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FEATURES OF THE MIXED SENSITIVITY H_∞-OPTIMIZATION FOR THE TWO-DEGREE-OF-FREEDOM CONTROLLER

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Abstract. *The paper is devoted to the actual issue of the robust two-degree-of-freedom controller design. The statement of the problem for the robust optimization of the two-degree-of-freedom controller by the method of mixed sensitivity is represented. The expression for the cost function of the mixed sensitivity method for the system with the two-degree-of-freedom controller, taking into consideration the influence of disturbances, is obtained. The components of the generalized system with the two-degree-of-freedom controller and principles of the weighting transfer functions introduction are defined. The transformation of the formulated problem to the* H_∞ *- optimization which may be implemented by the MATLAB software is carried out.*

Keywords: disturbances, generalized system, H_∞-optimization, method of mixed sensitivity, stabilization system, two-degree-of-freedom controller.

Introduction

Nowadays, the complexity of control processes of attendant exploitation of the vehicles grows. At the same time the important problem of stabilization of the information-measuring devices that provide measurement and determination of the information necessary for the vehicle's control, navigation and tracking is arised. The rigid requirements by accuracy are usually given to such processes. It is impossible to satisfy these requirements without stabilization of a base on which the appropriate information-measuring devices are mounted.

It is worth mentioning that the accuracy characteristics of the information-measuring devices are steeply improved by the last years. Such tendency requires the appropriate progress in stabilization systems during the exploitation of these devices at the vehicles.

The design of the robust controller with two-degree-of-freedom (2DOF) represents the modern approach for this problem solving.

Statement of problem

Nowadays, methods of control systems synthesis, based on the modern control theory, are the most widespread.

The choice of the synthesis method depends on the features of a system to be designed and conditions of its exploitation.

Firstly, these systems operate in the conditions of the external disturbances (the sea irregularities, the wind action and the disturbances due to irregularities of the road profile for ships, aircrafts and ground vehicles respectively).

Secondly, parameters of the control objects change significantly in time.

Taking into account all these circumstances, it is expedient to solve the problem of the studied systems stabilization based on the robust control. The main problem of the robust control system synthesis is the search of the stabilization law which is able to provide accuracy of the stabilization system in accordance with the given requirements in spite of presence of uncertainties in its mathematical description. This uncertainty may be caused by the different factors such as the external disturbances, errors of a system's transfer function determination and non-simulated dynamics.

One of the most widespread methods of the robust system design is the H_∞ -synthesis, which is described in [1]. This approach provides the robust performance and stability of the systems to be designed. In this case, the design problem is formulated as a problem of the mathematical optimization directed to the search of an optimal controller.

The advantage of this approach is simplicity of its application for the systems with the cross-connections between channels.

The disadvantages of this approach are the mathematical complexity and critical influence of the system's mathematical description adequacy on the efficiency problem solution.

In the modern problems of control systems design there is a concept of controllers with one and two degrees of freedom. In many practical applications the systems with both feedback and command signals are used, that result in the necessity to consider the 2DOF controller. Sometimes, the one degree-of-freedom (1DOF) controller may be used for this problem solution, for example, when control is implemented by the error signal representing a difference between the input and command signals. But the 1DOF controller does not allow to provide the rigid requirements to the transient quality indexes. Usually, the solution of the tracking problems is actual for control by the information-measuring systems mounted at the moving base requires usage of the 2DOF controllers.

Analysis of the last researches

The method of the mixed sensitivity belongs to the widespread methods of the H_∞-optimization. The statement of the robust synthesis problem by means of this method is represented in [2; 3]. The cost function and features of its algorithmic and program realizations are discussed in [4]. In all above mentioned papers the systems with the 1DOF controller have been considered. So, the problem of the 2DOF controller design is actual and requires the further research. Although some separate statements of this problem are considered in [3].

The goal of this paper is representation of the basic features of the H_∞-optimization for the stabilization systems of the information-measuring devices with the 2DOF controller on the basis of the method of mixed sensitivity taking into consideration the influence of the external disturbances.

The statement of the H_∞-synthesis problem

H_∞- synthesis is a powerful instrument for design of the feedback control systems based on determination of the bounded frequency responses as a functions of the singular numbers. There is an approach for robust systems design, when the sufficient condition of the robust stability is formulated in the form of norms, bounded by the weighting transfer functions.

