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# FINITE-ELEMENT MODELING OF SALT TECTONICS

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**Abstract.** *The two-dimensional thermal model of graben structure in the presence of salt tectonics on the basis of a finite elements method is constructed. The analysis of the thermal field is based on the solution of stationary equation of heat conductivity with variable boundary conditions. The high precision of temperatures distribution and heat flows is received. The decision accuracy is no more than 0,6 %.*

**Keywords:** finite elements method, heat flows, salt tectonics, temperatures.

## Introduction

Heat flow is the main and reliable source of information about the Earth thermal state.

Detailed calculated data of the surface heat regime are necessary for:

– classification of geothermal anomalies by the physic-mathematical principle;

– calculations of temperature in the earth's crust and upper mantle;

– building of a complex system of data interpretation.

The finite element method is an efficient numerical method of the solution of complicated problems of fundamental and applied geothermy.

Geological-structural factors may lead to the distortion of a stationary geothermal field. The geological-structural factors are as follows:

– bed of layers with inhomogeneous and anisotropic heat conductivity;

– alternation of structural forms and blocks with different thermal conductivity (salt diapir, intrusion of a different type, grabens, contacts heterogeneous blocks to faults).

The form of anomalies, dimensions and values in relation to the background can serve as a search signs of salt deposits.

## Analysis of researches

Salt tectonics is a widespread form of folded dislocations of sedimentary layer of the earth's crust.

Salt strata have a number of specific rheological properties [1]:

– the low density relative to other sedimentary rocks;

– the high plasticity, especially in conditions of high pressures and temperatures.

Salt tectonics manifests itself in different forms [1]:

– small swelling (salt pillows);

– salt diapir;

– salt mounds and anticline.

The length of the salt mounds and anticline makes up of dozens, sometimes over hundreds of kilometers.

Vaults of salt domes are often broken by frow of stretching. Due to the processes of stretching vaults of salt domes are complicated by the grabens (fig. 1) [1].

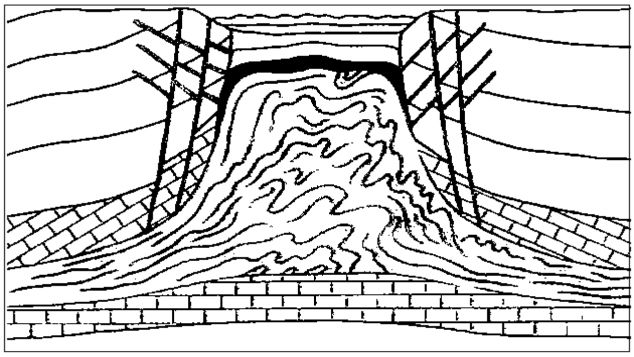


Fig. 1. Salt dome

The main factors that create the salt tectonics are [1]:

– gravitational – mechanism of inversion densities;

– tectonic − horizontal compression.

Under the action of gravitational factor salts come to the surface from under the covering her more dense sediment. Then salt pillows and diapirs-dome are formed, which is especially typical for depressions within the platforms. The existence of shallow anticlinal elevations or fault ledges in the sub salt bed creates the difference load above the salt strata.

The difference of such a load contributes to the growth of diapirs-domes. With the capacity of above-mentioned salt sediments in several hundred meters salt flow and its injection in the kernel salt structures begin [1].

Tectonic factor in the highest degree is displayed in the external areas of the folded structures and in their advanced and intermountain deflection. Under the influence of tectonic factors salt mound and anticline are arisen [1].

To the areas of salt tectonics include [1]:

– continental and intercontinental rifts and paleorifts, above rifts deep syneclise;

– advanced and intermountain deflections and the external zone of folded structures.

**Work purpose** – the method development of calculation of temperature and heat flows in graben structure with elements of salt tectonics.

### Problem statement

The geometry of the problem in the presence near bort salt dome, the stratum of salt and salt rod in the central part of the model is shown in fig. 2.

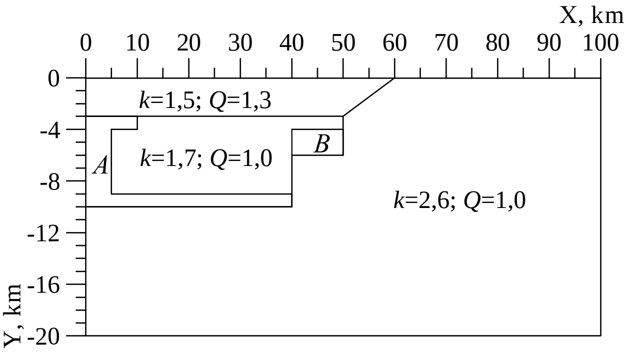



Fig. 2. Model graben structure in the presence of salt tectonics:
*A* – k = 5,0; Q = 1.0;
*B* – k = 5,0; Q = 1.0

The sizes of the model are presented in absolute units. Thermal conductivity $k$ and the heat generation $Q$, shown in fig. 2, are closed to the real values.

