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COLLISION PROBABILITY OF AIRCRAFT FLYING ON PARALLEL TRACKS

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Abstract. *Concepts of Air Traffic Management are focused on the geometry of airways. Today's navigation facilities are able to increase airspace capacity due to more accurate path keeping and providing reduced position error. It will increase capacity of airspace but decrease safety level. According to this collision probability between two aircraft flying on opposite tracks with reduced lateral separation is considered and compared with level of safety.*

Keywords: aircraft, collision probability, lateral separation, position error, safety.

1. Introduction

The commercial aviation industry is a significant component of the global economy. More than 4.5 percent of the world economic output is associated with the air transport component of civil aviation (International Civil Aviation Organization (ICAO), 1998).

The busiest airspace is congested for significant periods of time due to a combination of traffic volume, weather and other airspace restrictions. The problem is to achieve the highest levels of efficiency and safety. Airspace Management Concepts attempt to mitigate required traffic flow management and allow more user preference and traffic flexibility. According to Seven-Year Flight Forecast of Eurocontrol by 2018 anticipates 11.1 million Instrument Flight Rules movements in Europe. From 2014 it will grow in averages 1.9 % per year for the whole 2012-18 period. Flight delays rise with traffic growth. The cause for this delay is commonly referred to as a "lack of capacity" (Elefante 2001). Capacity of the airspace is constrained by many factors including number of runways, aircraft separation requirements, and the Air Traffic Controller workload limitations.

Any increase in capacity must be accompanied by a demonstration that such an increase will be safe (Fig. 1).

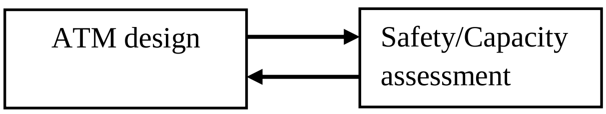


Fig. 1. Dependence between airspace structure, capacity and safety

As capacity of airspace increases, the relative safety must decrease to achieve the same absolute accident rate over time.

2. Literature overview

The existing Air Traffic Service route structure often involves mileage penalties, compared to the most economic routes, which may be great circle routes but which also take into account wind, temperature and other factors such as weight of the aircraft, charges and safety (International…2002).

According to navigation facilities one of the possibilities to increase airspace capacity is to reduce separation minima (Moek, Smeltink 2005).

Aircraft separation standards use space and time divisions to prevent conflict and confrontation (Thompson 1997). On one side Separation minima reductions will increase capacity of airspace, but on the other side growing air transport demand will provide rising risk of conflicts and collisions (Netjasov 2010).

Safety depends of the density and complexity of air traffic and overall air traffic states (for winter or summer, weather patterns etc.) (Eurocontrol Metrics…2000).

Concepts of "capacity", "safety" and "technology" are focused on the geometry of airways:

– capacity increases for smaller separation L;

– navigation and flight technology should provide a reduced root mean square (r.m.s.) position error σ;

– combination of L and σ should be such that the probability of collision does not exceed ICAO Target Level of Safety (TLS) of 5×10^{-9} per hour (Campos, Marques 2011; Masao, Masato 2008).

The actual motion of the aircraft is affected by uncertainty, due mainly to wind, but also to errors in tracking, navigation, and control (Netjasov, Milan Janic 2008). The deferent sources of uncertainty affecting the aircraft motion generally cause deviations from the nominal flight path (Moek, Smeltink 2005).

Thus the goal of this paper is to determine the collision probability between two aircraft flying on opposite tracks with reduced lateral separation, which compares with the ICAO TLS.

3. Aircraft collision probability on parallel flight tracks

Large aircraft flight deviations are rare events and the r.m.s. position error is an indirect safety metric because it specifies an upper bound for the probability of collision (Campos, Marques 2011). The r.m.s. position error is easy to implement as a safety metric: it just requires monitoring of the deviation between the actual trajectory and the intended trajectory.

$$F_k(x;\sigma) = A\exp(-a|x|^k),$$

where k is the weight.

Constant a is determined by the condition of unit total probability:

$$A = \frac{k a^{\frac{1}{k}}}{\left[2\Gamma\left(\frac{1}{k}\right)\right]}.$$

The r.m.s. position error σ or variance σ^2, where:

$$k = 1,$$

$$a = \frac{\sqrt{2}}{\sigma},$$

$$A = \frac{1}{\left(\sigma\sqrt{2}\right)}.$$

It leads to:

$$F_1(x;\sigma) = \left[\frac{1}{\sigma\sqrt{2}}\right]\exp\left(-\sqrt{2}\frac{|x|}{\sigma}\right).$$

For statistically independent aircraft deviations at position x:

$$P_k(x;L,\sigma_1,\sigma_2) = F_k(x;\sigma_1)F_k(L-x;\sigma_2).$$

Consider two aircraft flying at a constant lateral separation L in parallel flight paths (Fig. 2). In this collision scenario, the aircraft can at all times drift into positions less than a minimum separation distance.

