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ELECTROMAGNETIC PROCESSES IN THE PULSED INDUCTORS OF THE ELECTRONS ACCELERATORS

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There is investigated by numerical methods the process of magnetic field formation at its excitation in the cross section of the closed laminated core of pulsed inductor. Analysis of the field spreading from the borders of cross section has been done with using of two-dimensional finite-element model of wave equation written in the criterial form with equivalent values of magnetic permeability μ *and dielectric constant* ε *included. Two situations have been considered: 1) the medium of laminated package is anisotropic ideal magneto-dielectrics, i.e. has not electrical conductivity; 2) medium of package has isotropic electromagnetic parameters* μ *and* ε*, but has the losses of energy for eddy currents due to presence of electrical conductivity in the layers of ferromagnetics. It was established the character of wave phenomena and conditions of their essential influence on the field distribution. There is shown that clear diffusion picture of the field spreading occurs when the ratio* $K_W / \mathrm{Re}_M \le 0{,}01$ *(here* K_W *is a non-dimensional wave parameter,* Re_M *is the magnetic Reynolds' number) in the range of the values* $\mathrm{Re}_M = 1...6$*. Effectiveness of the core cross section using was estimated for the conditions of relatively low influence of wave phenomena when it was enough to use the speed of the field diffusion instead of phase velocity of electromagnetic waves in the medium of core.*

Key words: anisotropy; electrical conductivity; electromagnetic wave; field diffusion; inductor; magnetic Reynolds' number; mathematical simulation; multi-layer package; pulsed magnetic field; wave equation.

Characteristics of problem

The modern accelerators of electron beams of high power have the wide area of commercial applications. The next: are most typical of them:

– radiation therapy and radiation surgery;
– sterilization of medical materials, preparations and instruments;
– sterilization and pasteurization of food;
– disinfection of products in agricultural sphere;
– technology of volume and surface polymerization of materials without using of special chemical hardening;
– utilization and processing of technical rubber after using including the automobile tires;
– disinfection, sterilization and clearing of natural water and effluent in big volume.

In the induction accelerators of electrons the pulsed inductors is usable for the creation of longitudinal electric field which accelerates the electron beam. Namely inductors are under consideration in this paper. They are manufactured in view of ferromagnetic rings [1] with a primary winding which has several (mostly three or four) parallel segments and supplied from source of voltage at near rectangular form of pulses. Electron beam plays the role of secondary contour of inductor. The essential specifics of inductor consist of small duration of pulse (several tens or hundreds of nanoseconds) and of external shielding body presence manufactured of conductor. Electric field and corresponding currents induced at the internal surface of screen define the characteristics of working process and the parameters of energy efficiency for accelerator. During the pulse of voltage the magnetization current of inductor rises as quasi-linear function of time what is typical for the pulsed transformers.

The film ferromagnetic materials as Metglas (produced by company Honeywell in USA) are used in the pulsed and high frequency devices and they are manufactured as thin layer of ferromagnetics (thickness 12…22 μm) on the polymer band thickness 4…6 μm and width 1” or more. Winding package of this type presents the multi-layer structure at taking turns of magnetic and non-magnetic layers. Similar materials are in the production also in Russia under name Melta. The cost of amorphous magnetic materials is high enough, that is why the question about effective use of the cores volume is actual for the reduction of accelerators price.

Analysis investigations and printed works

The question about peculiarities of electromagnetic processes in the multi-layer media at the presence of non-magnetic inter-layers has been considered in the many scientific publications beginning from the classical works [4, 5]. This phenomenon has been investigated both from the point of view of definition of equivalent parameters of laminated packages and from the point of view of their frequency response definition. In particular the main feature of wave processes in the uniform electro-conducting medium have been considered in [6]. The frequency and pulsed modes of magnetization have been studied in the work [7]. The pulsed process of the plane electromagnetic wave was considered in the book [9] on the example of sea water. The paper [8] must be mentioned as first among the late publications, there are investigated the frequency features of equivalent permeability in laminated magnetic cores of power equipment. The pulsed excitation of magnetic field was studied by authors of present paper in application to finely laminated packages both in diffusion approximation [1] and on the base of wave equation analysis but without taking into account the affect of the electrical conductivity of medium [2]. Information about following steps of this (taking into account the electrical conductivity) was presented in the report on the international conference in Sebastopol [12].