Finite-element mesh contains 298 units and 527 of the elements. The problem was solved with the use of a linear approximation of a test function.

The defining equation for this task is the Poisson equation

$$k\left\{\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right\}+Q=0. \quad (1)$$

On the upper surface of the model Dirichle boundary condition is specified

$$T = 0^{\circ}\text{ C} \quad (2)$$

The side walls perfectly isolated

$$k\frac{\partial T}{\partial x}=0. \quad (3)$$

On the lower boundary of the model Neumann boundary condition is specified

$$q = 25\text{ mW/m}^2\,. \quad (4)$$

Equations (1) and the boundary conditions (2)-(4) are the only way to determine the problem.

Mathematical modeling is an inevitable component of scientific and technical progress. The question organization about the mathematical modeling reflects a clear plan of actions. It can be broken into three stages: model–algorithm–program [2].

The main idea of finite elements method is that any continuous value (for example, temperature) can be approximated by a discrete model. The discrete model is based on the multitude of piecewise-continuous functions defined on a finite number of subdomains. Piecewise-continuous functions are defined with the help of values of continuous value in a finite number of points of the considered region [3–5].

From the variational calculation it is known, that the decision $T(x,y)$ satisfying the equations (1)-(4) coincides with the function that minimizes the functional

$$\Pi=\iint_{\Omega}\left[\frac{1}{2}k\left\{\left(\frac{\partial T}{\partial x}\right)^2+\left(\frac{\partial T}{\partial x}\right)^2\right\}-QT\right]dx\,dy+ +\int_S qTdS, \quad (5)$$

where $T(x,y)$ is test function that belongs to the class of permissible functions.

Let's choose the triangular element $ijk$. Nodal values of the temperature $T$ define the function in each triangular element $e$ with nodes $i,j,k$.

The test function presentation $T(x,y)$ on the element $e$ through the basic functions can be written in the following way:

$$T(x,y)=\left[N_i,N_j,N_k\right]\{T\}^e; \quad N_i=\frac{a_i^e+b_i^e x+c_i^e y}{2\Delta}; \quad (6)$$

$a_i^e = x_j y_k - x_k y_j$;

$b_i^e = y_j - y_k$;

$c_i^e = x_k - x_j$;

where $\Delta$ is the area of the triangle $ijk$.

Constants $a_j$, $a_k$, $b_j$, $b_k$, $c_j$, $c_k$ can be defined by cyclic rearrangement of indices.

Nodal values $T$ uniquely determine the function in the whole domain. Functional $\Pi$ can be minimized in relation to these values.

Differentiating equation (5) and using the relation (6), it can be obtained:

$$\frac{\partial \Pi^e}{\partial T_i} = \frac{1}{4\Delta^2} \iint \Big[ k \big\{ [b_i, b_j, b_k] \{T\}^e b_i + [c_i, c_j, c_k] \{T\}^e c_i \big\} \Big] dxdy - \frac{1}{2\Delta} \iint Q (a_i + b_i x + c_i y) dxdy. \quad (7)$$

Each element gives the contribution only into three derivatives, associated with the nodes:

$$\left\{ \frac{\partial \Pi}{\partial T} \right\}^e = \begin{Bmatrix} \dfrac{\partial \Pi^e}{\partial T_i} \\ \dfrac{\partial \Pi^e}{\partial T_j} \\ \dfrac{\partial \Pi^e}{\partial T_k} \end{Bmatrix}.$$

Equation (7) and the other two equations for the nodes $j$ and $k$ can be written in the form

$$\left\{ \frac{\partial \Pi}{\partial T} \right\}^e = [h] \{T\}^e. \quad (8)$$

The expression (8) can be obtained for each element.

After the integration of the equation (7) can be get

$$[h] = \frac{k}{4\Delta} \left\{ \begin{bmatrix} b_i b_i & b_j b_i & b_k b_i \\ b_i b_j & b_j b_j & b_k b_j \\ b_i b_k & b_j b_k & b_k b_k \end{bmatrix} + \begin{bmatrix} c_i c_i & c_j c_i & c_k c_i \\ c_i c_j & c_j c_j & c_k c_j \\ c_i c_k & c_j c_k & c_k c_k \end{bmatrix} \right\}.$$

Note that the element stiffness matrix $[h]$ is symmetric.