This approach is accepted in such automated computer-aided facilities for the robust systems optimal design as the Robust Control Toolbox [1].

The generalized statement of the H_∞-synthesis problem [2] is shown in fig. 1.

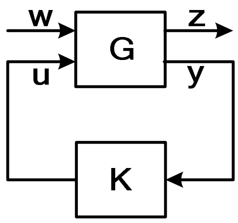

Fig. 1. The statement of the H_∞-synthesis problem

The synthesized system consists of the control object and controller described by the matrix transfer functions $\mathrm{G}(s)$, $\mathrm{K}(s)$ respectively. These transfer functions must be fractionally-rational and proper. The generalized control object represents a system with two inputs and two outputs. The vector $\mathbf{w}$ represents the external output, which, in the general case, consists of disturbances, measurement noise and command signals. The input vector $\mathbf{u}$ represents the control signals. The output vector $\mathbf{z}$ determines the quality of the control processes. For example, it may be characterized by the command signal tracking error, which must be equal to zero in the ideal case. The output vector $\mathbf{y}$ represents the vector of the observed signals, which are used for feedback organization. The transfer function from input $\mathbf{w}$ to output $\mathbf{z}$ is denoted $\mathbf{W}_w^z$. Respectively, the main task of the H_∞- synthesis is the choice of such controller $\mathbf{K}(s)$, which can minimize the $\|\mathbf{W}_w^z\|_\infty$ norm.

The control system, shown in fig. 1, can be described in the following way

$$\begin{bmatrix}\mathbf{z}\\ \mathbf{y}\end{bmatrix} = \mathbf{P}(s)\begin{bmatrix}\mathbf{w}\\ \mathbf{u}\end{bmatrix} = \begin{bmatrix}\mathbf{P}_{11}(s) & \mathbf{P}_{12}(s)\\ \mathbf{P}_{21}(s) & \mathbf{P}_{22}(s)\end{bmatrix}\begin{bmatrix}\mathbf{w}\\ \mathbf{u}\end{bmatrix};$$
$$\mathbf{u} = \mathbf{K}(s)\mathbf{y}.$$

In the state space this representation becomes

$$\dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{B}_1\mathbf{w}(t) + \mathbf{B}_2\mathbf{u}(t);$$
$$\mathbf{z}(t) = \mathbf{C}_1\mathbf{x}(t) + \mathbf{D}_{11}\mathbf{w}(t) + \mathbf{D}_{12}\mathbf{u}(t);$$
$$\mathbf{y}(t) = \mathbf{C}_2\mathbf{x}(t) + \mathbf{D}_{21}\mathbf{w}(t) + \mathbf{D}_{22}\mathbf{u}(t);$$
$$\mathbf{u}(t) = \mathbf{K}\mathbf{y}(t).$$

The state equation in the matrix form looks like

$$\begin{bmatrix} \dot{\mathbf{x}}(t) \\ \mathbf{z}(t) \\ \mathbf{y}(t) \end{bmatrix} = \begin{bmatrix} \mathbf{A} & \mathbf{B}_1 & \mathbf{B}_2 \\ \mathbf{C}_1 & \mathbf{D}_{11} & \mathbf{D}_{12} \\ \mathbf{C}_2 & \mathbf{D}_{21} & \mathbf{D}_{22} \end{bmatrix} \begin{bmatrix} \mathbf{x}(t) \\ \mathbf{w}(t) \\ \mathbf{u}(t) \end{bmatrix}.$$

The solution of the H_∞-synthesis problem is based on the solutions of the Riccati equations. In this case is necessary to satisfy the following conditions [2; 5].

1. The pair of the matrices $\mathbf{A}, \mathbf{B}_1$ must be stabilizable and the pair of the matrices $\mathbf{A}, \mathbf{C}_1$ must be detectable.
2. The pair of the matrices $\mathbf{A}, \mathbf{B}_2$ must be stabilizable and the pair of the matrices $\mathbf{A}, \mathbf{C}_2$ must be detectable.
3. The following equality must take place $\mathbf{D}_{12}^{\mathrm{T}}[\mathbf{C}_1 \quad \mathbf{D}_{12}] = [\mathbf{0} \quad \mathbf{I}]$.
4. The following expression must be true $\begin{bmatrix} \mathbf{B}_1 \\ \mathbf{D}_{21} \end{bmatrix} \mathbf{D}_{21}^{\mathrm{T}} = \begin{bmatrix} \mathbf{0} \\ \mathbf{I} \end{bmatrix}$.

