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FEEDFORWARD SEMI-ACTIVE MODEL-BASED CONTROL OF A PLATE CARRYING CONCENTRATED MASSES

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Abstract. *The multiobjective task of optimal control of vibration response of an elastic plate is considered. An application of a genetic algorithm for determination of the optimum compensating force frequency dependence and parameters of concentrated masses for different boundary conditions is described. The principle of virtual work and Ritz approach are employed for investigation of dynamic behaviour of mass-loaded plates, which are subjected to any number of forces. The optimisation problem is formulated as a constrained task. Optimization provided the reduction of both total acceleration level and compensating force. Numerical results show the appropriateness of the model for optimization of concentrated masses values and their location on a plate. Interpolation of optimal compensating force parameters frequency dependence is used for the design of feedforward control system.*

Keywords: concentrated mass; feedforward control; semi-active method; vibration of plates.

1. Introduction

Plates are often used in practical engineering work, e.g. aircraft, spacecraft and ships. In some cases it is necessary to mount devices on elements of a construction under condition of the minimal acoustical and vibrating loading.

To provide effective work of these devices it is necessary to prevent resonance of the structural elements. One of the methods, which can be used in order to solve the problem of acoustical and vibrating loading reduction for engineering construction, is based on applying optimum distribution of the concentrated masses. Passive methods possess a large number of advantages compared to active structural acoustic or vibration control.

The use of passive methods is usually less susceptible to errors, cheaper and requires no additional power supply.

Chen and Handelman [1956] carried out a study on the determination of the fundamental natural frequency of a rectangular plate with a rigid mass under certain boundary conditions using the Rayleigh–Ritz method.

Stokey and Zorowski [1963] developed a general method for determining approximately the natural frequencies of a rectangular plate with arbitrarily located masses.

Laura et al. [1987] calculated the fundamental frequency of a plate carrying several concentrated masses using the optimized Rayleigh method. The effect of attached masses on free vibrations of rectangular plates is studied in paper [Amabili et al. 2006] by considering rotary inertia of concentrated masses and geometric imperfections of the plate.

The works written by Low and Dubey [Low, Dubey 1997; Chai, Low 1993; Low 1997; Low 2003] summarise the existence of three different methods for eigenfrequency determination.

The first is based on Rayleigh quotient [Chai, Low 1993; Low 1997; Low 2003]. It is used only for an approximation of the fundamental frequency and requires the shape function to be known.

The investigation done by Ciancio et al. [2007] is based on the usage of Ritz variational method. This method is employed by the authors in the current study too. A number of other researchers have continued the trend of seeking harmonic solutions to the plate-mass problem.

For the more complicated boundary conditions and structures, carrying concentrated masses, the finite element method is utilised in order to calculate mode shapes and eigenfrequencies.

Employing this method, Ranjan [2006] made a parametric study of mass value and location influence on the fundamental natural frequency of circular plate with clamped (CC) and simply supported (SS) boundary conditions. In this paper two types of classical boundary conditions are investigated: SS and CC condition.

The dynamic behaviour of the rectangular plate excited by a harmonic force at a certain point is studied.

At the decision of an optimizing problem additional restriction was used: the total weight of plates with concentrated masses and plate without masses remained a constant (mass-loaded plate had a smaller thickness).

Unlike filtered signal least mean squares algorithm, which is usually used for feedforward control [Preumont 2003], proposed semi-active control methods gives possibility to control vibration response of flexible structure over its entire surface.

The article is structured as follows. The task of optimal feedforward control is formulated, after that presentation of the governing equations for flexible structure is given.

2. Problem statement

In the case, when the frequency of oscillations varies with the time t, it is reasonable to apply control system (Fig. 1) to the flexible structure for proper adjustment of compensating force value F_C and phase φ_C.

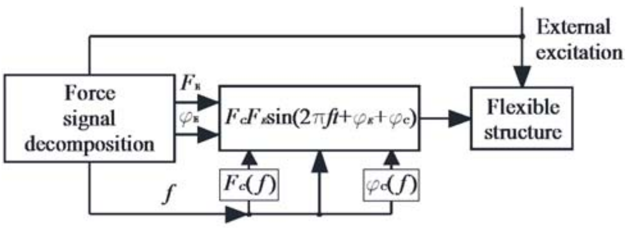


Fig. 1. Feedforward control system

Other parameters, which includes mass values m_m and their location on flexible structure x_m and y_m, cannot be varied during exploitation of flexible structure. These parameters have to be determined prior to the application of control system to the flexible structure. If the location of exciting force is excluded from the set of feasible locations of additional masses and compensating force, then determination of optimum values for these parameters requires the application of optimization algorithm, which is able to find global optimum. Because in this case noise and vibration objective functions dependance on parameters m_m, x_m, y_m, x_c, y_c possess several local minima.

Multiobjective optimization task for semi-active method is formulated as follows:

$$\begin{cases} \min \quad L_{\Sigma a} = 10\lg \dfrac{1}{2a_0^2}\sum_{\{n\}}^{N} a_{n\Sigma}^2, \\ \min \quad F_{cOpt} = \dfrac{1}{N}\sum_{\{n\}}^{N} F_{cnOpt}, \end{cases} \quad (1)$$

where $L_{\Sigma a}$ – r.m.s. acceleration level;

F_{cnOpt} – values of F_{cn} found from the optimization in first objective function.

F_{cn} and φ_{cn} are the value and phase of compensating force at mode n, which is characterized with the combination of n_x and n_y numbers, where n_x is the number of halfwaves, which is contained in plate length a, n_y is the number of halfwaves, which is contained in plate width b.

