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## RESEARCHES OF METROLOGY DESCRIPTIONS OF MEAN MEASURING OF MOMENT INERTIA ROTORS ELECTROMOTORS

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Static metrology descriptions of mean of measuring of moment of inertia of rotors of electro motors are investigational on the basis of got by decomposition in the row of Taylors functions of transformation of analytical expressions, which describe a sensitiveness, nominal function of transformation, functions of coefficients of influence of influencing parameters and speeds of their change, additive and multiplicative errors

## Statement of the problem

In tests electric motor on regulatory compliance a key parameter is the rotational moment of inertia. Based on the moments of inertia are calculated many other rotational parameters of an electric motor (EM). The most important of these are dynamic moment, the moment of resistance, and so on [1].

The development of measurement and study of their metrological characteristics is an actual scientific problem for every exact measurement (test), especially during the tests, the EM and the study of their quality. In this regard, there is a need to develop a qualitatively new means of measurement (MM), the moment of inertia, which correspond to the modern development of science and technology, as well as studies of their basic metrological parameters for the synthesis of this type of MM from normalized metrological characteristics.

### Analysis of research and publications

Given the well-known analytical expression for finding the moment of inertia, which is given in references [1, 2], we write the transformation of MM inertia as

$$N_{J} = \frac{2 \ln \left(\frac{M_{k} loh^{3}}{er^{4} C \mathcal{I} \mathcal{I} \mathcal{I}}\right)}{P T_{0}} J, \qquad (1)$$

where  $N_J$  – number of pulses at the input timercounter MM moment of inertia for a time interval of free oscillations of the rotor EM (initial value);

 $M_k$  – torque at slip S = 1, which is described by Kloss [1];

l – length of the measuring arm;

 $\xi$ - modulus of elasticity of the membrane sensor efforts (SE);

h – thickness of the membrane SE;

$$e = 0.17;$$

r - the radius of the membrane SE;

C – stiffness SE;

 $\Box \Box = -$  Normalized value absolute error transducer (in particular, the accuracy SE);

J – moment of inertia is measured (input value);

P – coefficient of damping;

 $T_0$  – period model pulses, which is filled with time measurement of the moment of inertia after the completion of the transition process (with and without power and electric motor torque reduction of the value of M<sub>k</sub> to zero).

The aim is to create models of basic metrological static characteristics of MM inertia and research on the basis of their metrological characteristics that will identify the reasons for the increased errors, correct them by taking into account deviations investigated and synthesized MM inertia with normalized metrological characteristics.

# Metrological models of basic static characteristics

In operating conditions pollutants inertia carries functional transformation informative parameter J (moment of inertia) to the number of pulses, which are calculated over a time interval meter free vibration rotor EM N<sub>I</sub>. In addition to an informative signal on him are influencing variables which include all the other parameters that have a direct connection with the original value and attract emergence uninformative component transformation. From equation of (1) that the process of converting information parameter J in output affects a large number of influencing factors. But due to the fact that almost all the values are the same, and shall be measured at the EM deenergized when the value of the torque value changes from  $M_k$ to zero, then it is reasonable to study the torque is affected as the influence quantity on the measured moment of inertia (information parameter).

To obtain metrological models of basic static characteristics of MM inertia expand transformation equation (1) in a Taylor series, which will take the form

$$N_{J} = N_{J_{0}} + \left[\frac{\partial N_{J}}{\partial J}\right]_{0} J + \frac{1}{2!} \left[\frac{\partial^{2} N_{J}}{\partial J^{2}}\right]_{0} J^{2} + \frac{1}{3!} \left[\frac{\partial^{3} N_{J}}{\partial J^{3}}\right]_{0} J^{3} + \dots + \left[\frac{\partial^{2} N_{J}}{\partial J \partial M_{k}}\right]_{0} J \mathcal{A} M_{k} + \left[\frac{\partial N_{J}}{\partial M_{k}}\right]_{0} \mathcal{A} M_{k} + \frac{1}{2!} \left[\frac{\partial^{2} N_{J}}{\partial M_{k}^{2}}\right]_{0} \mathcal{A} M_{k}^{2} + \dots, (2)$$

where  $N_{Jo}$  - free term of the expansion of space equal to the output signal at zero input value (  $N_{Jo} =$ 0 for J = 0);

$$\frac{\partial N_{J}}{\partial J} - \text{Sensitivity MM inertia (S_{J});}$$
$$\frac{\partial^{2} N_{J}}{\partial J^{2}} - \text{Changing the MM of inertia (S_{J}');}$$
$$\frac{\partial^{3} N_{J}}{\partial J^{3}} - \text{The rate of change of sensitivity MM}$$

moment of inertia  $(S_J'')$ ;

$$\frac{\partial^2 N_J}{\partial J \partial M_k}$$
 - The combined effect of the influence

quantity the rated sensitivity ( $\delta_{Mk}$ );

 $AM_k$  - Change (deviation) of the normalized influence quantity (graded) torque  $M_k$ ;

 $\frac{\partial N_{J}}{\partial M_{k}}$  - influence factor affecting the value to

the output  $(B_J)$ ;

$$\frac{\partial^2 N_J}{\partial M_k^2}$$
 - The rate of change of the coefficient of

influence affecting the value on the output signal  $(B_J)$ .