The conditions 1 and 2 guarantee the absence of imaginary eigenvalues of the Hamilton matrices, which correspond to the Riccati equations. The condition [3] means orthogonality of the signals $\mathbf{C}_1\mathbf{x}(t)$ and $\mathbf{D}_{12}\mathbf{u}(t)$. This condition for the H_2-problem means that the weighting control matrix in the norm of the vector $\mathbf{z}(t) = \mathbf{C}_1\mathbf{x}(t) + \mathbf{D}_{12}\mathbf{u}(t)$ is unitary and the components $\mathbf{z}(t)$ of the state vector $\mathbf{x}(t)$ and control vector $\mathbf{u}(t)$ do not influence on this norm. The condition 4 shows the orthogonality of the signals $\mathbf{B}_1\mathbf{w}(t)$ and $\mathbf{D}_{21}\mathbf{w}(t)$. So, the conditions 3, 4 are usual for the H_2-problem and spread on the case of the H_∞-optimization.

It is worth noting, that in such statements of problems the control object is believed to be the set of some devices and units. This set of devices and units consists of the control object, actuator, measuring system and some additional units [6].

The H_∞-optimization by the method of mixed sensitivity

The transfer function from input $\mathbf{w}$ to output $\mathbf{z}$ can be determined by means of the linear fractional transformation [3]

$$\mathbf{z} = [\mathbf{P}_{11} + \mathbf{P}_{12}\mathbf{K}(\mathbf{I} - \mathbf{P}_{22}\mathbf{K})^{-1}\mathbf{P}_{21}]\mathbf{w} = \\ = \mathbf{F}_{\mathrm{L}}(\mathbf{P}, \mathbf{K})\mathbf{w}, \qquad (1)$$

where $\mathbf{F}_{\mathrm{L}}(\mathbf{P}, \mathbf{K})$ is the lower linear fractional-transformation of $\mathbf{P}$ and $\mathbf{K}$.

The purpose of the H_∞-optimization is the synthesis of such a controller $\mathbf{K}(s)$, which can minimize the ∞-norm of the lower linear fractional transformation of $\mathbf{P}$ and $\mathbf{K}$

$$\min_{\mathbf{K}_{\mathrm{per}}} \| \mathrm{F}_{\mathrm{L}}(\mathbf{P}, \mathbf{K}) \|_\infty . \qquad (2)$$

The choice of an optimal controller is implemented on the set of all controllers that satisfy the closed system $\mathbf{W}_w^z$ internal stability. This set is called the set of stabilizing or permissible controllers.

In the practical applications it is convenient to search a controller, for which the H_∞- norm of the closed system transfer function does not exceed some given positive number [2]:

$$\| \mathrm{F}_{\mathrm{L}}(\mathbf{P}, \mathbf{K}) \|_\infty < \gamma,$$

where

$$\gamma > \gamma_0 = \min_{\mathbf{K}_{\mathrm{per}}} \| F_{\mathrm{L}}(\mathbf{P}, \mathbf{K}) \|_\infty .$$

As a rule, during the real control systems design it is necessary to achieve some different goals. Providing of the high quality tracking processes, limitations of the energy expenses by control and rejection of disturbances belong to such goals. It is known, that on the set of the controllers, that provide the internal stability of the system, the different characteristics of the system may be provided in the conditions of providing the minimal H_∞ - norms of the following functions [3]:

− the accuracy of the tracking processes is $\min \| (\mathbf{I} + \mathbf{GK})^{-1} \|_\infty$;

− rejection of the disturbance is $\min \| (\mathbf{I} + \mathbf{GK})^{-1} \|_\infty$;

− attenuation of the measurement noise is $\min \| \mathbf{GK}(\mathbf{I} + \mathbf{GK})^{-1} \|_\infty$;

− decreasing of the control energy $\min \| \mathbf{K}(\mathbf{I} + \mathbf{GK})^{-1} \|_\infty$.

The function

$$\mathbf{S} = (\mathbf{I} + \mathbf{GK})^{-1}$$

represents the transfer function by an error and is called the sensitivity function.