Optimization is done with the following control parameters: m_m, x_m, y_m, x_c, y_c and constrains:

1) $\sum_{m}^{M} m_m \le \zeta M_{plate}$,

2) $x_{gap} < x_m < a - x_{gap}$, $x_{gap} < x_c < a - x_{gap}$;

3) $y_{gap} < y_m < b - y_{gap}$, $y_{gap} < y_c < b - y_{gap}$;

4) $S_{olap} = 0$,

where ζ=0.3;

M_{plate} is mass of plate part, which oscillates;

x_{gap} and y_{gap} are gaps from the plate boundaries;

S_{olap} is area of attached to the plate objects' overlap.

Evaluation of $a_{n\Sigma}^2$ in the first objective of eq. (1) includes optimization of the following expression with control parameters F_{cn}, φ_{cn}:

$$\min \ a_{n\Sigma}^2 = \omega_n^4 \int_0^a \int_0^b w(\omega_n) w^*(\omega_n) dxdy \quad (2)$$

Optimization in eq. (2) is performed separately for every mode n. The value of transverse motion of flexible structure w in (2) can be estimated based on analytical model of flexible structure.

3. Simplified analytical modelling of plate

Consider transverse motion of a plate with finite dimensions of its area $a \times b \times h$ with concentrated masses m_m attached at the points (x_m, y_m) (Fig. 2).

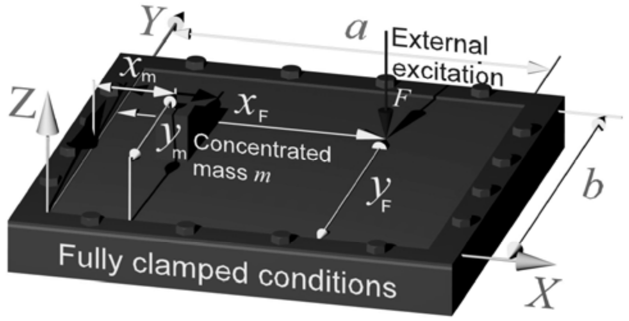


Fig. 2. Setup and geometry of the investigated plate

External force $F_F(t)$ is located in the point (x_F, y_F).

The mathematical model of plate oscillation can be constructed on the basis of a principle of virtual work [Chen, Handelman 1956]. The principle of virtual work for the considered model of plate can be written in the following form:

$$\sum_{F=1}^{K} F_F \delta \vec{w}(x_F, y_F)\, \vec{n} \exp(i\phi_F) =$$

$$= \rho h \int_0^a \int_0^b \ddot{\vec{w}}(x,y)\, \vec{n}\, \delta \vec{w}(x,y)\, \vec{n}\, dxdy +$$

$$+ \sum_{m=1}^{M} m_m \ddot{\vec{w}}(x_m, y_m)\, \vec{n}\, \delta \vec{w}(x_m, y_m)\, \vec{n} +$$

$$+ \sum_{m=1}^{M} \rho_m I_m \Big[\ddot{\vec{w}}_x(x_m, y_m)\, \vec{n}\, \delta \vec{w}_x(x_m, y_m)\, \vec{n} +$$

$$+ \ddot{\vec{w}}_y(x_m, y_m)\, \vec{n}\, \delta \vec{w}_y(x_m, y_m)\, \vec{n} \Big] +$$

$$+ D \int_0^a \int_0^b \Big\{ \Big[\vec{w}_{xx}(x,y)\, \vec{n} + \nu\, \vec{w}_{yy}(x,y)\, \vec{n} \Big] \delta \vec{w}_{xx}(x,y)\, \vec{n} +$$

$$+ \Big[\vec{w}_{yy}(x,y)\, \vec{n} + \nu\, \vec{w}_{xx}(x,y)\, \vec{n} \Big] \delta \vec{w}_{yy}(x,y)\, \vec{n} +$$

$$+ 2(1-\nu)\, \vec{w}_{xy}(x,y)\, \vec{n}\, \delta \vec{w}_{xy}(x,y)\, \vec{n} \Big\} dxdy +$$

$$+ \sum_{F=1}^{K} k_{\mathrm{sh}\,F}\, \vec{w}(x_{\mathrm{sh}\,F}, y_{\mathrm{sh}\,F})\, \vec{n}\, \delta \vec{w}(x_{\mathrm{sh}\,F}, y_{\mathrm{sh}\,F})\, \vec{n} \qquad (3)$$

where $\vec{w}(x, y, t)$ is a vector of transverse displacement at plate oscillation, dots above $\ddot{w}$ denote derivative with respect to time and indices w_{xy} denote the derivative with respect to plate coordinates *x* and *y*;

m_m – the masses which are in points (x_m, y_m);

ρ is density of plate material;

ρ_m, I_m – density and moment of inertia of the *m*-th mass;

$k_{\mathrm{sh}\,F}$ is shaker stiffness;

M and *K* are respectively the number of masses and forces.

$\vec{n}$ – a vector normal to the plate;

D – plate stiffness:

$$D = \frac{Eh^3}{12(1-\nu^2)};$$

E – complex Young's modulus:

$E = E' \cdot (1 + i\,\eta)$;

E' – Young's modulus;

ν – Poisson's ratio;

η – damping loss factor;