The purpose of work

The goal of this paper is to show the process of the field penetration into laminated package in comparison of diffusion phenomena and wave processes whose role is growing along with a rise of the re-magnetization frequency or with reduction of the pulses of field duration. The mathematic simulation of the process of the field setting up is the main tool of this investigation at using of given equivalent parameters of medium: magnetic permeability μ and dielectric constant ε. Unlike to work [2] there is taken into account the effect of electrical conductivity of medium σ and the combined criteria of this effect are defined. The pulsed field is considered in the cross section of the circular inductors wounded by the band of amorphous ferromagnetics as Metglass or Melta on the polymer electro-insulating underlayer.

The formulation of task

The most full analysis of the magnetic field penetration into multi-layer core can be performed on the base of the equation for induction with account of currents of conductivity, currents of displacement and possible vibration of ferromagnetic layers when their velocity $V \neq 0$. This equation can be written at use of equivalent parameters of medium μ, σ, ε as

$$\Delta\vec{H} = \frac{\partial}{\partial t}(\mu\,\sigma\,\vec{H}) + rot\left[(\mu\,\sigma)\vec{V}\times\vec{H}\right] + \frac{\partial^2}{\partial t^2}\left(\mu\,\varepsilon\,\vec{H}\right), \quad (1)$$

If it is possible to neglect the possible vibration of layers (V = 0) and to assume that equivalent parameters are not depending on the time and coordinates then equation for z-component of induction will be looking as

$$\frac{\partial^2\vec{B}}{\partial t^2} + \frac{\sigma}{\varepsilon}\frac{\partial\vec{B}}{\partial t} = V_{em}^2\Delta\vec{H}\,, \qquad (2)$$

where $V_{em} = \dfrac{1}{\sqrt{\mu\varepsilon}}$ is a velocity of electromagnetic wave propagation in the equivalent medium.

After choice of scaling factor for the time and space coordinates of process (T and L respectively) the equation (2) can be written with normalized variables:

$$\frac{\partial^2\vec{B}}{\partial\tilde{t}^2} + \frac{\sigma T}{\varepsilon}\frac{\partial\vec{B}}{\partial\tilde{t}} = \frac{V_{em}^2 T^2}{L^2}\tilde{\Delta}\vec{B}\,, \qquad (3)$$

here $\tilde{\Delta} = L^2\Delta = \dfrac{\partial^2}{\partial\tilde{x}^2} + \dfrac{\partial^2}{\partial\tilde{y}^2} + \dfrac{\partial^2}{\partial\tilde{z}^2}$ is the Laplace's operator in normalized coordinates,

$$\tilde{t} = \frac{t}{T}\,, \quad \tilde{x} = \frac{x}{L}\,, \ \tilde{y} = \frac{y}{L}\,, \ \tilde{z} = \frac{z}{L}\,.$$

That is reasonable to accept a duration T of the current pulse supplied the inductor as the time scale of process. The path of electromagnetic wave run during the pulse duration can be accepted as the space scale of calculation model:

$L = V_{em}T$.

Now equation (3) for one-dimensional problem is reduced to equation (4) known from [3] (below in the equation (4) and later a designation for normalized variables "~" is omitted):

$$\frac{\partial^2\vec{B}}{\partial t^2} + k\frac{\partial\vec{B}}{\partial t} = a^2\frac{\partial^2\vec{B}}{\partial x^2}\,, \qquad (4)$$

where for situation under consideration

$$k = \frac{\sigma T}{\varepsilon}\,; \quad a^2 = \frac{V_{em}^2 T^2}{L^2} = 1\,. \qquad (5)$$

Correlations (5) define the relative role of wave process and diffusion process at the pulsed field setting up in the core.

