the influence of perturbations of various origins is evaluated. The model is based on hydrodynamics of flows with free boundaries and the theory of small perturbations. Specific analysis is provided for cavitational flows behind cones.*

Keywords: angle of attack; cavitation number; cavitator; cone perturbation; current fluid cavity; differential equations; drag coefficient; Froude number; gravity; kinetic energy; Laplace equation; potential.

1. Introduction

The theory of cavitational flows is widely used in various applications, including aerodynamics of aircraft wings, elements of marine vessels, and liquid media flows in industrial processes. This article aims to show the possibility of improving analysis of the flow perturbations caused by conical cavitators.

The basis of this theory was established in the previous studies [Logvinovich 1969; Zhuravlev 1973]. Similar methods were used previously to calculate the deformation of spherical cavities [Plesset, Shaffer 1948]. These approaches first established in [Logvinovich 1969; Zhuravlev 1973] were subsequently further developed in [Buivol 1980]. Relevant to these studies, an experimental method of measuring cavity geometries including perturbed cavity geometries [Epshtein, Lapin 1975; Epshtein, Lapin 1980] has been developed. The main results of these studies were obtained for disc-shaped cavitator, thickness of which could be neglected.

However, the methods developed in [Buivol 1980] can be improved so that it becomes possible to apply them to calculate cavities in case of cavitator-cones. In the present method, axisymmetric flow behind a cavitator-cone is used as an unperturbed flow [Logvinovich, Serebriakov 1975; Serebriakov 2008]. The method is illustrated for a case of simultaneous effects of gravity and the angle of attack of the cavitator.

2. Mathematical model

In presenting a mathematical model we follow [Buivol 1980]. It is based on the model of a planar nonstationary potential flow of an ideal fluid. The coordinate system xOy is arranged so that negative axis Ox is directed against the cavitator velocity vector V_0, while the intersection of jets with cone edges occurs at the time $t=0$ in the observation plane at $x=0$. In this case, the problem reduces to the integration of the Laplace equation in the polar coordinate system:

$$\frac{\partial^2\Phi}{\partial r^2}+\frac{1}{r}\frac{\partial\Phi}{\partial r}+\frac{1}{r^2}\frac{\partial^2\Phi}{\partial\vartheta^2}=0 \qquad (1)$$

and satisfying impermeable cavity wall conditions:

$$\frac{\partial R}{\partial t}+\frac{1}{R^2}\frac{\partial\Phi}{\partial\vartheta}\frac{\partial R}{\partial\vartheta}=0, \qquad (2)$$

for a given or constant pressure difference at the cavity boundary:

$$\frac{\partial\Phi}{\partial t}+\frac{1}{2}\left[\left(\frac{\partial\Phi}{\partial r}\right)^2+\left(\frac{1}{r}\frac{\partial\Phi}{\partial\vartheta}\right)^2\right]-\frac{P_\infty-P_c}{\rho}=0, \qquad (3)$$

as well as the condition of solution equals zero at infinity (Φ – the velocity potential, R – axisymmetric cavity radius, P_∞, P_c – pressure values at infinity and in the cavity, respectively, and ρ – the density of the liquid).

An asymptotic solution of the problem for the velocity potential and the radius of the cavity cross-section are found in accordance with the requirements of the theory of small perturbations as follows

$$R=R_0(x,t)+f(x,t,\vartheta),$$

$$\Phi=\Phi_0(x,t,r)+\varphi(x,t,r,\vartheta),$$

where the perturbation φ and f are sufficiently small relative to the ground state and

$$\Phi_0=R_0\dot{R}_0\ln r-\frac{uR_0^2}{r}\cos\vartheta, \qquad (4)$$

$$\varphi=\sum_{n=0}^{\infty}a_n(t)r^{-n}\cos n\vartheta. \qquad (5)$$

Φ_0 in equation (4) is the first member of velocity potential in cylindrical expansion, the second term is the flow potential around a cylinder with speed u, and (5) is a general solution of Laplace equation .