The function

$$\mathbf{T} = \mathbf{GK}(\mathbf{I} + \mathbf{GK})^{-1}$$

represents the closed system transfer function and is called the complementary sensitivity function.

The function

$$\mathbf{R} = \mathbf{K}(\mathbf{I} + \mathbf{GK})^{-1}$$

is called the sensitivity function by control [4].

So, in practical situations it is convenient to use the combination of the cost functions. This approach is used in the mixed sensitivity method. It is worth noting, that in many cases the requirements to the control system can not be satisfied simultaneously. But the situation can be significantly simplified, taking into consideration the frequency ranges, for which the fulfillment of the concrete requirement is important. To limit the frequency ranges of the sensitivity characteristics the weighting transfer functions may be used.

The features of the H_∞-optimization by the method of mixed sensitivity can be considered on the example of the control system for which requirements of the high quality tracking and bounding of the control signals energy by the levels are given at the same time. The structural chart of such system is represented in fig. 2.

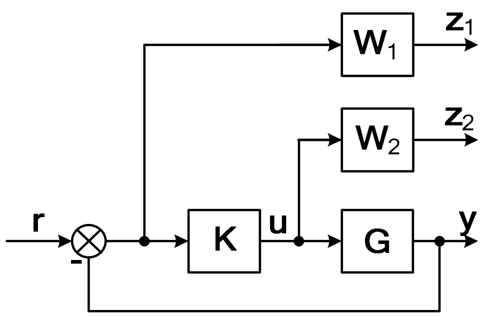

Fig. 2. The structural chart of the control system with the weigh transfer functions

To use the method of mixed sensitivity for this problem solution it is necessary to introduce the weighting transfer functions as it is shown in fig.2.

Based on the above mentioned requirements, the cost function of the method of mixed sensitivity looks like [3]

$$\min_{\mathbf{K}_{\text{per}}} \left\| \begin{matrix} \mathbf{W}_1(\mathbf{I}+\mathbf{GK})^{-1} \\ \mathbf{W}_2(\mathbf{I}+\mathbf{GK})^{-1} \end{matrix} \right\|_\infty . \qquad (3)$$

To correlate this task with the statement of the H_∞- synthesis problem represented in fig.1, it is necessary to introduce corresponding notations for the input and output signals of the system to be studied

$$w = r,$$

$$\mathbf{z} = \begin{bmatrix} z_1 \\ z_2 \end{bmatrix} = \begin{bmatrix} e \\ u \end{bmatrix},$$

$$y = e,$$

$$u = u.$$

Then the expressions for determination of the output signals as follows

$$z_1 = r - Gu;$$

$$z_2 = u;$$

$$e = r - Gu.$$

According to this set of equations, the generalized system $\mathbf{P}$ looks like

	r	u
$z_1(e)$	1	$-G$
$z_2(u)$	0	1
$y(e)$	1	$-G$

The components of the matrix $\mathbf{P}$ may be determined in the following way

$$\mathrm{P}_{11} = \begin{bmatrix} 1 \\ 0 \end{bmatrix};$$

$$\mathrm{P}_{12} = \begin{bmatrix} \mathrm{G} \\ 1 \end{bmatrix};$$

$$\mathrm{P}_{21} = [1];$$

$$\mathrm{P}_{22} = [\text{-G}].$$

Applying the linear fractional transformation (1) to this system we can obtain

$$F_{\mathrm{L}}(P,K) = \begin{bmatrix} 1 \\ 0 \end{bmatrix} + \begin{bmatrix} -G \\ 1 \end{bmatrix} K(1+GK)^{-1} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} +$$

$$+ \begin{bmatrix} -GK(1+GK)^{-1} \\ K(1+GK)^{-1} \end{bmatrix} = \begin{bmatrix} (1+GK)^{-1} \\ K(1+GK)^{-1} \end{bmatrix}.$$

The obtained result confirms the coincidence of the expressions (2) and (3).