The plate is loaded by a harmonic force

$$F_F(t) = F_F \exp(-i\omega t + \varphi_F),$$

which has a phase φ_F and angular frequency ω. For harmonic oscillations of a plate the equation (3) takes a form:

$$\delta\Phi = 0, \qquad (4)$$

where

$$\Phi = \sum_{F=1}^{K} F_F \vec{w}(x_F, y_F)\, \vec{n} \exp(i\varphi_F) +$$

$$+ \frac{\rho h \omega^2}{2} \int_0^a \int_0^b (\vec{w}(x,y)\vec{n}\vec{w}(x,y)\vec{n}) dxdy +$$

$$+ \frac{\omega^2}{2} \sum_{j=1}^{M} \rho_m I_m [\vec{w}_x(x_m, y_m)\vec{n}\vec{w}_x(x_m, y_m)\vec{n} +$$

$$+ \vec{w}_y(x_m, y_m)\vec{n}\vec{w}_y(x_m, y_m)\vec{n}] -$$

$$- D \int_0^a \int_0^b \{ \frac{1}{2} \big[\vec{w}_{xx}(x,y)\vec{n}\vec{w}_{xx}(x,y)\vec{n} +$$

$$+ \vec{w}_{yy}(x,y)\vec{n}\vec{w}_{yy}(x,y)\vec{n} \big] + \nu \vec{w}_{xx}(x,y)\vec{n}\vec{w}_{yy}(x,y)\vec{n} +$$

$$+ (1-\nu)\vec{w}_{xy}(x,y)\vec{n}\vec{w}_{xy}(x,y)\vec{n} \} dxdy +$$

$$+ \frac{\omega^2}{2} \sum_{m=1}^{M} m_m [\vec{w}(x_m, y_m)\vec{n}\vec{w}(x_m, y_m)\vec{n}] -$$

$$- \frac{1}{2} \sum_{F=1}^{K} k_{sh\,F} \big[\vec{w}(x_{sh\,F}, y_{sh\,F})\vec{n}\vec{w}(x_{shF}, y_{sh\,F})\vec{n} \big].$$

In order to receive the solution of equation (4) Ritz method is used. In accordance with Ritz method the form of transverse motion of plates is expressed by means of functions, which $\varphi_n(x)$ satisfy the geometrical boundary conditions.

For example, displacement for plate oscillation can be presented in the form:

$$w = \sum_{\{n\}}^{\infty} A_n X_{nx}(x) Y_{ny}(y), \qquad (5)$$

where $\{n\} = (n_x, n_y)$ is combination of modal numbers, A_n are the unknown coefficients.

For CC plate $X_{nx}(x)$ and $Y_{ny}(y)$ are beam functions which satisfy boundary conditions of the CC beam:

$$X_{nx}(x) = \cos\alpha - \cosh\alpha - \gamma_{nx}(\sin\alpha - \sinh\alpha),$$

$$Y_{ny}(y) = \cos\beta - \cosh\beta - \gamma_{ny}(\sin\beta - \sinh\beta),$$

$$\alpha = \frac{\lambda_{nx} x}{a};$$

$$\beta = \frac{\lambda_{ny} y}{b};$$

$$\cos\lambda_i ch\lambda_i = 1;$$

$$\gamma_i = \frac{\cos\lambda_i - \cosh\lambda_i}{\sin\lambda_i - \sinh\lambda_i}, \quad i = n_x, n_y.$$

For SS plate $X_{nx}(x)$ and $Y_{ny}(y)$ are beam functions which satisfy boundary conditions of the SS beam:

$$X_{nx}(x) = \sin\frac{n_x \pi}{a};$$

$$Y_{ny}(y) = \sin\frac{n_y \pi}{b}.$$

Due to orthogonally of accepted beam functions, function Φ (4) can be simplified. Orthogonal properties of used CC and SS beam functions can be found in [Berthelot 1999].

Because δA_n is finite and arbitrary quantity, the expression (4) can be executed only in the following case:

$$\frac{\partial\Phi}{\partial A_n} = 0 \quad \text{for} \quad n = 1 \ldots N. \qquad (6)$$

Substitution of (5) into (6) leads to system of the linear equations with quantity unknown variables A_n equal to number of modes N:

$$(\mathbf{B}_f + \omega^2 \mathbf{B}_\omega)\mathbf{A} = \mathbf{C}, \qquad (7)$$

where

$$C_n = -\sum_{F=1}^{K} F_F X_{n1}(x_F) Y_{n2}(y_F).$$

For $n \neq s$:

$$b_{\omega ns} = \sum_{m=1}^{M} m_m X_{nx}(x_m) X_{sx}(x_m) Y_{ny}(y_m) Y_{sy}(y_m) +$$

$$+\rho_m I_m \left[\frac{dX_{nx}(x_m)}{dx_m} \frac{dX_{sx}(x_m)}{dx_m} Y_{ny}(y_m) Y_{sy}(y_m) + \right.$$

$$\left. + X_{nx}(x_m) X_{sx}(x_m) \frac{dY_{ny}(y_m)}{dy_m} \frac{dY_{sy}(y_m)}{dy_m} \right];$$

$$b_{fns} = b_{fnsB} - \sum_{F=1}^{K} k_{sh\,F} \times$$

$$\times X_{nx}(x_{sh\,F}) X_{sx}(x_{sh\,F}) Y_{ny}(y_{sh\,F}) Y_{sy}(y_{sh\,F}),$$

where b_{fnsB} depends on boundary conditions:

for CC plate:

$$b_{fnsB} = -\frac{2D}{ab} I^{11}_{nx\,sx} I^{11}_{ny\,sy};$$

for SS plate:

$$b_{fnsB} = 0.$$

For $n = s$:

$$b_{\omega nn} = \sum_{m=1}^{M} m_m X^2_{nx}(x_m) Y^2_{ny}(y_m) +$$

$$+\rho_m I_m \left[\left(\frac{dX_{nx}(x_m)}{dx_m} \right)^2 Y^2_{ny}(y_m) + \right.$$

$$\left. + X^2_{nx}(x_m) \left(\frac{dY_{ny}(y_m)}{dy_m} \right)^2 \right] + abh\rho_p;$$

$$b_{fnn} = b_{fnnB} - \sum_{F=1}^{K} k_{sh\,F} X^2_{nx}(x_{sh\,F}) Y^2_{ny}(y_{sh\,F}),$$

where b_{fnnB} depends on boundary conditions:

for CC plate:

$$b_{fnnB} = -\frac{2D}{ab} I^{11}_{nx\,nx} I^{11}_{ny\,ny} - D\left(\frac{b}{a^3}\lambda^4_{nx} + \frac{a}{b^3}\lambda^4_{ny} \right);$$

for SS plate:

$$b_{fnnB} = -D\left(\frac{b\pi^4}{4a^3}n_x^4 + \frac{a\pi^4}{4b^3}n_y^4 + \frac{\pi^4}{2ab}n_x^2 n_y^2\right).$$

The system of equation (7) can be solved numerically after evaluation of all necessary coefficients.