Analytical expression for the sensitivity characteristics of MM research inertia is given by

$$S_{J} = \frac{\partial N_{J}}{\partial J} = \frac{2 \ln \left(\frac{M_{k} loh^{3}}{er^{4} C \mathcal{I} \mathcal{I}}\right)}{PT_{0}}.$$
 (3)

Change factors and the rate of change of the sensitivity of the MM inertia are zero.

Analytical expression for the joint effect of the coefficient of informative parameter J and  $M_k$  influence quantity the rated sensitivity MM moment of inertia is given by

$$\delta_{Mk} = \frac{\partial^2 N_J}{\partial J \partial M_k} = \frac{2}{M_k P T_0}.$$
 (4)

To study the effect of affecting the value of the coefficient on the output and the rate of change of the coefficient of influence affecting the value on the output signal using partial analytical expressions of the last two terms of the Taylor series, which is represented in equation (2), which are relative to the rate of change and influence coefficients, respectively:

$$B_{J} = \frac{\partial N_{J}}{\partial M_{k}} = \frac{2J}{M_{k}PT_{0}}, \qquad (5)$$

$$B_{J}' = \frac{1}{2} \frac{\partial^2 N_{J}}{\partial M_{k}^{2}} = -\frac{J}{M_{k}^{2} P T_{0}}.$$
 (6)

In equation (2) the second, third and fourth terms on the nominal transfer function of MM inertia. Given the fact that the second and third partial derivatives of the transformation equations (1) are zero, the analytical expression for the study of nominal transfer function of MM moment of inertia of the form

$$N_{J_{\rm H}} = \frac{2\ln\left(\frac{M_{\rm k}loh^3}{er^4 C \mathcal{I} \mathcal{I}}\right)}{PT_0} J.$$
 (7)

Metrological additive model error in the changing influence quantity  $M_k$  by value (value)  $\square M_k$  is the sum of the last two terms of the Taylor series, presented in equation (2) and is described by the following analytical expression

$$\mathcal{A}N_{aMk} = B_{J}\mathcal{A}M_{k} + \frac{1}{2}B_{J}^{'}\mathcal{A}M_{k}^{2} =$$
$$= \frac{2J}{M_{k}PT_{0}}\mathcal{A}M_{k} - \frac{J}{M_{k}^{2}PT_{0}}\mathcal{A}M_{k}^{2}.$$
 (8)

Metrological model multiplicative uncertainty in the changing influence quantity  $M_k$  to the value (value)  $\square M_k$  is the fifth term of the Taylor series, which is represented in equation (2) and is described by the product of the combined effect of (4) to reject the influence quantity  $AM_k$  the normalized value and the input value (moment of inertia J)

$$\mathcal{A}N_{mMk} = \mathcal{K}_{Mk}\mathcal{A}M_{k}J = \frac{2J}{M_{k}PT_{0}}\mathcal{A}M_{k}.$$
 (9)

## Study of the characteristics of metrological change models

Based on the meteorological models (1 - 9) of the basic static characteristics of MM inertia study their characteristics (surface) changes in nominal values of influence quantities that are shown in Figure 1 - 12.



Figure 1 - Static characteristic MM moment of inertia:  $N_J$ - the number of pulses that corresponds to the moment of inertia in the measurement range of  $10^{-5}$  Hm<sup>2</sup> to 0.02 Hm<sup>2</sup>; J – moment of inertia



sensitivity in measuring range inertia; J - moment of inertia

Characteristics of changes in the coefficient of influence (5) and its rate of change (6) when the effect of the self-locking mode  $M_k$  50 to 1 Hm when measuring the moment of inertia at the lower limit of measurement  $J = 2 \cdot 10^{-5}$  Hm<sup>2</sup> are shown in Figure 4

and Figure 7, respectively. And the characteristics of changes in the coefficient of influence and its rate of change when the influence quantity  $M_k$  mode of selfbraking from 50 to 1 Hm when measuring the moment of inertia at the upper limit of measurement  $J = 0.02 \text{ Hm}^2$  are shown in Figure 5 and Figure 8, respectively.

The surface, which characterizes the rate of change of the coefficient  $M_k$  influence on the output signal (6) MM moment of inertia is shown in Figure 6.





Figure 3 - The surface coefficient variation influence quantity  $M_k$  influence on the output signal:  $\beta_J$  - coefficient of influence; J - moment of inertia;  $M_k$  - torque-slip S = 1



Figure 4 - The characteristic changes in the coefficient of influence influence quantity  $M_k$  on the output signal at  $J = 2 \cdot 10^{-5} \text{ Hm}^2$ :  $\beta_J$  - coefficient of influence;  $M_k$  – torque

Characteristic changes in the coefficient of joint influence (4) informative parameter J and  $M_k$  influ-

ence quantity the rated sensitivity MM moment of inertia is shown in Figure 9.