There are the different statements of the tasks, which can be solved by the method of mixed sensitivity. One of these problems may be described by the generalized system and the cost function [4] which look like

$$\mathbf{P} = \begin{bmatrix} \mathbf{W}_1 & -\mathbf{W}_1\mathbf{G} \\ 0 & \mathbf{W}_2 \\ 0 & \mathbf{W}_3\mathbf{G} \\ \mathbf{I} & -\mathbf{G} \end{bmatrix};$$

$$\min_{\mathbf{K}_{\text{per}}} \left\| \begin{bmatrix} \mathbf{W}_1(\mathbf{I}+\mathbf{GK})^{-1} \\ \mathbf{W}_2\mathbf{K}(\mathbf{I}+\mathbf{GK})^{-1} \\ \mathbf{W}_3\mathbf{GK}(\mathbf{I}+\mathbf{GK})^{-1} \end{bmatrix} \right\|_\infty .$$

In this problem the necessity to limit an error of the command signal tracking, the control signal and the output signal respectively are taking into account.

The structural chart, which explains the statement of this problem, is represented in fig. 3.

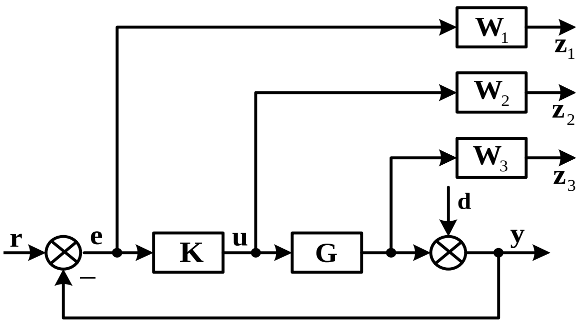

Fig. 3. The structural chart for the method of mixed sensitivity

The singular numbers of the closed transfer matrix functions from the command signal $\mathbf{r}$ to the signals of an error, input signals and output signal $\mathbf{e}$, $\mathbf{u}$, $\mathbf{y}$ [7] can be used for the quantitive estimation of the stability margins and frequency responses of the system.

Method of the mixed sensitivity for the 2DOF controller design

The important requirement to the stabilizing system of the information-measuring devices is to provide good tracking of the given command signals. In such situation the 2DOF controller is frequently applied [2, 3]. Such controller consists of two controllers $\mathbf{K}_1, \mathbf{K}_2$.

The feedforward controller $\mathbf{K}_1$ provides requirements to tracking quality. In other words, it minimizes a difference between a system's output signal and a reference model's signal. The feedback controller $\mathbf{K}_2$ provides the internal and robust stability and rejection of disturbances. The statement of the problem of the 2DOF controller synthesis taking into account the influence of the external disturbances which can be solved by means of the method of mixed sensitivity is represented in the fig. 4.

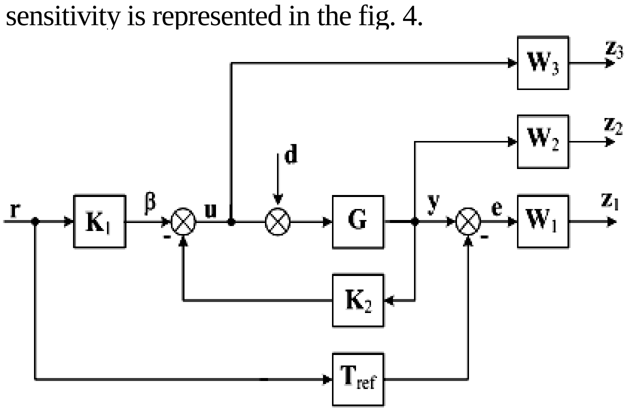

Fig. 4. The problem statement of the 2DOF controller synthesis by the method of mixed sensitivity

According to this chart, the basic goals of the synthesis along with providing of the internal stability are minimization of the error signal $\mathbf{e}$, control signal $\mathbf{u}$ and output signal $\mathbf{y}$ under influence of the external disturbances. These goals require introduction of the weighting transfer functions $\mathbf{W}_1$, $\mathbf{W}_2$, $\mathbf{W}_3$.