Problems (1) – (3) can be linearized if all equations are expanded in Taylor series in the vicinity of undeformed (axisymmetric) cavity surface and only linear expansion terms are considered. The linearized mathematical model of the problem is as follows:

$$\nabla^2 \varphi = 0, \quad \text{outside the cavity}$$

$$-\frac{\partial f}{\partial t} = (\nabla\Phi_0 - \vec{c})\nabla f - \nabla\varphi\nabla F_0 - f\frac{\partial(\nabla\Phi_0 - \vec{c})\nabla F_0}{\partial n},$$

$$\frac{\partial\varphi}{\partial t} + (\nabla\Phi_0 - \vec{c})\nabla\varphi + f\left[\frac{\partial^2\Phi_0}{\partial n \partial t} + (\nabla\Phi_0 - \vec{c})\frac{\partial\nabla\Phi_0}{\partial n}\right] =$$

$$= -\frac{\partial\Phi_0}{\partial t} + \vec{c}\nabla\Phi_0 - \frac{1}{2}(\nabla\Phi_0)^2 - \frac{p_k - p_\infty}{\rho}, \tag{6}$$

at the cavity boundary:

$$\vec{c} = \{u\cos\vartheta, -u\sin\vartheta, 0\},$$

where u – the vertical component of the cavity velocity;

n – the vector perpendicular to the surface of the cavity.

Satisfying the first condition (6), we obtain the dependence of the modes a_n of the potential φ of deformational modes f_n of cross-sectional radii of the cavity

$$\frac{na_n}{R_0^{n+1}} = -\dot{f}_n - \dot{R}_0\frac{f_n}{R_0} - un\frac{f_{n-1} - f_{n+1}}{R_0}.$$

After satisfying condition (6), we get an infinite system of nonlinear differential equations of the second order with respect to deformational modes, which after some transformations can be expressed as

$$\ddot{f}_1 = -2\dot{f}_1\frac{\dot{R}_0}{R_0} + 2\dot{f}_1\frac{\dot{f}_2}{R_0} + U,$$

$$\ddot{f}_2 = -\frac{2\dot{R}_0}{R_0}\dot{f}_2 + \frac{1\cdot\ddot{R}_0}{R_0}f_2 - \frac{2\cdot 2u}{R_0}(\dot{f}_1 - \dot{f}_3),$$

$$\text{.......................................}, \tag{7}$$

$$\ddot{f}_n = -\frac{2\dot{R}_0}{R_0}\dot{f}_n + (n-1)\frac{\ddot{R}_0}{R_0}f_n - \frac{2nu}{R_0}(\dot{f}_{n-1} - \dot{f}_{n+1}),$$

$n \geq 3$.

Here $U = \dfrac{1}{F_{rL}^2}$ where the Froude number $F_{rL} = \dfrac{V_0}{\sqrt{gL_k}}$ is defined by the half-length of the cavity, L.

Froude number $Fr = Fr_{d_n}$, which is defined by the break-off diameter d_n.

If appropriate initial conditions for the deformations of the radii and the rate of such deformations are added to (7), one obtains the Cauchy problem

$$f_n(0) = f_{n0}, \quad \dot{f}_n(0) = \dot{f}_{n0}.$$

In the absence of angle of attack and zero initial conditions, the solution of the Cauchy problem determines the influence of gravity on the movement and shape of the cavity. The problem can readily be solved in Mathcad, Matlab or Maple.

3. Cavities behind cones

The case of a disc- shaped cavitator was analyzed previously in [Buivol 1980]. Therefore, without considering this case, we proceed to deriving an equation for the radius of the cavity by a circular cone. In this case, we use an approximate *formulation* of the problem, which is the integration of the generalized differential equation S.S. Grigoryan

$$-4\frac{\partial^2 R^2}{\partial t^2}\ln\frac{\psi(x,t)}{R} + \frac{1}{R^2}\left(\frac{\partial R^2}{\partial t}\right)^2 = 4\sigma, \tag{8}$$

where σ – the cavitational number.

