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<sup>1</sup>Victor Bocharnikov, Prof.  
<sup>2</sup>Illya Bocharnikov, Postgraduate

## OPTIMAL DISCRETE FUZZY FILTER OF UAV'S FLIGHT PARAMETERS

National Aviation University

<sup>1</sup>E-mail: bocharnikovvp@yandex.ru<sup>2</sup>E-mail: kronos\_88b@mail.ru

**Abstract.** *The robust optimal discrete filter for flight information parameters estimation of Unmanned Aerial Vehicle in conditions of nonstationary and not additive disturbance influence, with unknown parameters, is synthesized. The filter based on the theory of fuzzy measure and fuzzy-integral calculus. An estimation of the signal is determined by fuzzy images of the signal estimated value at the previous step of the measured signal and by selection of filtration function. . The investigations of the optimality of synthesized fuzzy filter are performed.*

**Keywords:** fuzzy estimate, fuzzy integral, fuzzy measure, robust control theory.

### Introduction

As it was previously shown in [1] to ensure the measurement accuracy of Unmanned Aerial Vehicle (UAV) flight under the implementation of robust control in the condition of fuzzy information about the noises it is reasonable to use discrete fuzzy filter.

In that paper it was shown that for the effectiveness of the filter it is reasonable to use two-dimensional filtering function as the Cartesian product of fuzzy measures in the state space of the filter and time space of its work.

However, the work was not precisely defined the procedure and the basic relations for the two-dimensional filtering function, which provides optimal filtered result signal in the sense of some criterion. Thus, the scientific challenge, which is discussed in this paper is to obtain the optimal parameters of fuzzy discrete filter and to investigate the effectiveness of the results obtained on the example.

### Optimization Problem Statement

In [1; 2] it is showed that the fuzzy evaluation process  $\hat{s}(\omega)$  will be the best way to describe the true fuzzy process  $s_u(\omega)$ , if the condition of the form is satisfied:

$$P = (s) \int_{\Omega} [\hat{s}(\omega) \wedge s_u(\omega)] \circ g(\cdot) \rightarrow \max_{\hat{s}}, \quad (1)$$

where  $(s) \int_{(\cdot)} [\cdot]$  is the designation of Sugeno fuzzy integral of fuzzy measure.

As was shown in [1] in the discrete case the fuzzy observer defines by fuzzy-integral dependence on the form:

$$\hat{s}_{t+1}(\omega) = (s) \int_T \hat{s}_t(\omega) \circ \partial_{\tau}(\cdot | \omega, t), \quad (2)$$

where  $\partial_{\tau}(\cdot | \omega, t) : 2^{(T \times \Omega) \times T} \rightarrow [0, 1]$  is discrete conditional fuzzy measure [3] on a time space  $T$ , which determines the credibility of fuzzy process evaluation in previous time moments, for a fixed state-space process  $\omega \in \Omega$ . In practice, the conditional measure  $\partial_{\tau}(\cdot | \omega, t)$  defines a time  $\tau \subseteq T$  window on  $T$ ;

$\hat{s}_t(\omega)$  is defined as a slice of the estimated fuzzy process in discrete time  $t \in T$ , which is under state filtering function, is determined by the relations of the form:

$$\hat{s}_t(\omega) = \sigma_t(\omega) \wedge \varphi_R^t(\omega),$$

$$\sigma_t(\omega) = s_M^t(\omega) \wedge R_p(p_A^t(\omega) | s),$$

$$\varphi_R(\omega) = \begin{cases} [R, 1], \omega \in B_R(\omega), R \in [0, 1] \\ \varphi_R^{\min}, \omega \notin B_R(\omega), \varphi_R^{\min} \leq R. \end{cases} \quad (3)$$

In the above expressions (3) used the notation, considered in detail in [1], which physically define the following components of a fuzzy digital filter:

$s_M^t(\omega)$  is fuzzy process state of model of dynamical fuzzy system;

$R_p(p_A^t(\omega) | s)$  is fuzzy measurement of the process state in discrete time moment  $t \in T$ ;

$\varphi_R^t(\omega)$  is filtrating function in the state space  $\omega \in \Omega$ .

Form of this function and its physical meaning discussed in detail in [2]. Function, defined by the Cartesian product of fuzzy measures:

$$W_t(\omega, \tau) = \varphi_R^t(\omega) \times \partial_\tau(\cdot | \omega, t).$$

Filtering function  $W_t(\omega, \tau)$  "cuts" a certain sub domain of the space  $\Omega \times T$  and practically defines the characteristics of a fuzzy filter.

The objective of this study is to determine the optimal parameters of two-dimensional filter function for the discrete fuzzy filter of the type (2) in the sense of criterion (1).