It is worth mentioning, that the choice of the weighting matrices is an ambiguous problem, which requires the use of heuristic methods for its solution the, for example the method of trials and errors, which takes into account the experience of the designers of a system. According to the structural chart represented in fig.4, the relationships between input and output signals of the system look like

$$z_1 = -W_1 T_{ref} r + W_1 G u ;$$
$$z_2 = W_2 u ;$$
$$y_1 = r ;$$
$$y_2 = Gu .$$

Based on these relationships, the structure of the generalized system $\mathbf{P}$ may be described in the following way

$$\mathbf{P} = \begin{bmatrix} -W_1 T_{ref} & W_1 G \\ 0 & W_2 \\ I & 0 \\ 0 & G \end{bmatrix} .$$

The cost function of this problem, which is lying in search of the optimal H_∞ -controller $\left[K_1 \text{ - } K_2\right]$, may be defined in the following way

$$\min_{\mathbf{K}_{\text{per}}} \left\| \begin{matrix} -\mathbf{W}_1\mathbf{T}_{ref} + \mathbf{W}_1\mathbf{G}\mathbf{K}_1(\mathbf{I}+\mathbf{G}\mathbf{K}_2)^{-1} \\ \mathbf{W}_2\mathbf{K}_1(\mathbf{I}+\mathbf{G}\mathbf{K}_2)^{-1} \end{matrix} \right\|_\infty .$$

Notice that the statement of the problem of H_∞ -optimization by the method of mixed sensitivity depends essentially on the features of the system to be synthesized and conditions of its exploitation. For example, for the problems of the inertial stabilization of the information-measuring devices, assigned for exploitation at the ground vehicles, it is necessary to provide the good tracking and to take into consideration that they are exploited in conditions of the external disturbances. In contrast to [2; 3] it seems more convenient to carry out direct introduction of the disturbance $\mathbf{d}$ in the generalized system. The most important disturbances for such systems are the torques caused by the unbalanced state and the angular rate of the object on which the stabilization system is mounted.

This angular rate depends on irregularities of the profile of the surface on which the vehicle moves. For this problem statement of the H_∞-optimization by the method of mixed sensitivity the relationships between input and output signals look like

$$z_1 = -W_1 T_{ref} r + W_1 G_d d + W_1 G u;$$

$$z_2 = W_2 u;$$

$$z_3 = W_3 G u;$$

$$y_1 = r;$$

$$y_2 = Gu + G_d d.$$

The appropriate generalized system may be described by the matrix

$$\mathbf{P} = \begin{bmatrix} -W_1 T_{ref} & W_1 G_d & W_1 G \\ 0 & 0 & W_2 \\ 0 & 0 & W_3 G \\ I & 0 & 0 \\ 0 & G_d & 0 \end{bmatrix},$$

where

$$\mathbf{P}_{11} = \begin{bmatrix} -W_1 T_{ref} & W_1 G_d \\ 0 & 0 \\ 0 & 0 \end{bmatrix};$$

$$\mathbf{P}_{12} = \begin{bmatrix} W_1 G \\ W_2 \\ W_3 G \end{bmatrix};$$

$$\mathbf{P}_{21} = \begin{bmatrix} I & 0 \\ 0 & G_d \end{bmatrix};$$

$$\mathbf{P}_{22} = \begin{bmatrix} 0 \\ G \end{bmatrix};$$

here

$$w = r;$$

$$\mathbf{z} = \begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix} = \begin{bmatrix} W_1 e \\ W_2 u \\ W_3 G u \end{bmatrix};$$

$$u = u;$$

$$\mathbf{y} = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} \beta \\ y \end{bmatrix}.$$

The cost function for the H_∞-optimization by means of the method of mixed sensitivity for the stabilizing system of the information-measuring devices may be determined in the following way

$$\min_{\mathbf{K}_{\text{per}}} \left\| \begin{matrix} -W_1 T_{ref} + & W_1 G_d (1+GK_2)^{-1} \\ +W_1 G K_1 (1+GK_2)^{-1} & \\ W_2 K_1 (1+GK_2)^{-1} & -W_2 K_2 G_d (1+GK_2)^{-1} \\ W_3 G K_1 (1+GK_2)^{-1} & W_3 G_d (1+GK_2)^{-1} \end{matrix} \right\|_\infty .$$

To pass on to the statement of the H_∞-optimization problem, which can be solved by the MATLAB toolboxes, it is necessary to determine the generalized system in the state space. In this case it is necessary to take into consideration the series and parallel connections of the system. Notice that in a system with the 2DOF controller it is necessary to take into account the desirable reference transfer function $\mathbf{T}_{ref}$, which is chosen by the designer to form the desirable gain-frequency responses of the system.