In equation (8) normalized values are used:

$$t = \frac{V_0 t^*}{L}, \quad R = \frac{R^*}{L}.$$

By introducing designation

$$\mu = \ln\frac{\psi(x,t)}{R},$$

one obtains

$$-4\mu\frac{\partial^2 R^2}{\partial t^2} + \frac{1}{R^2}\left(\frac{\partial R^2}{\partial t}\right)^2 = 4\sigma. \tag{9}$$

The two terms in the left part of equation (9) are not of the same order as the original $\dot{R}$, since the tangent of the angle between the tangent to the meridian and the axis of the body cavity is of small order $\delta > 0$. Thus, the second term in this equation is of the order δ^2 so this part can be neglected, which leads to the equation

$$\mu\frac{\partial^2 R^2}{\partial t^2} + \sigma = 0. \tag{10}$$

To integrate equation (10), where the value μ is still unknown, we need to determine initial conditions. To determine the initial conditions, we note that at $t=0$ the observation plane is intersected by a solid object with cross-section S_n, therefore

$$S(t)\big|_{t=0} = S_n, \quad \text{or} \quad \bar{S}(\bar{t})\big|_{\bar{t}=0} = \frac{S(\bar{t})}{S_n} = 1.$$

Below we will omit the dash denoting normalized values.

In order to determine the derivative and simplify the model, we assume that all the kinetic energy is converted into energy of expanding rings and will neglect its part corresponding to the flow along the cavity in addition to the part defined by equation (10). Then the kinetic energy can be calculated by the formula

$$T = \frac{\rho}{2}\int_0^{2\pi}\int_R^{\psi} v_r^2 r\,dr\,d\vartheta,$$

where R – the internal radius of the ring;

ψ – the value of its outer radius corresponding to all the energy dissipated;

v_r – the radial velocity component, which is defined in a symmetric expansion formula $v_r = \dfrac{R\dot{R}}{r}$, where r is a current polar radius in the polar coordinate system (r, ϑ), and $\dot{R} = \dfrac{dR}{dt}$ – the rate of expansion of the inner radius.

Thus, the kinetic energy is determined by a fairly simple formula

$$T = \frac{\rho}{2}(R\dot{R})^2 \int_0^{2\pi} d\vartheta \int_R^{\psi} \frac{dr}{r} = \pi\rho\left(R\dot{R}\right)^2 \ln\left(\frac{\psi}{R}\right) = \pi\mu\rho\left(R\dot{R}\right)^2. \tag{11}$$

Since the kinetic energy equals zero at the initial point of time, the expression (11) is the change in kinetic energy equal to the work of external forces over time *t*. External force here is the resistance of the body, which is calculated by the formula [Zhuravlev 1973]

$$W_0 = c_x S_n \frac{\rho V_\infty^2}{2}.$$

Therefore, $T = W_0$ and

$$\mu\left(\frac{1}{2}\frac{dR^2}{dt}\right)^2 = c_x \frac{V_0^2}{2} R_n^2 \Rightarrow \frac{dR^2}{dt} = \frac{2c_x V_0^2}{\mu R^2},$$

where $c_x = c_x(\sigma)$ – coefficient of resistance of the body.

Thus, using the condition $T = W$, one finds an expression for the derivative at the initial time

$$\left.\frac{dR^2}{dt}\right|_{t=0} = \sqrt{\frac{2c_x}{\mu}},$$

and Grigoryan problem reduces to the integration of equation (10) for the following conditions

$$R^2(0) = R_n^2,$$

$$\left.\frac{dR^2}{dt}\right|_{t=0} = \sqrt{\frac{2c_x}{\mu}}.$$

This leads to the expression for the square of the radius

$$R^2 = -\frac{\sigma}{2\mu}t^2 + t\sqrt{\frac{2c_x}{\mu}} + 1.$$

Using $\dfrac{dR^2}{dt} = 0$ one finds the coordinates of the mid-point intersection