Solution is valid for any number of forces and masses.

In order to find the eigenfrequencies the following generalized eigenvalue problem has to be solved

$$\mathbf{B}_f \mathbf{V} = \lambda \mathbf{B}_\omega \mathbf{V},$$

where eigenfrequencies can be expressed in terms of eigenvalue λ as follows: $\omega = \sqrt{-\lambda}$.

4. Comparison of the received solution to experimental results

Reduction of acoustic radiation and vibration response of constructions due to the increase of mass, when the sizes of its active area are preserved, is obvious. In the case, when weight of construction remains unchanged, the efficiency of passive method of reduction of vibration response and acoustic radiation by the concentrated masses needs to be proved.

Properties of the investigated CC plate are resulted in a Table 1.

Table 1. Properties of the investigated plate

Property	Value	Dim.
Length, a	0.864	m
Width, b	0.562	m
Thickness, h	$1.97\cdot10^{-3}$	m
Density, ρ	7970	kg/m^3
Mass, suspended on plate, m_F	$31\cdot10^{-3}$	kg
Young's modulus, E	$171\cdot10^{9}$	Pa
Poisson coefficient, ν	0.3	—
Material (DIN 17100)	Steel 37	
Damping loss factor, η	0.01	—

Two plates are used for research. In an experiment with the first plate, which has 1.97 mm thickness (Table 1), the impedance head Brüel and Kjær 8001 attached to the plate represents only one point mass. The second, slightly thinner plate is equipped with additional point masses to remain the same overall weight.

Experimental researches of the second plate with a thickness of 1.47 mm is executed after the calculation of optimum parameters of additional mass by a genetic algorithm. On the basis of the known solution for w , objective function for a genetic algorithm is created:

$$L_{\Sigma obj} = 10\lg \int_{f_{\min}}^{f_{\max}} \omega^2 \int_0^a \int_0^b w(x,y) w^*(x,y)\, dx dy df. \tag{8}$$

This function takes into account vibration speed over the whole active area of plate. When optimization was performed, the integration over the plate area was conducted with relative accuracy of 0.1%.

Integration over the frequency was conducted with accuracy of 0.05%.

This provides sufficient quantity points in order to take into account all modes of vibrations in the considered frequency range. In each of these points integrand is evaluated.

Total maximal mass of the additional masses is limited by the difference of the masses of active areas between plates, which have thicknesses of 1.97 and 1.47 mm.

In such way we provide equality between mass of more thick plate and mass of thin plate with additional masses.

With the purpose of measurements' simplification by the laser scanning head of Polytec PSV-400 (LSV) the additional masses are attached from the side of plate, to which the modal exciter Brüel and Kjær 4809 is fastened. Thus, the top side of the plate remains flat, simplifying the optical measurement.

Optimization was conducted for the five additional masses. For some masses the results of optimization have shown very small values of masses with no practical relevance (weight less than production accuracy).

Thus, the amount of the masses was limited to three, and then to one. During optimization the stiffness of modal exciter is taken into account, in accordance to its specifications.

During optimization, "overlap conditions" are used in order to exclude these positions of the point masses, where they would clash with the impedance head.

Setting the task in this way, eliminates evident solution in accordance with which mass must be placed in position of force application.

Checking for implementation of conditions was inserted in the Matlab function “isTrialFeasible”. This function is used by the functions of genetic algorithm, which create an initial population, and functions of crossover. Also there is a mutation function “mutationadaptfeasible”, which calls “isTrialFeasible” function. Implementation of discrete limiting conditions is thus provided during the whole process of evolution.

During optimization, individuals, which do not have physical sense, are not created saving calculation time.

The result of optimization, which is received from genetic algorithm, was deepened with the hybrid optimization function. As a hybrid function the intrinsic Matlab function “patternsearch” is used, that also satisfies to all mentioned above constraints.

During optimization the followings modes shapes are taken into account: 1-1, 2-1, 1-2, 2-2, 3-1, 1-3, 2-3, 3-2, 4-1, 3-3, 2-4, 4-2.

The range of frequencies, in which optimization was conducted, is 20-200 Hz.

The result of optimization is a set of parameters containing the value of the optimal masses and their position, which are represented on Fig. 3.

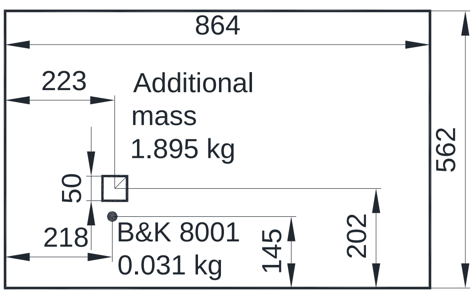


Fig. 3. Scheme of mass location on the active plate area (values given in millimeters)

The dependence of the objective function on the frequency of the modal exciter is shown in Fig. 4 (Numbers mark the modes’ numbers. res. denotes resonance of experimental setup).