### Assumptions And limitations Adopted in the Article

In this paper, under the synthesis of optimal parameters of fuzzy discrete filter, the following assumptions and limitations were adopted:

a) It is assumed that the fuzzy discrete filter is constructed for the case of one previous value accounting estimates  $\hat{S}_{t-1}(\omega)$ , that is, the depth of history accounted equal to 1. In this case the conditional fuzzy measure of the filtering function of time  $\partial_\tau(\cdot | \omega, t) : 2^{(T \times \Omega) \times T} \rightarrow [0,1]$  is determined by a time interval  $\tau \subseteq T$  of power  $\text{Card } \tau = 2$ .

b) It is assumed that the filtering function of time is considered as uniform fuzzy measure, which implies "equal contribution" to the estimation of the previous estimation and measurement at the current step, that is

$$\partial_\tau(\cdot | \omega, t) = \partial_\tau(\cdot | \omega, t-1) = \partial_\tau(\omega) \in [0,1].$$

c) Based on the results obtained in [1], the equation of the fuzzy logic filter for the case can be represented as follows:

$$\hat{S}_t(\omega) = (1 - \alpha)\hat{S}_{t-1}(\omega) + \alpha[\sigma_t(\omega) \wedge \varphi_R^t(\omega)]. \quad (4)$$

In this dependence parameter  $\alpha \in [0,1]$  is uniquely dependent on the filtering function of time  $\partial_\tau(\omega) \in [0,1]$  [2].

d) In [2] it was shown that the optimality criterion (1) can be converted to the form:

$$P = \min_\tau \left\{ (s) \int_\Omega \sigma_\tau(\omega) \circ \left\langle (s) \int_\Omega \hat{S}_t(\omega) \circ g \right\rangle \right\} \rightarrow \max_{\hat{S}_t(\omega)}, \quad (5)$$

where  $g(\cdot) : 2\Omega \rightarrow [0,1]$  is fuzzy measure on the state space of signal  $\Omega$ , and the triangular brackets denote the extended fuzzy measure on the state space [2] generated by the fuzzy process  $\hat{S}_t(\omega)$ . In the sequel to optimize the parameters of fuzzy discrete filter, we consider the optimality criterion (5).

### Synthesis of optimal fuzzy discrete filter

Parameters of two-dimensional filtering function  $W_t(\omega, \tau)$  to be optimized. To solve this problem it is necessary to determine the two-dimensional filter function  $W_t(\omega, \tau)$  satisfying the optimality criterion (5). To do this it is necessary to define the parameters of filtering functions  $\varphi_R^t(\omega)$  on the state space and the temporary space  $\partial_\tau(\cdot | \omega, t)$ .

In accordance with the dependence (3) filtering function over the states space  $\varphi_R^t(\omega)$  is uniquely determined by the value of parameter  $R \in [0,1]$ . For the most "stiffness" case filtering function  $\varphi_R^t(\omega)$  will look like:

$$\varphi_R^t(\omega) = \begin{cases} [R, 1], & \omega \in B_R(\omega) \subseteq \Omega, \quad R \in [0,1] \\ 0, & \omega \notin B_R(\omega) \end{cases} \quad (6)$$

where subset  $B_R(\omega) \subseteq \Omega$  is determines in accordance with graph fig. 1.

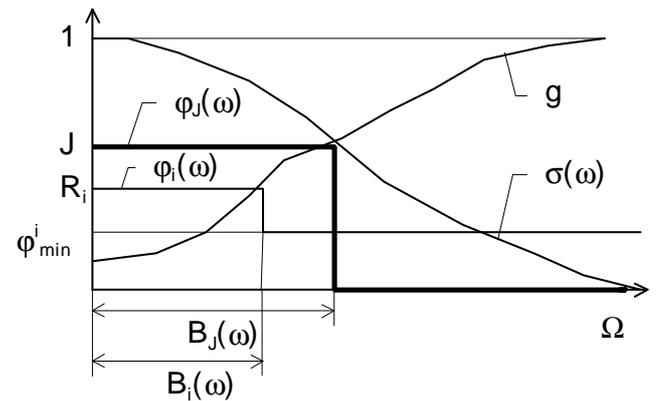


Fig. 1. Filtering function for the state space  $\varphi_R^t(\omega)$

In its turn, under our earlier assumptions about the filter function on a time space  $\partial_\tau(\cdot | \omega, t)$ , this function for a particular state  $\omega \in \Omega$  is uniquely determined by the parameter  $\alpha(\omega) \in [0,1]$  in accordance with the expression:

$$\partial_t(\omega) = (1 - \alpha(\omega)) \cdot \hat{s}_{t-\tau}(\omega) + \alpha(\omega) \times \\ \times [\sigma_t(\omega) \wedge \varphi_R^t(\omega)].$$

Hence, the parameter  $\alpha(\omega) \in [0,1]$  will be determined by:

$$\alpha(\omega) = \frac{\partial_t(\omega) - \hat{s}_{t-1}(\omega)}{(\sigma_t(\omega) - \varphi_R^t(\omega)) - \hat{s}_{t-1}(\omega)}.$$

In the equation of fuzzy digital filter (4) to reduce the computational cost single  $\alpha \in [0,1]$  parameter value is used that can be obtained in several variants.