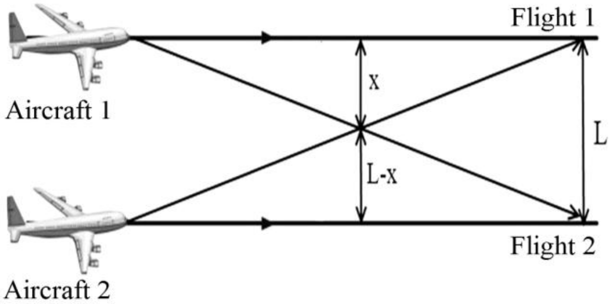


Fig. 2. Lateral separation between aircraft

A coincidence will occur if position errors are x (aircraft 1) and L-x (aircraft 2).

An integral over all positions along the line joining the two aircraft:

$$Q_k(L;\sigma_1,\sigma_2) = \\ = \int_{-\infty}^{+\infty} P_k(x;L,\sigma_1,\sigma_2)dx = \int_{-\infty}^{+\infty} F_k(x;\sigma_1)F_k(L-x;\sigma_2)dx.$$

The collision probability has the dimensions of inverse length. The ICAO TLS of 5×10^{-9} per hour can be converted for a maximum airspeed V_0 = 625 kt to an ATLS.

The safety criterion is $\overline{Q_0} = 5\times10^{-9}$. Upper bound for $V_0 \leq 625$ kt, is $Q \leq Q_0 = \frac{\overline{Q_0}}{V_0}$ which apply the ATLS 8×10^{-12} nm^{-1}.

Two aircraft with dissimilar r.m.s. position errors follow the next probability distribution:

$$Q_0 \geq Q_1(L;\sigma_1,\sigma_2) = \\ = \left[\frac{1}{2\sigma_1\sigma_2}\right]\int_{-\infty}^{+\infty}\exp\left[-\sqrt{2}\left(\frac{|x|}{\sigma_1}+\frac{|L-x|}{\sigma_2}\right)\right]dx.$$

The appearance of modulus in the argument of the exponential requires that the range of integration $-\infty;+\infty$, be split in three parts.

1. The flight paths between of the two aircraft at $0 \leq x \leq L$:

$$2\sigma_1\sigma_2 Q_{11} = \int_0^L \exp\left[-\sqrt{2}\left(\frac{x}{\sigma_1}+\frac{(L-x)}{\sigma_2}\right)\right]dx = \\ = \exp\left(\frac{-\sqrt{2}}{\sigma_2}\right)\int_0^L \exp\left[-\sqrt{2}x\left(\frac{1}{\sigma_1}-\frac{1}{\sigma_2}\right)\right]dx$$

and involves an integration:

$$2\sigma_1\sigma_2 Q_{11} = \exp\left(\frac{-\sqrt{2}L}{\sigma_2}\right)\times \\ \times\left\{1-\exp\left[-\sqrt{2}L\left(\frac{1}{\sigma_1}-\frac{1}{\sigma_2}\right)\right]\right\}\left[\sqrt{2}\left(\frac{1}{\sigma_1}-\frac{1}{\sigma_2}\right)\right]^{-1},$$

and simplifies to:

$$Q_{11} = \left[2\sqrt{2}(\sigma_2 - \sigma_1)\right]^{-1} \times$$
$$\times \left[\exp\left(\frac{-\sqrt{2}L}{\sigma_2}\right) - \exp\left(\frac{-\sqrt{2}L}{\sigma_1}\right)\right].$$