The state and observation equations for the generalized system look like

$$\dot{\mathbf{x}}_G = \mathbf{A}_G \mathbf{x}_G - \mathbf{B}_G \mathbf{u} + \mathbf{B}_{G_d} \mathbf{d};$$

$$\dot{\mathbf{x}}_{T_{ref}} = \mathbf{A}_{T_{ref}} \mathbf{x}\mathbf{T}_{ref} + \mathbf{B}_{T_{ref}} \mathbf{r};$$

$$\dot{\mathbf{x}}_{W_1} = \mathbf{A}_{W_1} \mathbf{x}_{W_1} - \mathbf{B}_{W_1} \mathbf{u} + \mathbf{B}_{W_1} \mathbf{r};$$

$$\dot{\mathbf{x}}_{W_2} = \mathbf{A}_{W_2} \mathbf{x}_{W_2} + \mathbf{B}_{W_2} \mathbf{u};$$

$$\dot{\mathbf{x}}_{W_3} = \mathbf{A}_{W_3} \mathbf{x}_{W_3} - \mathbf{B}_{W_3} \mathbf{u} + \mathbf{B}_{W_3} \mathbf{r};$$

$$\mathbf{z}_1 = \mathbf{C}_{W_1} \mathbf{x}_{W_1} + \mathbf{D}_{W_1} \mathbf{r};$$

$$\mathbf{z}_2 = \mathbf{C}_{W_2} \mathbf{x}_{W_2} + \mathbf{D}_{W_2} \mathbf{u};$$

$$\mathbf{z}_3 = \mathbf{C}_{W_1} \mathbf{x}_{W_3} + \mathbf{D}_{W_3} \mathbf{u} + \mathbf{D}_{G_d} \mathbf{d};$$

$$\mathbf{y}_1 = \mathbf{r};$$

$$\mathbf{y}_2 = \mathbf{D}_G \mathbf{u} + \mathbf{D}_{G_d} \mathbf{d}.$$

Then, taking into consideration the series and parallel connections in the state space, the generalized model of a system as follows

$$
\mathbf{P}=\begin{bmatrix}
A_G & 0 & 0 & 0 & 0 & 0 & B_{G_d} & B_G & \\
0 & \mathbf{A}_{T_{ref}} & 0 & 0 & 0 & \mathbf{B}_r & 0 & 0 & \\
-\mathbf{B}_{W_1} & 0 & \mathbf{A}_{W_1} & 0 & 0 & \mathbf{B}_{W_1} & 0 & -\mathrm{B}_{W_1}\mathbf{D}_G & \\
0 & 0 & 0 & \mathbf{A}_{W_2} & 0 & 0 & 0 & \mathbf{B}_{W_2} & \\
\mathbf{B}_{W_3}\mathbf{C}_G & 0 & 0 & 0 & \mathbf{A}_{W_3} & \mathbf{B}_{W_3} & 0 & \mathbf{B}_{W_3}\mathbf{D}_G & \\
-\mathbf{D}_{W1}\mathbf{D}_G & 0 & \mathbf{C}_{W_1} & 0 & 0 & \mathbf{D}_{W_1} & 0 & \mathbf{D}_{W_1}\mathbf{D}_G & 0 \\
0 & 0 & 0 & \mathbf{C}_{W_2} & 0 & 0 & 0 & \mathbf{D}_{W_2} & \\
\mathbf{D}_{W_3}\mathbf{C}_G & 0 & 0 & 0 & \mathbf{C}_{W_3} & 0 & \mathbf{D}_{G_d} & \mathbf{D}_{W_3}\mathbf{D}_G & \\
0 & 0 & 0 & 0 & 0 & \mathbf{I} & 0 & 0 & \\
\mathbf{C}_G & 0 & 0 & 0 & 0 & 0 & \mathbf{D}_{G_d} & \mathbf{D}_G &
\end{bmatrix}.
$$

Now, the H_∞-synthesis can be realized in the MATLAB, for example, by means of the Robust Toolbox function *hinfopt* [4], which defines the equation of the controller minimizing the H_∞- norm of the generalized control object by the way of an optimal value γ search.

Conclusions

The statement of the H_∞-synthesis problem by the method of mixed sensitivity for the robust systems with the 2DOF-controller taking into account the influence of disturbances is represented. The expression for this problem cost function is derived. The appropriate model of the generalized system, which allows the implementation of the H_∞- optimization by means of MATLAB software is defined.

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