Abbreviation res. on Fig. 4 marks resonance of experimental setup, which was accompanied by transverse motion of CC edges of plate.

Peaks on the spectrum with optimum mass on frequencies 65.4 and 76.8 Hz correspond to two different combinations of modes’ pair 1-2 and 3-1.

Other unsigned on Fig. 4 modes cannot be referred to any known forms of vibrations. The modes 5-1, 5-2, 4-3 were not included in the set of modes, for which the optimization was performed.

Appearance of these modes in the frequency range of optimization, leads to additional peaks on the spectrum of objective function. These peaks increase objective on frequencies, which exceed 160 Hz.

The wavelengths of higher modes are shorter, so any inaccuracy in gluing mass or force is more significant for that modes rather than for lower modes.

On low frequencies we have another reason of reduction of method’s efficiency. Value of objective function for a mode 1-1, that is measured on a plate without the additional masses makes -11, on a plate with the optimum masses the objective function is equal -11.1.

It means that on mode 1-1, at the use of optimum parameters of additional mass, take place a slight increase of objective function on 0.1. Due to the inclusion of mode in the frequency range of optimization and in the set of modes, which is taken into account during optimization, the increase objective function is insignificant.

Because mode 1-1 has no nodes, diminishing of vibrations on a mode 1-1 is impossible at the use of method of reduction of vibration by the concentrated masses. Especially, if we take into account that in this case, there were no changes of total mass of construction. In the case when an increase of mass is allowed the reduction of vibration is possible also on the mode 1-1.

Form of vibrations on the mode determines mode contribution to the total vibration response and acoustic radiation of plate. Both modes 2-1 and 1-2 have one nodal line. Mode 2-1 has a greater value of objective function for a plate without mass than mode 1-2. Consequently the reduction of objective function is more important for a mode 2-1 than for a mode 1-2.

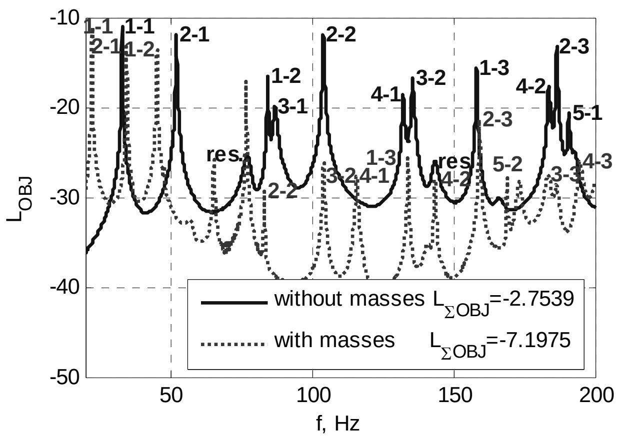


Fig. 4. Dependence of objective function (8) from frequency

During optimization in the wide frequency range an algorithm gives advantage to the modes which have a greater value of objective function. Unlike on the mode 1-2, there is reduction of objective function on the mode 2-1. The use of optimum mass results in displacement of nodal line for a mode 2-1 in position of force. In any case effect of objective function reduction is achieved due to the change of mode shape in such way, that the vibration in the point of force application is decreased. Displacement of nodal line in position of force takes place not always. So for a mode 1-2 a nodal line for optimum configuration vice versa was remote from the point of force application. This results in the growth of objective function on this mode.

At the use of method of vibration response and acoustic radiation attenuation with the concentrated masses the reduction of eigenfrequencies is typical. So on Fig. 4 we can find displacement of all present modes in the considered range. Peaks, which correspond to eigenfrequencies, are differently shifted along the frequency.

The example of such phenomenon is a location of peaks of modes 4-1 and 3-2 on a spectrum represented on Fig. 4. Investigations shows that mass do not cause the change of vibration shape and accordingly do not cause the shift of eigenfrequency, only in the case of its location in the node of mode shape. The closer is the mass location to the peak of the mode shape the greater is the shift of eigenfrequency. Let us consider mode 4-1, its nodes are much closer to the location masses than nodes of mode 3-2, that caused greater shift along frequency of mode 3-2 than the mode 4-1.

In the range of frequencies from 60 Hz the modes of vibrations of plate have sufficient quantity of nodes for the reduction of objective function for each of them. Total reduction of objective function, which is evaluated on the basis of experimental data, is 4.4436.

Intensity of acoustic radiation was measured on the distance 50-55 mm from the acoustic centre of lower microphone to the surface of plate. Measurements are done with the use of signal with frequency, which changes linearly with speed 20 Hz/s.

The received data are the result of 3 averaged sweeps without overlap. Fig. 5 shows the 1/12-oktave acoustic power spectrum for two CC plates with parameters summarized in a Table. 1. On thinner plate optimum mass is placed in accordance with Fig. 5.

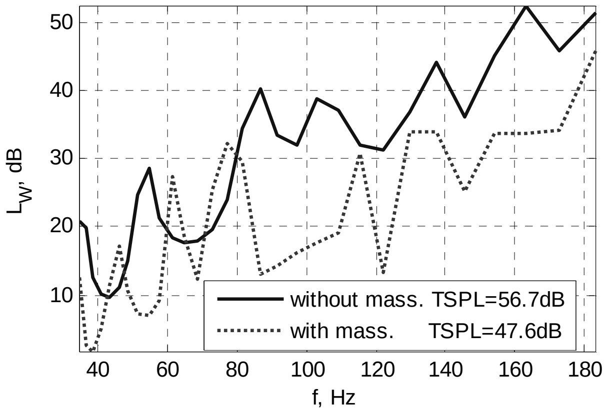


Fig. 5. Sound power level of CC plate

It follows from Fig. 5, that reduction of total level of sound power (TSPL) in the range of frequencies from 35 to 180 Hz composes 9.1 dB. The conducted experimental researches proved the efficiency of the method application for vibration response and acoustic radiation reduction with the concentrated masses at the preservation of total mass.