2. The remaining contributions at a point $x \ge L$ outside the path of second aircraft:

$$2\sigma_1\sigma_2 Q_{12} = \int_L^{\infty} \exp\left\{-\sqrt{2}\left[\frac{x}{\sigma_1} + \frac{x-L}{\sigma_2}\right]\right\} dx$$

leads to an integral:

$$2\sigma_1\sigma_2 Q_{12} =$$
$$= \exp\left(\frac{\sqrt{2}L}{\sigma_2}\right)\int_L^{\infty} \exp\left\{-\sqrt{2}x\left(\frac{1}{\sigma_1} + \frac{1}{\sigma_2}\right)\right\} dx =$$
$$= \exp\left(\frac{\sqrt{2}L}{\sigma_2}\right)\left[\sqrt{2}\left(\frac{1}{\sigma_1} + \frac{1}{\sigma_2}\right)\right]^{-1} \exp\left[-\sqrt{2}L\left(\frac{1}{\sigma_1} + \frac{1}{\sigma_2}\right)\right]$$

which simplifies to:

$$Q_{12} = \left[2\sqrt{2}(\sigma_1 + \sigma_2)\right]^{-1} \exp\left(\frac{-\sqrt{2}L}{\sigma_1}\right).$$

3. The coincidence $-\infty < x < 0$ outside the flight path of the first aircraft:

$$2\sigma_1\sigma_2 Q_{13} =$$
$$= \int_{-\infty}^{0} \exp\left\{\sqrt{2}\left[\frac{x}{\sigma_1} - \frac{(L-x)}{\sigma_2}\right]\right\} dx =$$
$$= \exp\left(\frac{-\sqrt{2}L}{\sigma_2}\right)\int_0^{\infty} \exp\left\{-\sqrt{2}x\left(\frac{1}{\sigma_1} + \frac{1}{\sigma_2}\right)\right\} dx$$

then

$$Q_{13} = \left[2\sqrt{2}(\sigma_2 + \sigma_1)\right]^{-1} \exp\left(\frac{-\sqrt{2}L}{\sigma_2}\right).$$

The sum of Q_{11}, Q_{12}, Q_{13} where:

$$Q_1(L;\sigma_1,\sigma_2) = \left[2\sqrt{2}(\sigma_2 - \sigma_1)\right]^{-1} \times$$
$$\times \left[\exp\left(\frac{-\sqrt{2}L}{\sigma_2}\right) - \exp\left(\frac{-\sqrt{2}L}{\sigma_1}\right)\right] +$$
$$+ \left[2\sqrt{2}(\sigma_2 + \sigma_1)\right]^{-1}\left[\exp\left(\frac{-\sqrt{2}L}{\sigma_1}\right) + \exp\left(\frac{-\sqrt{2}L}{\sigma_2}\right)\right]$$

for distribution:

$$Q_1(L;\sigma_1,\sigma_2) = Q_{11} + Q_{12} + Q_{13} \le Q_0 = 8 \times 10^{-12} \text{ nm}.$$

The preceding safety-separation criteria are applied to lateral separation in controlled airspace with standard and reduced separation.

The standard deviation for VOR/DME is taken as 0.3 NM. The GNSS navigation is highly accurate in the lateral path keeping.

Let compare the collision probability for lateral separation in 5, 4, 3 nm in controlled airspace with the ATLS probability of collision per nautical mile in controlled airspace with $\sigma_1, \sigma_2 \le 0.3$ nm.

The r.m.s. position error of two aircraft where $\sigma_1 = \sigma_2 \equiv \sigma$.

1. The flight paths between two aircraft when $0 \le x \le L$:

$$\overline{Q}_{11} = \left(2\sigma^2\right)^{-1}\int_0^L \exp\left(\frac{-\sqrt{2}L}{\sigma}\right) dx =$$
$$= \left[\frac{L}{2\sigma^2}\right]\exp\left(\frac{-\sqrt{2}L}{\sigma}\right).$$

2. The remaining contributions at a point $x \ge L$ outside the path of second aircraft:

$$\overline{Q}_{12} = \left(2\sigma^2\right)^{-1}\exp\left(\frac{-\sqrt{2}L}{\sigma}\right)\int_L^{\infty}\exp\left(-2\frac{\sqrt{2}x}{\sigma}\right) dx =$$
$$= \left(4\sqrt{2}\sigma\right)^{-1}\exp\left(\frac{-\sqrt{2}L}{\sigma}\right).$$

3. The coincidence $-\infty < x < 0$ outside the flight path of the first aircraft

$$\overline{Q_{13}} = \left(2\sigma^2\right)^{-1}\exp\left(\frac{-\sqrt{2}L}{\sigma}\right)\int_{-\infty}^{0}\exp\left(-2\frac{\sqrt{2}x}{\sigma}\right) dx =$$
$$= \left(4\sqrt{2}\sigma\right)^{-1}\exp\left(\frac{-\sqrt{2}L}{\sigma}\right).$$