At k >>1 the effect of the wave process can be neglected, then diffusion equation can be considered in the form [1]

$$\frac{\partial \vec{B}}{\partial t} = D \Delta \vec{B},$$

where $D = \frac{1}{\mu\sigma}$ is a diffusion coefficient for the field penetration into multi-layer medium.

If the condition k >>1 is not satisfied the wave process must be included into consideration due to damping of wave in the process of its propagation what is stipulated by energy loss on Joule's heating in the electro-conducting layers of ferromagnetics.

A fundamental solution of equation (4) has view of movable Heaviside's function θ of variable amplitude which can be described by modified Bessel's function $I_0(\xi)$ and exponential law of damping as

$$E(x,t) = \frac{1}{2}\theta(t - |x|)\, e^{-kt/2}\, I_0(\xi), \qquad (6)$$

where $\xi = \frac{1}{2}k\sqrt{t^2 - x^2}$.

After substitution $B(x,t) = \exp(-\frac{1}{2}kt)b(x,t)$ it is possible to exclude the first derivative on time. In the 1D case that leads to one of simple form of Clein and Gordon equation [3] which is well studied in the quantum theory of field and other applications:

$$\frac{\partial^2 b}{\partial t^2} = \frac{\partial^2 b}{\partial x^2} + qb, \qquad (7)$$

where $q = \frac{1}{4}k^2$.

Equation (7) has the same fundamental solution as (6).

At the short pulses (with T < 10^{-7}sec) it is impossible to neglect the wave phenomena without proper ground.

Electrical conductivity of ferromagnetics causes a damping of waves which propagate from the borders of the core cross section to the central part of core. At the same time the role of diffusion phenomena at the setting up of the field in the cross section of core is growing.

A detail analysis of wave processes is known for the alternating sinusoidal fields [6], in particular for multi-layer packages [8]. Concerning of the pulsed electromagnetic waves propagation in the electro-conducting media, this phenomena have been studied in the literature rather from qualitative side than quantitative one.

The last versions of program software Comsol [10] or more accessible for all users the specialized program EMP-3 of company Field Precision (USA) [11] can be used for the numerical simulation of the wave process of the field propagation in the electro-conducting medium.

The equation (2) can be written for simulation via normalized variables as

$$\Delta \overline{H} = \mathrm{Re}_M \frac{\partial \overline{H}}{\partial t} + K_W \frac{\partial^2 \overline{H}}{\partial t^2}, \qquad (8)$$

with next designations:

$$\mathrm{Re}_M = \mu\sigma V_{bas} L_m X_m,$$

$$V_{bas} = \frac{L_m X_m}{T_m},$$

$$K_W = \left(\frac{V_{bas}}{V_{em}}\right)^2,$$

$$V_{em} = \frac{1}{\sqrt{\mu\varepsilon}},$$

where Re_M is a magnetic Reynolds' number;

V_{bas} is the basic velocity which is necessary for passage of electromagnetic wave along full length of the cross section transversal dimension;

K_W is the square of ratio of basic velocity V_{bas} to the phase velocity of electromagnetic waves V_{em} in the medium of package;

L_m is the basic dimension of model in non-dimensional coordinates;

X_m is the scaling coefficient for transfer of non-dimensional basic length or width of cross section into real dimensional values;

T_m is real duration of process in seconds.

Equation (8) can be written also in the form

$$\Delta \vec{H} = \mathrm{Re}_M \left(\frac{\partial \vec{H}}{\partial t} + \beta \frac{\partial^2 \vec{H}}{\partial t^2} \right), \qquad (9)$$

$$\beta = \frac{K_W}{\mathrm{Re}_M} = \frac{\varepsilon}{\sigma T_m},$$

where β is a parameter which define the relative role of the wave process in the field propagation on the background of diffusion process.

Simulation of wave process in ideal magneto-dielectrics

To study the role of wave process it is reasonable to consider first the ideal magneto-dielectrics $(\sigma = 0)$,

i.e. to believe we investigate the core without loss of energy, for which the equation (8) is true at $\mathrm{Re}_M = 0$.