5. Semi-active control of forced vibration

Orthogonal properties of the beam functions make it possible to simplify eq. (2):

$$a_{n\Sigma}^2 = \omega_n^4 \int_0^a \int_0^b ww^* \, dxdy =$$

$$= \omega_n^4 \int_0^a \int_0^b \left(\sum_{\{n\}}^N A_n X_{nx} Y_{ny} \right) \left(\sum_{\{s\}}^N A_s^* X_{sx} Y_{sy} \right) dxdy =$$

$$= \omega_n^4 ab \sum_{\{n\}}^N A_n(\omega_n) A_n^*(\omega_n),$$

where A_n^* are complex conjugate Ritz coefficients.

The problem is solved with the help of controlled elitist genetic algorithm with non-dominated sorting that is a modified version of the NSGA-II, implemented in Matlab function “gamultiobj”. Principle of this algorithm is shown in Fig. 6.

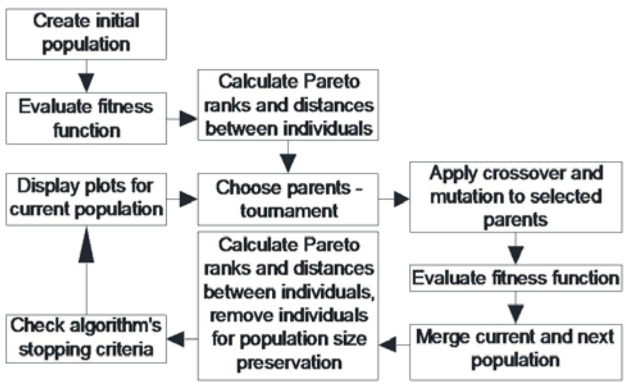


Fig. 6. The scheme for illustration of the principle of multi-objective optimization using a genetic algorithm

The calculation procedure for the block “Calculation of objective functions” is depicted in Fig. 7.

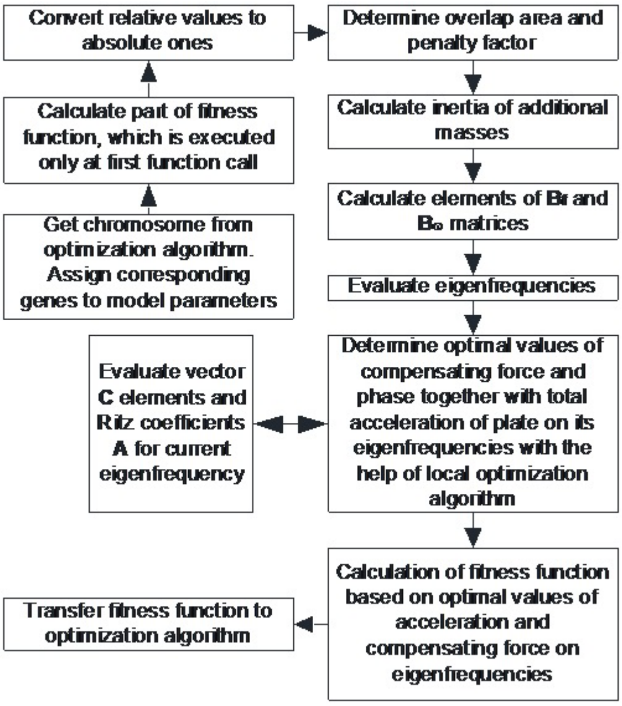


Fig. 7. The algorithm of calculation of objective functions for the semi-active method of vibration reduction

Chromosome of individual consists of the following sequence of genes $\left[\overline{m}_m, \overline{x}_m, \overline{y}_m, \overline{x}_c, \overline{y}_c\right]$, which are transmitted to the objective function of genetic algorithm. The dash over variables designates that the values are relative to $\left[M_{plate,} a, b, a, b\right]$. Under the part of the objective function, which performs only at the first function call, is meant the calculation of values, which are the same for all individuals. These calculations include: assignment of plate parameters, mode numbers set, constant weight location and their values, mass of the plate and compensating force joint, plate density, density of additional masses, offsets from objects on the plate, parameters of the external excitation, beam function constants and calculation of constant intermediate values.

Any optimization algorithm, which is able to quickly find a local optimum function of two variables, is suitable for finding of the optimal values of the function $a^2_{n\Sigma}$ and parameters F_{cn} and φ_{cn}.

Nelder-Mead simplex method is used for optimization of the constrain of type 1 in equation (1). This method is implemented in Matlab functions “fminsearch”.

The multi-objective genetic algorithm NSGA-II is not intended for use with nonlinear constraints. The objective functions $L_{\Sigma a}$ and F_{cOpt} are multiplied by the penalty factor $10^{50*Solap}$ for exclution of overlapping from the set of feasible solutions.

Optimization was performed for the parameters of the plate shown in Table 2 for the oscillation modes: 1-1, 2-1, 1-2, 2-2, 3-1, 1-3, 3-2, 2-3, 4-1, 3-3, 1-4, 4-2, 2-4, 5-1, 4-3, 3-4, 1-5, 5-2.

These modes cover the frequency range 15-250 Hz, for which the optimization was performed.