By combining all derivatives of the functional $\Pi$ and equating them to zero, can be obtained the final equations minimization process:

$$\frac{\partial \Pi}{\partial T_i} = \sum \frac{\partial \Pi^e}{\partial T_i} = 0.$$

With the help of equations (8) the last expression can be written in the form

$$\frac{\partial \Pi}{\partial T_i} = \sum \sum h_{ik} T_k,$$

where the summation is carried out on all elements and nodes.

Thus, the equation of thermal conductivity (1) with the help of the finite elements method was transformed to the equation

$$\mathbf{KT} = 0,$$

where $\mathbf{K} = \sum \sum h_{ik}$ is the general system stiffness matrix.

Let's denote by $\{F\}^e$ the second term in the right-hand side of the expression (7). If to accept, that the heat generation $Q$ does not change inside the element, then integrating

$$F_i = -\frac{Q}{2\Delta} \iint (a_i + b_i x + c_i y) dxdy,$$

can be get the final result

$$\{F\}^e = -\frac{Q\Delta}{3} \begin{Bmatrix} 1 \\ 1 \\ 1 \end{Bmatrix}.$$

Heat flow in the vertical direction for each node finite-element mesh can be found from the equation (6):

$$q = -k \{gradT\} = -\frac{\partial T}{\partial y} = -\frac{k}{2\Delta} \{c_i \; c_j \; c_k\} \begin{Bmatrix} T_i \\ T_j \\ T_k \end{Bmatrix}.$$

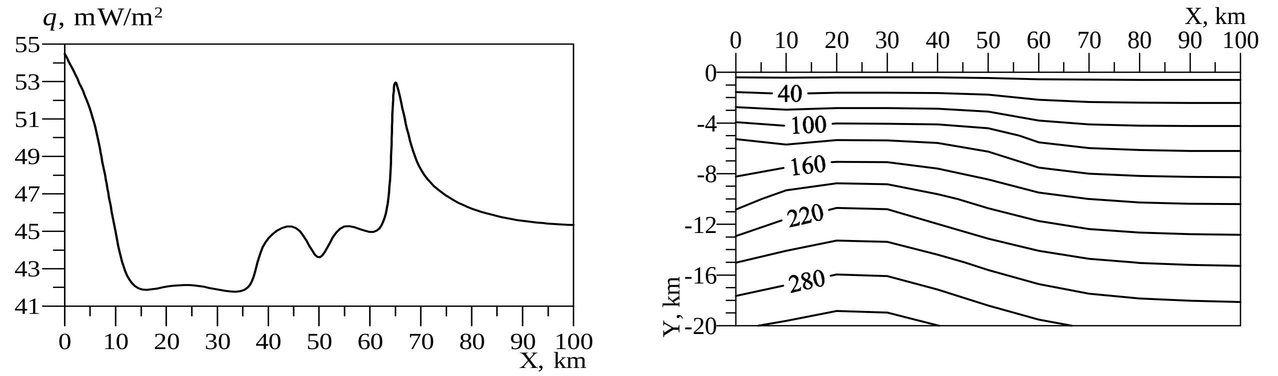
In some problems of applied geothermy the geophysical experiment formulation causes certain difficulties. In such cases it is necessary to conduct a computational experiment, which can partly replace the expensive natural experiments.

With the help of computational experiment one can create a different combination of natural factors. On the basis of experimental data it can be selected the range of values of thermal fields parameters. Computational experiment allows to get effective mathematical model, adequate to the geophysical process [6–8].

## Analysis of received results

The distribution curve of the surface heat flow clearly reflects the availability of the contact zones with different thermophysical properties. The high thermal conductivity of salt is the cause of the sharp highs of the surface heat flow (fig. 3, *a*).

The distribution of temperature is shown in fig. 3, *b*.




*a* *b*

Fig. 3. Thermal model for graben structure in the presence of salt tectonics:
*a* – distribution of surface heat flow;
*b* – temperature distribution

In lower thermal conductivity blocks isolines of temperature are thicken, and in the areas of contact the highest gradients of temperature are observed.

**Conclusions**

Computer programs are created for the solution of stationary heat conduction problem with the Dirichle and Neumann boundary conditions on the basis of linear approximation of the trial function. The approximation choice depends on the specific geothermal task.

Physic-geological and mathematical model of graben structure in the presence of salt tectonics on the basis of the method of finite elements are constructed. Analysis of the thermal field is based on the solution of the stationary equations of heat conduction with variable boundary conditions.

To obtain accurate results computational experiment was used. With a help of computational experiment the necessary combination of the geometrical factors and geothermal parameters are created. The precision of the solution when using a linear approximation of the basis function is not more than 0.6%.

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