The laminated package of core is presented in simulation model as continuum with anisotropic equivalent parameters μ and ε what is result of layered structure of package. Anisotropic properties of package were introduced by matrix coefficient

$$D_W = \frac{1}{K_W} = \frac{1}{\mu\varepsilon V_{bas}^2} = \begin{bmatrix} D_W(x) & 0 \\ 0 & D_W(y) \end{bmatrix}.$$

The phase velocity of electromagnetic waves in the medium of core depends on the relative values of parameters $\mu_r = \frac{\mu}{\mu_0}$ and $\varepsilon_r = \frac{\varepsilon}{\varepsilon_0}$:

$$V_{em} = \frac{\sqrt{\mu_0\,\varepsilon_0}}{\sqrt{\mu_r\,\varepsilon_r}} = \frac{v_r}{c},$$

here $\nu_r = \frac{\nu}{c} = \frac{1}{\sqrt{\mu_r \varepsilon_r}}$, $c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$,

c is a velocity of electromagnetic waves in the vacuum.

Hence coefficient D_W is

$$D_W = \frac{v_r^2}{(cV_{bas})^2}.$$

The field propagation was considered in the plane *A-A* of the model cross section (Fig.1), i.e. along the normal direction to the plates of ferromagnetics. A boundary condition on the external border of the core cross section was taken as $H_z(t_0)=1$. During simulation in the software FemLab the relative value of the propagation velocity v_r was gradually reduced at the fixed basic velocity V_{bas}. At reduction of parameter v_r the process of the field setting up was located in the zones near the border, what can be called as the "wave skin-effect'.

Account of electromagnetic properties enabled to study the coordinate specifics of this process [2].

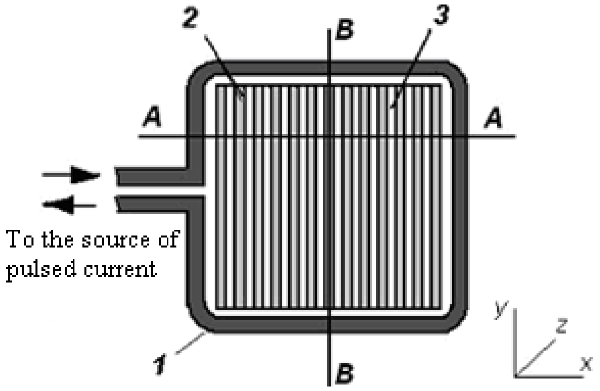

Fig.1. Schematic view of the core cross section of the pulsed inductor:
1 – a turn of winding fot the field excitation;
2 – the layers of ferromagnetics;
3 – the layers of insulation.

For the relative electromagnetic parameters μ and ε which values slightly exceed 1, wave-skin distribution of field near the borders is seen only in case of very fast processes (duration less than 10^{-9} sec) for the cores of big enough dimensions, when the needed basic velocity $V_{баз}$ more than velocity of the wave propagation in this medium, i.e. when coefficient $K_W>1$.

Distribution of the pulsed magnetic field taking into account the electrical conductivity

For making clear the influence of electrical conductivity of ferromagnetics layer on the character of wave process at the field excitation it was studied the field distribution according to equation (9).From the above-going definition (5) we have $k = 1/\beta$. It means that effect of wfve process is very small at $\beta << 1$ but must be essential at $\beta \geq 1$. At use of software package Comsol v.3.5 the two-dimensional simulation of the pulsed field was performed in the cross section of core with non-dimensional values 0.8 ×0.8 for uniform and isotropic equivalent parameters μ and ε. Boundary condition was given as the pulsed switching on the field along the perimetric line of cross section [12]. Variable criteria of problem are the next: $\mathrm{Re}_M = \mathrm{var}$ at the constant $\beta = 1$, or $\beta = \mathrm{var}$ at the constant $\mathrm{Re}_M = 5$.