Table 2. Properties of the investigated plate from steel Ст3 and parameters of invariable masses

Property	Value	Dim.
Active area dimensions, $a\times b$	0,392×0,351	m×m
Thickness, h	$0{,}53\cdot10^{-3}$	m
Density, ρ	7362,6	kg/m^3
Young’s modulus, E'	210e9	Pa
Poisson coefficient, ν	0,3	–
Damping loss factor, η	0,01	–
Mass of compensating force joint, m_c	$2{,}47\cdot10^{-3}$	kg
Mass of exciting force joint, m_e	$7{,}55\cdot10^{-3}$	kg
Accelerometer masses (KB11), m_a	$14\cdot10^{-3}$	kg
Cross-section area of additional mass, $b_m\times b_m$	0,02×0,02	m×m
Additional mass density, ρ_m	7800	kg/m^3

6. Results of problem (1) solution for semi-active control method

The result of the optimization is the Pareto front, depicted in Fig. 8.

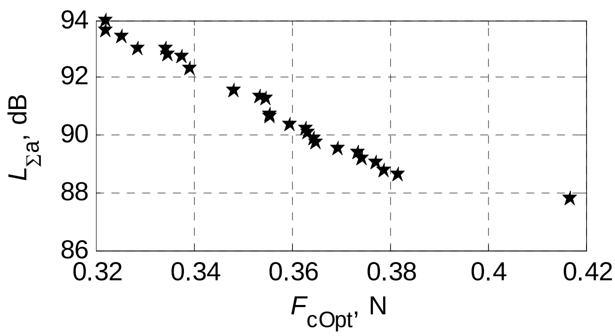


Fig. 8. The set of Pareto optimal solutions found with genetic algorithm

Above the line, consisting of the points of the front, the solution of optimization problem is not optimal. Under the Pareto front there are no solutions. Let us chose solution with objective functions $[F_{cOpt}; L_{\Sigma a}]$ = [0,417H; 87,8 dB], because the difference in the values of the objective function F_{cOpt} is not significant compared with the difference in objective function $L_{\Sigma a}$ values.

Selected solution in the parameter space (i.e. in relative coordinates on the plate) is shown on Fig. 9.

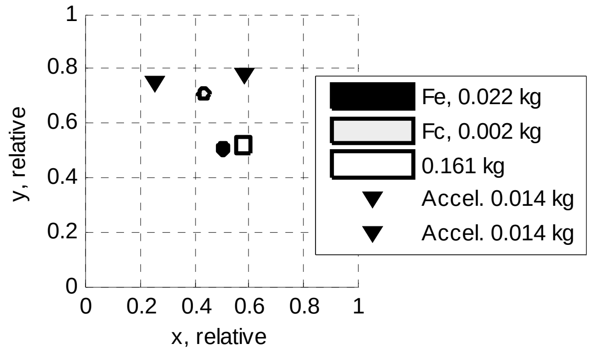


Fig. 9. The layout of the optimal mass (square), the exciting and the optimal compensating forces (circle) and accelerometers (points) in relative coordinates on the CC plate

Spectra for the selected optimal solution are found in a similar way to the optimization problem solution in eq. (2). Algorithms of these calculation differ only in a set of frequencies, for which acceleration is computed. Solutions on the Pareto front are close to each other not only in the objective function space, but also in parameter space. The difference between the maximum and minimum values of the genes on the front are $[\Delta\overline{m}_m = 0{,}0049;\ \Delta\overline{x}_m = 0{,}0016;\ \Delta\overline{y}_m = 0{,}0198;$ $\Delta\overline{x}_c = 0{,}0527;\quad \Delta\overline{y}_c = 0{,}0331]$. The largest difference is observed in the location of compensating force x_c.

Optimal spectra are shown in Fig. 10, 11. I.e. when the compensating force with the parameters shown in Fig. 11 is applied to the plate then we get spectrum marked as "Semi-active" in Fig. 10. Fig. 10 shows the decrease of the total acceleration level on all modes of oscillation. However, there are frequencies (53,3 Hz, 58.9 Hz, 68 Hz, Hz 89.5 etc), where there is almost no reduction in the total acceleration level. These frequencies correspond to the zero values of compensating force on the spectrum of optimum compensating force values (Fig. 11, *a*).

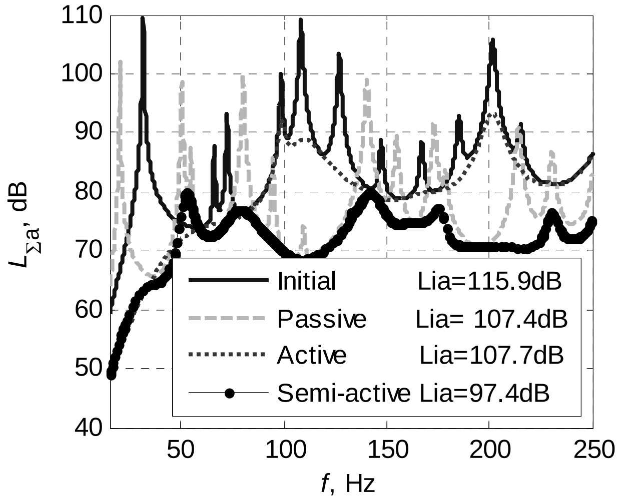
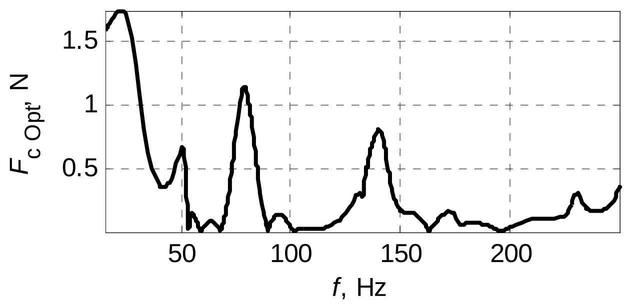
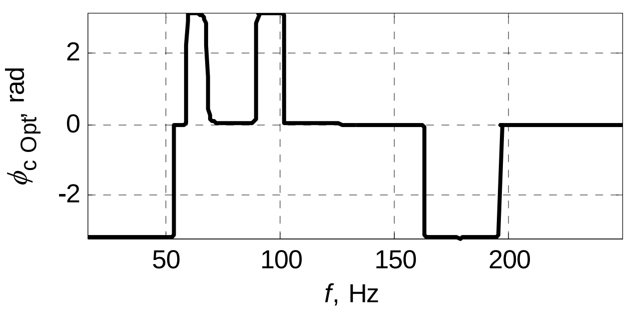