$$-\frac{\sigma t}{\mu} + \sqrt{\frac{2c_x}{\mu}} = 0 \Rightarrow x_m = \frac{\mu}{\sigma}\sqrt{\frac{2c_x}{\mu}},$$

so that the final cavity radius for the cone can be represented as

$$R = R_k\sqrt{1 - a(t-1)^2},$$

$$t = \frac{x}{L},$$

$$L = \frac{\mu}{\sigma}\sqrt{\frac{2c_x}{\mu}},$$

$$\mu = \frac{1}{2}\ln\left[\frac{\ln\left(\dfrac{2}{\sigma}\right)}{e\sigma}\right],$$

$$R_k = R_n\sqrt{\frac{c_x + \sigma}{\sigma}}, \qquad a = \frac{c_x}{c_x + \sigma}.$$

Study [Serebriakov 2008] gives slightly different asymptotic formulas. Therefore, it is recommended to calculate the radius using the formula

$$R_0^2 = \frac{1}{\lambda^2}(x+a)[1-(x+a)],$$

$$a = \frac{(c_d - k\sigma)}{c_d},$$

where $\lambda^2 = \dfrac{2\mu}{\sigma}$ is squared cavity elongation.

To calculate the resistance coefficient of the cone at zero cavitation number, one can use the formula

$$c_{d0} = 2\left[1-(1+\varepsilon^2)\left(1-\frac{\varepsilon^2}{4}\ln\frac{\varepsilon^2}{4}\right)\right],\ \varepsilon = \tan\gamma\,,$$

where γ – a half cone angle.

The solution of the Cauchy problem was implemented using Matlab, for which files with functions were compiled to integrate the system of equations, while a script file was generated to execute the necessary calculations, produce output graphics, etc.

If cavitator has an angle of attack α, it leads to a non-zero derivative of cavity buoyancy already present in the break-off cross-section at the initial point of time. In case of sufficiently thick cones, formula to calculate $\frac{df_1}{dt}$ is the same as for a disc:

$$\dot{h}(x) = -\frac{c_x R_n^2 \sin\alpha\cos\alpha}{2R_0(x)}\bigg|_{x=0} = -\frac{c_x}{4}\sin 2\alpha\,.$$

For thin cones similar expression is simplified:

$$\frac{dh}{dt}\bigg|_{t=0} = -\alpha\,.$$

Fig. 1 shows the vertical displacement $h(t)/R_k$ of the cavity in the cavitation flow over a cone with $\gamma = 12^0$, $\sigma = 0.0125$, Fr = 30 at different angles of attack ($\alpha = 0^0, 3^0, 6^0, 12^0$).

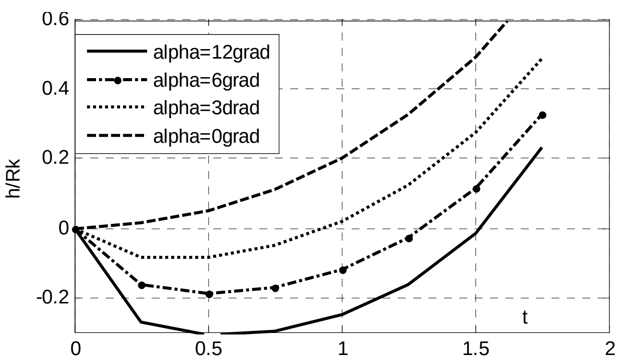

Fig. 1. Floating cavity s = 0.0125; Fr = 30

The presence of the angle of attack causes the from of the cavity to "dive" (negative $h(t)$). The greater the angle of attack, the greater part of the cavern that dives and just the "stern" of the cavity experiences upward displacement due to buoyancy. Therefore, gravity has a strong influence on the angle of attack only in the stern.