For $\mathrm{Re}_M = 1$ the influence of wave process is small if $\beta = 0{,}01 \cdots 0{,}5$, but with rise of parameter β up to meaning 0.5 … 1.0 the motion of the stepping waves on induction is seen from the borders of cross section to the center, and at their convergence in the central zone the oscillation of induction occur up to 3 …5 times more in value than at the border, and gradually this oscillations reduce and induction becomes close to the boundary value.

The instantaneous picture of the field distribution via time step $0.1T_m$ are shown in the Fig. 2 for parameters $\mathrm{Re}_M = 1,\ \beta = 0{,}5$.

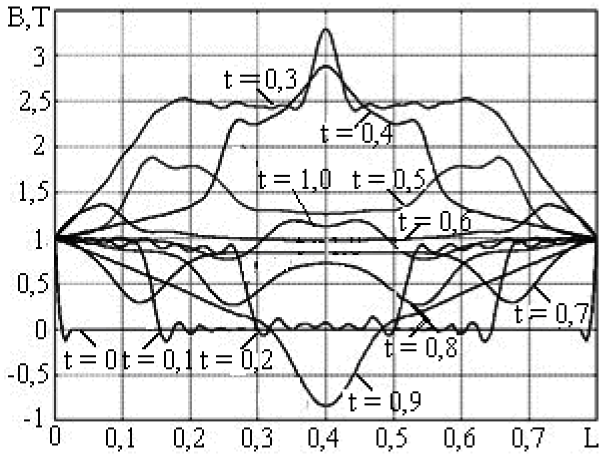

Fig.2. Distribution of field along the middle line of the core cross section ($\mathrm{Re}_M = 1,\ \beta = 0{,}5$).

In the Fig. 3 with use of grey scale a two-dimensional distribution of field is shown in the cross section of core near the end of process ($t/T_m = 1$) for meanings of parameters $\mathrm{Re}_M = 1,\quad \beta = 1$. While boundary value of field is equal 1 (Fig. 3) the field in the central zone is equal 0.7 from boundary value, and in the transition areas field has oscillation in the range 0.3 … 0.7.

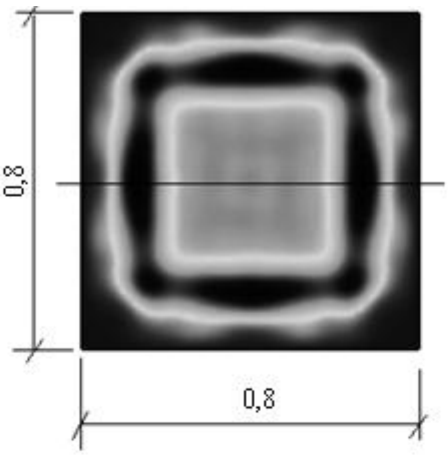

Fig.3 Two-dimensional picture of field in the cross section of core for meanings of parameters $\mathrm{Re}_M = 1,\quad \beta = 1$.

Rising of parameter Re_M leads to more intensive damping of wave during their propagation from borders to the center, their speed reduces. For values $\mathrm{Re}_M = 3 \ldots 5$ and $\beta = 1$ the oscillations of the field in the central part of package cannot be finished during the pulse duration through lack of time after convergence of waves from the borders of core.

The process of field setting up in the cross section of core is shown in time in the Fig. 4. for parameters $\mathrm{Re}_M = 5,\quad \beta = 1$.