Fig. 10. Optimal spectra of the total acceleration level

On the spectrum of the optimal phase of compensating force (Fig. 11, *b*) at these frequencies takes place a sharp change in the phase on π rad.



a



b

Fig. 11. The optimum spectrum of the compensating force value (*a*), phase (*b*) as a result of optimization for semi-active control method

Such phenomena are caused by the fact that with the change in frequency occur change of the oscillation shape of the plate. Previous parametric studies of the active noise reduction methods have shown small effect of concentrated compensating forces at their placement in the nodes of the oscillation shapes. There are no points on the plate for the entire range of frequencies, through which the nodal line do not pass on some frequencies.

That's why optimization lead to solution, in which there is no reduction of the vibration of the individual intermediate frequencies.

Fig. 10 shows that the semi-active method is the most effective. For accurate determination of the noise reduction methods' efficiency the concept of integral level of acceleration L_{ia} is introduced as follows:

$$L_{ia} = 10\lg \frac{1}{2a_0^2} \int_{f\min}^{f\max} \omega^4 ab \sum_{\{n\}}^{N} A_n A_n^* \,\mathrm{d}f.$$

Fig. 10 shows values of acceleration level for initial configuration, configuration without compensating force (passive method), configuration without additional mass (active method) and configuration with optimal compensating force and mass (semi-active method). Significant reduction of the acceleration level in the frequency bands 85-140 Hz and 180-220 Hz is caused by the usage of the concentrated mass. At the same time the placement of the compensating force and application of optimal spectra of compensating force can significantly reduce each single peak in the spectrum of the total acceleration level.

Optimization shifts the location of compensating force so that its intersections with the nodal lines are at frequencies that do not make a significant contribution to the integral acceleration level.

It can be concluded from the analysis that the total acceleration level compensation must be done with several compensating forces. Applying at least two compensating forces at different positions on the plate, we can get reduced total acceleration at each frequency in the low frequency band under consideration.

For optimum feedforward control of flexible structures optimum compensating force amplitude and phase frequency dependence (Fig. 11) should be included in control system.

Author suggests piecewise cubic Hermite polynomial interpolation of optimal frequency dependences with matlab function "pchip". Received piecewise polynomial representation of $F_c(f)$ and $\varphi_c(f)$ is then embedded into the control system (Fig. 1) in Simulink with "MATLAB Function" block type.

7. Conclusions

The change of the optimal parameters of the control signal, which is caused by the change of external disturbance frequency, led to the necessity of automatic control system design in a wide band of lower frequencies covering the first 10-15 resonances of the elastic structure.

Formulated and solved optimization problems allowed us to establish the optimal parameters of the control signal for integral vibration criteria of the flexible mechanical structures. Multicriteria optimization task of takes into account the criterion of vibration and criteria, the reduction of which leads to a decrease in the control signals values. Experimental verification of the proposed models of control objects, showed a satisfactory correspondence between the results of calculations and measured data.

Application of optimization results to the elastic CC plate in experimental conditions demonstrated decrease of root-mean square velocity level integrated over the plate surface on the 5.5 dB and reduction of total sound power level on 9.1 dB.

Combination of active and passive methods for reduction of the vibrations of flexible structures allowed us to obtain additional reduction of vibration integrated over the entire plate surface by 10 dB.

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Розглянуто багатокритеріальну задачу оптимального управління вібраційною відповіддю пружної пластини. Описано застосування генетичного алгоритму для визначення оптимальної залежності компенсуючої сили від частоти і параметрів концентрованих мас для різних граничних умов. Використано принцип віртуальної роботи і підхід Рітца для дослідження динаміки пластини з приєднаною масою, що знаходиться під дією довільної кількості сил. Задачу оптимізації, яка забезпечує зниження як сумарного рівня віброприскорення, так і компенсуючої сили, сформульовано як задачу з обмеженнями. Зазначено, що числові результати показують придатність моделі для оптимізації величин концентрованих мас та їх розміщень на пластині. Інтерполяцію залежності компенсуючої сили від частоти використано для синтезу системи управління з прямим зв’язком.

Ключові слова: вібрація пластин; комбінований метод; концентрована маса; управління з прямим зв’язком.

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Рассмотрена многокритериальная задача оптимального управления вибрационным ответом упругой пластины. Описано применение генетического алгоритма для определения оптимальной зависимости компенсирующей силы от частоты и параметров концентрированных масс для различных граничных условий. Использован принцип виртуальной работы и подход Ритца для исследования динамики пластины с присоединенной массой, находящейся под действием произвольного количества сил. Задача оптимизации, которая обеспечивает снижение как суммарного уровня виброускорения, так и компенсирующей силы, сформулирована как задача с ограничениями. Отмечено, что численные результаты показывают применимость модели для оптимизации величин концентрированных масс и их размещений на пластине. Интерполяция зависимости компенсирующей силы от частоты использована для синтеза системы управления с прямой связью.

Ключевые слова: вибрация пластин; комбинированный метод; концентрированная масса; управление с прямой связью.

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