However, as shown in Fig. 2, the deformation of the lower part of the cavity is quite weak, since it compensates for the effect of gravitational influence of the angle of attack.

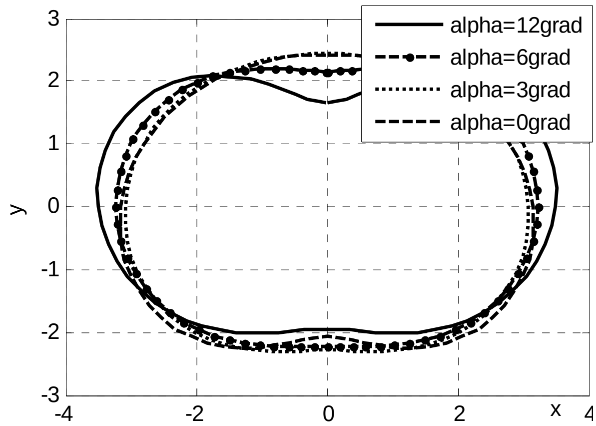

Fig. 2. Section t = 1.5 cavity s = 0.0125; Fr = 30

But the effect of gravity on the upper part of the cavity angle of attack increases by creating a movement of this part down, which leads to a depression in the top of the cavity (Fig. 2).

4. Conclusions

The method of cavitational flow analysis for a cone-shaped cavitator in presence of gravity is presented. It uses the method of small perturbations of an axisymmetric cavity. Differential equations of this problem are linearized in vicinity of undisturbed cavity surface. A mathematical model is presented as an infinite system of nonlinear differential equations of the second order. The Cauchy problem for this system is solved using Matlab. Specific results are presented and analyzed to illustrate deformations of the cavity under the influence of gravity.

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В.М. Буйвол. Нелінійний метод розрахунку збурень вісесиметричних кавітаційних течій

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Наведено математичну модель формування каверни при дії збурень різної природи, в основі якої лежать теорії потенціальних течій з вільними границями і малих збурень. Виконано аналіз результатів конкретних кавітаційних течій за конусами. Розглянуто метод розрахунку кавітаційних течій за конусами з урахуванням поля гравітації. Використано метод малих збурень вісесиметричної каверни. Диференціальні рівняння задачі лінеаризуються в околі незбуреної поверхні каверни. Побудовано математичну модель задачі у вигляді нескінченної системи нелінійних диференціальних рівнянь другого порядку. Задачу Коші для цієї системи розв'язано за допомогою пакету прикладної математики Matlab. На конкретних течіях показано процес деформування форми каверни під впливом поля гравітації.

Ключові слова: гравітація; диференціальні рівняння; збурення; каверна; кавітатор; кінетична енергія; коефіцієнт опору; конус; кут атаки; кут піврозхилу; перетин зриву; потенціал; рівняння Лапласа; течія рідини; число кавітації; число Фруда.

В.Н. Буйвол. Нелинейный метод расчета возмущений осесимметричных кавитационных течений

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Приведена математическая модель формирования каверны при действии возмущений различной природы, в основе которой лежат теории потенциальных течений со свободными границами и малых возмущений. Выполнен анализ результатов расчетов конкретных течений за конусами. Рассмотрен метод расчета кавитационных течений за конусами с учетом поля гравитации. Использован метод малых возмущений осесимметричной каверны. Дифференциальные уравнения задачи линеаризованы в окрестности невозмущенной поверхности каверны. Построена математическая модель задачи в виде бесконечной системы нелинейных дифференциальных уравнений второго порядка. Задача Коши для этой системы решена с помощью пакета прикладной математики Matlab. На примерах конкретных течений показан процесс деформирования формы каверны под влиянием поля гравитации.

Ключевые слова: возмущение; гравитация; дифференциальные уравнения; каверна; кавитатор; кинетическая энергия; конус; коэффициент сопротивления; потенциал; сечение срыва; течение жидкости; угол атаки; угол полураствора; уравнение Лапласа; число кавитации; число Фруда.

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