Figure 5 - The characteristic changes in the coefficient of influence influence quantity  $M_k$  on the output signal at  $J = 0.02 \text{ Hm}^2$ :  $\beta_J$  - coefficient of influence;  $M_k$  - torque



Figure 6 - The surface of the rate of change rate effects influence quantity  $M_k$  by the output:  $_{B'_J}$  - the rate of change rate exposure; J - moment of inertia;  $M_k$  - torque

Surface, characterizing the change in the nominal transfer function (7) MM moment of inertia of rotors EM is shown in Figure 10.

Surface, characterizing the change of the additive (8) and multiplicative (9) errors MM moment of inertia in changing influence quantity  $M_k$  are presented in Figures 11 and 12, respectively.

As can be seen from Figure 1, the static characteristic of the MM moment of inertia, which is described by equation (1) is linear over the entire range of measurements. Sensitivity MM moment of inertia (see Figure 2) is a constant, and hence the scale of the tool is uniform.

The coefficient of influence influence quantity  $M_k$  (5) to an output signal when measuring low moment of inertia of rotors EM has little effect (Figures 3, 4). And when measuring the moment of inertia at the upper limit of measurement (Figure 5) the impact factor of influence on the output signal MM is essential, especially with a decrease in torque to the lower range of values (from 10 to 1 Hm). It is in the value of the coefficient of the boundaries of influence should be considered when analyzing the results of measurement by adjusting them in a microprocessor (multiplication measurement results on correction factors).



Figure 7 - Characteristics of the rate of change koeffitsientaenta influence quantity  $M_k$  influence on the output signal with  $J = 2 \cdot 10^{-5} \text{ Hm}^2$ :  $B'_J$  - the rate of change of the coefficient of impact;  $M_k$  - torque



Figure 8 - Characteristics of the rate of change rate effects influence quantity  $M_{\rm k}$  on the output signal with J =

= 0.02 Hm<sup>2</sup>:  $B'_J$  - the rate of change of the coefficient of impact;  $M_k$  - torque



Figure 9 - The characteristics of changes in the coefficient of joint influence of the inertia and the influence quantity  $M_k$  a nominal sensitivity of MM:  $\overline{O}_{Mk}$  - The combined effects;  $M_k$  - torque



Figure 10 - The surface of the change in the nominal transfer function of MM moment of inertia:  $N_{\rm IH}$  -

nominal number of pulses corresponding to the moments of inertia in the measurement range; J - moment of inertia;  $M_k$  - torque

The value of the rate of change of the coefficient of influence (Figures 6 - 8) for measuring low moment of inertia of rotors EM is negligible. The rate of change of the coefficient influence at the upper limit of measurement shows the moment of inertia (see Figure 8), which affects a significant impact on the value of  $M_k$  the measurement results J occurs at reduced torque  $M_k$  5 to 1 Hm. Therefore, when measuring the moment of inertia EM at the top of measurements (and close to the upper limit of measurement) must be considered a significant impact affecting the value and make the correction in the resulting output.



Figure 11 - The surface of the change in the absolute accuracy of the additive MM moment of inertia in changing influence quantity  $M_k$ :  $\prod N_{aMk}$  - the number of pulses, the latter corresponding to the absolute error of the additive; J - moment of inertia;  $M_k$  - torque



Figure 12 - Surface changes multiplicative error MM moment of inertia in changing influence quantity  $M_k$ :  $\square N_{mMk}$  - the number of pulses, the latter corresponding to the multiplicative error; J - moment of inertia;  $M_k$  – torque

Characteristic changes in the coefficient combined effect (Figure 9) also shows that the influence quantity  $M_k$  significant effect on the result is when you change it from 10 to 1 Hm.

Nominal transfer function (Figure 10) is linear over the entire range of measurement when the moment of inertia torque 50 to 10 Hm. Inconsistency nominal static characteristic transfer function appears when the torque from 10 to 1 Hm.

From the resulting characteristics of the changes of the additive error (Figure 11) MM moment of inertia shows that the absolute value of the additive error in the measuring range of the moment of inertia of  $2 \cdot 10^{-5}$  to 0.02 Hm<sup>2</sup> no more than  $80 \cdot 10^{3}$  pulses at a specified value of the number of pulses output  $800 \cdot 10^3$ , which corresponds to the summary of the additive error of the SI, which does not exceed 10%. However, knowing the value of the additive error and the multiplicative error (Figure 12) in the whole range of measurements can be automatically excluded from the measurement results by modification or multiplication by the correction coefficients for the values corresponding to the values of the additive and multiplicative errors and moment of inertia, according to figures 11 and 12. This can be done in the microprocessor to display the results of measurements on the indicating device.

### Findings

Received metrological model (1 - 9) allow us to study the behavior of basic static metrological characteristics of MM moment of inertia, determine the causes of increased uncertainty MM indicate ways to reduce or eliminate, as well as synthesize MM moment of inertia with normalized metrological characteristics.

### Literature

1. *Vasilevskyi O.M.* Zasib vimihyuvannya dinamichnogo momentu elektromotoriv ta analaz yogo tochnosti / O.M. Vasilevskyi // Vimiryuvalna tekhnika ta metrologiya. – 2012. -  $N_{\odot}$ 73. – P. 52 – 56.

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