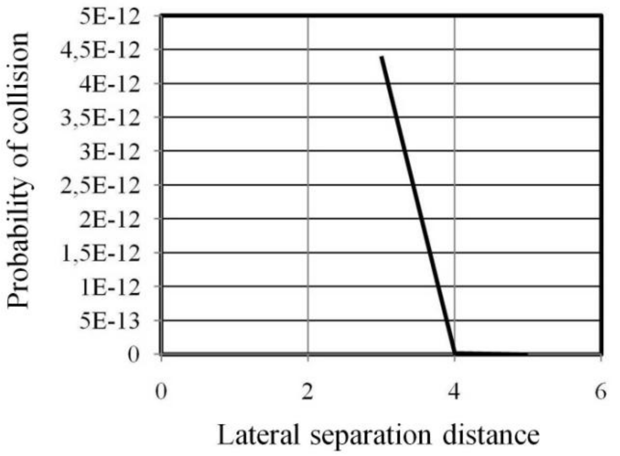
The sum specifies:

$$Q_1(L;\sigma) = \exp\left(\frac{-\sqrt{2}L}{\sigma}\right)(2\sigma)^{-1}\left(\frac{L}{\sigma} + \frac{1}{\sqrt{2}}\right)$$

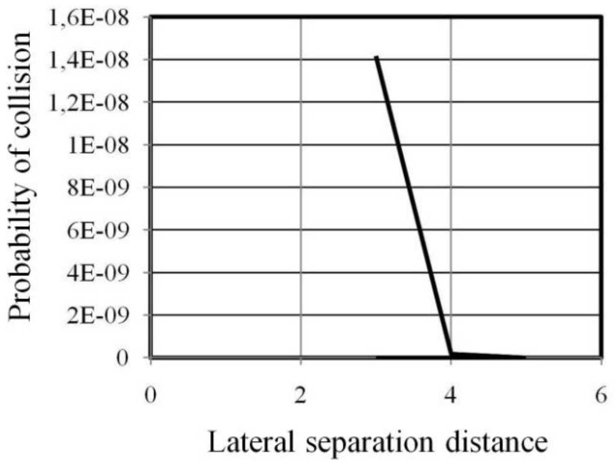
safety criterion:

$$Q_1(L;\sigma) = \overline{Q}_{11} + \overline{Q}_{12} + \overline{Q}_{13} \le Q_0 = 8 \times 10^{-12}\,\text{nm}^{-1}.$$

Probability of collision for L<4 nm exceed the ICAO TLS both dissimilar (Fig. 3, *a*) and equal (Fig. 3, *b*) r.m.s. position errors.



a



b

Fig. 3. Probability of collision aircraft with dissimilar (*a*) and equal (*b*) r.m.s. position errors

It is mean that navigation aids should provide r.m.s. position error $\sigma < 0.3$ nm of flight path deviations to reduction lateral separation.

4. Conclusions

Safety standards of ICAO require probability of collision of the odder 5×10^{-9} per hour. In the case of aircraft flying in opposite directions in parallel tracks, if the r.m.s. position error will be less than one tenth of the minimum separation distance, the probability of collision will be less then ICAO TLS.

The standard deviation for VOR/DME won't allow maintaining safety lateral separation when it will be less 5 nm. The GNSS navigation is highly accurate in the lateral path keeping. Thus, implementation of more accurate navigation procedures will allow to reduce separation minima and increase capacity of airspace.

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Розглянуто концепції управління повітряним рухом, зосереджені на геометрії повітряних трас. Показано, що збільшення пропускної здатності повітряного простору за рахунок більш точного витримування траєкторії та надання зменшеної помилки місцеположення, знизить рівень безпеки польотів. Проаналізовано ймовірність зіткнення двох повітряних кораблів на паралельних повітряних трасах при скороченому бічному ешелонуванні.

Ключові слова: безпека, бічне ешелонування, вірогідність зіткнення, повітряний корабель, помилка місцеположення.

В.П. Харченко[1], О.Н. Алексеев[2], Е.Н. Тапиа[3]. Вероятность столкновения воздушных судов на параллельных маршрутах
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Рассмотрены концепции управления воздушным движением, сосредоточенные на геометрии воздушных трасс. Показано, что увеличение пропускной способности воздушного пространства за счет более точного выдерживания траектории и предоставлении уменьшенной ошибки местоположения, снизит уровень безопасности полетов. Проанализирована вероятность столкновения двух воздушных судов на параллельных воздушных трассах при сокращенном боковом эшелонировании.

Ключевые слова: безопасность, боковое эшелонирование, вероятность столкновения, воздушное судно, ошибка местоположения.

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