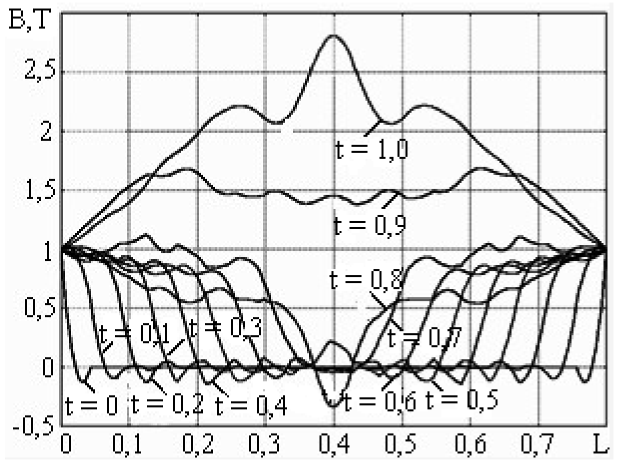

Fig. 4. The field distribution along the middle line of the core cross section ($\mathrm{Re}_M = 5,\quad \beta = 1$).

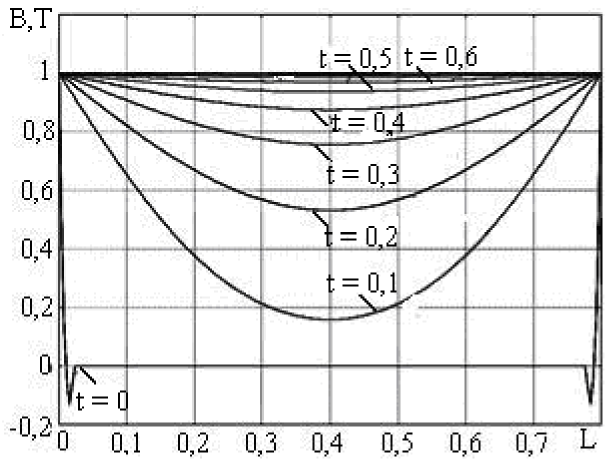

Fig. 5. Diffusion process of the field setting up in the cross section of core for $\mathrm{Re}_M = 5$, $\beta = 0{,}01$.

At the bigger values $\mathrm{Re}_M = 3 \ldots 5$ the conditions for transition from specific wave picture of field propagation to the diffusion picture have been investigated. Thus, for $\mathrm{Re}_M = 5$ and $\beta \le 0{,}01$ the picture of the field setting up has character of pure diffusion distribution (Fig. 5).

Conclusion

Investigation of the wave equation solution for electromagnetic field in the anisotropic magneto-dielectrics with no losses of energy (electrical conductivity is absent) for the wide range of electromagnetic parameters μ, ε (beginning from $\mu_0\varepsilon_0 = 1/c^2$ up to $\mu\varepsilon = 10^6\mu_0\varepsilon_0 = 10^6/c^2$) shows the existence of specific modes of the field setting up near the borders of rectangular cross section of core during the pulsed excitation of field.

The specifics of this process is defined by the criterial value $D_w = 1/K_W$ which is the square of ratio of the electromagnetic waves velocity V_{em} in the medium which is equivalent of the laminated package to the basic velocity V_{bas} which is necessary for the running of wave along the cross section of package during the pulse duration.

At the meanings of D_w going down to the value 0.01 it is seen clearly the location of the wave process in the near the borders zones with formation of so called “wave skin-effect”.

Electrical conductivity of ferromagnetics was taken into account at the consideration of the wave phenomena in the package of core laminated by sheets of micrometers thickness. There was used the assumption about isotropic properties of parameters μ and ε what gave a possibility to define the relative role of wave process and diffusion process of the field propagation in connection with magnetic Reynolds' number Re_M and parameter $\beta = \varepsilon/\sigma T_m = K_W/\mathrm{Re}_M$. The results of numerical simulation certify that in the range of magnetic Reynolds' number $\mathrm{Re}_M = 1...5$ for $\beta \le 0{,}01$ the picture of the field propagation has pure diffusion character and influence of the wave phenomena can be neglected. In general mathematic simulation with use of finite-element software FemLab and Comsol had resulted detailed representation about features of electromagnetic processes in the laminated packages wounded by extremely thin sheets of amorphous ferromagnetics. On the base of simulation results it is possible to estimate the effectiveness of the amorphous ferromagnetics volume using as the factor which define the high cost of inductor.

References

1. Chemerys V.T., 2008. *Diffusion of Electromagnetic Field Into the Core of Inductor at Induction Accelerator of Electrons* / V.T. Chemerys, I.O. Borodiy // Proceedings of National Aviation University – N 2. – P. 44 – 51.

2. *Чемерис В.Т.* Хвильовий скін-ефект в анізотропному магнітодіелектрику/ В.Т.Чемерис, І.О. Бородій // Наукоємні технології. – 2011. – № 3 – 4. – С. 99–104.

Chemerys V.T. 2011. *Wave skin-effect in anisotropic magneto-dielectrics* / V.T. Chemerys, I.O. Borodiy // Science-Based Technologies – N 3-4: 99-104.

3. *Полянин А.Д.* Справочник по линейным уравнениям математической физики /А.Д. Полянин. – М.: Физматлит. – 2001. – 576 с.

Polyanin A.D. 2001. *Guidebook for linear equations of mathematical physics /* A.D. Polyanin Moscow: PhysMatLit., 576 p.

4. *Бреховских Л. М.* Волны в слоистых средах. / Л.М.Бреховских. Из–во. АН СССР. – 1957. – 503 с.

Brehovskih L.M.1957.*Waves in the laminated media* / L.M. Brehovskih. *USSR Academy of Science Publ.*, 503 p.

5. *Поливанов К.. М.* Магнитные спектры материала, обусловленные макроскопической структурой// К.М. Поливанов. Электродинамика вещественных сред (сб. статей).– М.: Энергоатомиздат.– 1988.– С. 101 – 118.

Polivanov K.M. 1988. *Magnetic spectra of material stipulated by its macroscopic structutre* // K.M. Polivanov. Electrodynamics of substantial media. Moscow: Energoatomizdat. – P.101 – 118.

6. *Матвеев А.Н.* Электричество и магнетизм / А.Н. Матвеев – М.: Высш. шк. – 1983. – 464 с.

Matveev A.N. 1983. *Electricity and magnetism*/ A.N. Matveev .Moscow: High School Publ., 464 p.

7. *Поливанов К.М.* Намагничивание магнито-диэлектрических стержней прямоугольного сечения // *К.М. Поливанов*. Электродинамика вещественных сред (сб. статей).– М. Энергоатомиздат.– 1988.– С. 45 – 79.

Polivanov K.M. 1988. *Magnetization of magneto-dielectric cores at rectangular cross section* // K.M. Polivanov. Electrodynamics of substantial media. Moscow: Energoatomizdat .– P.45 – 79.

8. Abeywickram K.G. N.B. 2007. *Computations of Parameters of Power Transformer Windings for Use in Frequency Response Analysis.*/ K. G. N. B Abeywickrama, A.D. Podoltsev, Yu.V. Serdyuk, S.W. Gubanski // IEEE Trans. on Magnetics. Vol.43. N 5. – P. 1983 – 1990.

9. Rothwell E. J. 2001. *Electromagnetics* / E. J. Rothwell, M. J. Cloud. Boca Raton – London – New York – Washington, D.C.: CRC Press, 540 p.

10. *Comsol Multiphysics: Modeling guide, v.3.4.* – Available from internet: http: // www. chemeng.gtua.gr/courses/comp_tp/modeling_guide. pdf.

11. *EMP-3 – 3D Electromagnetic Fields Software.* – Available from internet: http://www.fieldp.com/ /emp3.html.

12. *Чемерис В.Т.* Моделирование волновых процессов в индукторных системах с учетом потерь энергии на вихревые токи / В.Т. Чемерис, И.О. Бородий // Проблемы повышения эффективности электромеханических преобразователей в электроэнергетических системах. – Севастополь: СевГТУ, 2012. – С.148 – 149.

Chemerys V.T. 2012. *Simulation of the wave processes in inductor systems taking into account the energy losses for eddy currents*/ V.T.Chemerys, I.O. Borodiy // The problems of effectiveness enchance for electromechanical converters at electrical energetic systems. – Sebastopol: Sebastopol State Techn. Univ. – P.148 – 149 .