In particular  $\alpha \in [0,1]$  parameter can be defined as an average, or as a fuzzy expected value:

$$\alpha = \frac{\sum_{\omega \in \Omega} \alpha(\omega)}{\text{Card} \Omega};$$

$$\alpha = (s) \int_{\Omega} \alpha(\omega) \circ g(\cdot), \quad (7)$$

where  $g(\cdot): 2^{\Omega} \rightarrow [0,1]$  is a fuzzy measure defined on the state space  $\Omega$ . In the case of the definition  $\alpha \in [0,1]$  in equation (7) when considering the uncertain impacts of disturbing fuzzy measure  $g(\cdot)$  can be presented as a measure of the possibility of complete uncertainty [4], that is  $\forall \omega \in \Omega, g(\omega) = 1$ . In this case, the parameter  $\alpha$  of the fuzzy filter is determined by the form:

$$\alpha = \max_{\omega \in \Omega} \alpha(\omega). \quad (8)$$

Thus, to optimize the two-dimensional filter function  $W_t(\omega, \tau)$ , we need to define the parameters  $\alpha \in [0,1]$  and  $R \in [0,1]$  which meet the criterion of optimality condition (5).

### Optimization of filtering function parameters $W_t(\omega, \tau)$

For the case when  $\text{Card} \tau = 2$ , the correlation for criterion (6) will be:

$$(s) \int_{\Omega} \sigma_t(\omega) \circ \left\langle (s) \int_{\Omega} \hat{s}_t(\omega) \circ g \right\rangle \wedge \\ \wedge (s) \int_{\Omega} \sigma_{t-1}(\omega) \circ \left\langle (s) \int_{\Omega} \hat{s}_{t-1}(\omega) \circ g \right\rangle \rightarrow \max_{\hat{s}_t(\omega)}. \quad (9)$$

In (9), there is an unknown fuzzy state of estimation process  $\hat{s}_t(\omega)$  for the current time moment. Lets make the necessary changes for the exception of  $\hat{s}_t(\omega)$ .

We represent the fuzzy integral for time in the following equivalent form:

$$(s) \int_{\Omega} \sigma_t(\omega) \circ \left\langle (s) \int_{\Omega} \hat{s}_t(\omega) \circ g \right\rangle = \\ = \left[ (s) \int_{\Omega} \sigma_t(\omega) \circ \left\langle (s) \int_{\Omega} \hat{s}_{t-1}(\omega) \circ g \right\rangle \vee \gamma \right] \wedge \beta,$$

where  $\gamma, \beta \in [0,1]$  are unknown parameters.

We substitute this relation in (9):

$$\left\{ (s) \int_{\Omega} \sigma_t(\omega) \circ \left\langle (s) \int_{\Omega} \hat{s}_{t-1}(\omega) \circ g \right\rangle \vee \gamma \right\} \wedge \\ \wedge \left\{ (s) \int_{\Omega} \sigma_{t-1}(\omega) \circ \left\langle (s) \int_{\Omega} \hat{s}_{t-1}(\omega) \circ g \right\rangle \right\} \wedge \beta \rightarrow \max_{\gamma, \beta}.$$

Since for optimal evaluation of a fuzzy process is necessary to maximize the criterion (6) at each step, then the following relationship is true:

$$(s) \int_{\Omega} \sigma_t(\omega) \circ \left\langle (s) \int_{\Omega} \hat{s}_{t-1}(\omega) \circ g \right\rangle \leq \beta.$$

Then above mentioned relation will be:

$$\left\{ (s) \int_{\Omega} \sigma_t(\omega) \circ \left\langle (s) \int_{\Omega} \hat{s}_{t-1}(\omega) \circ g \right\rangle \vee \gamma \right\} \wedge \\ \wedge \left\{ (s) \int_{\Omega} \sigma_{t-1}(\omega) \circ \left\langle (s) \int_{\Omega} \hat{s}_{t-1}(\omega) \circ g \right\rangle \right\} \rightarrow \max_{\gamma}.$$

If you enforce this condition, the criterion (5) takes the maximum value. Let us transform this relationship. Based on the properties of the fuzzy integral [2] holds:

$$(s) \int_{\Omega} \{ [\gamma \vee \sigma_t(\omega)] \wedge \sigma_{t-1}(\omega) \} \circ \left\langle (s) \int_{\Omega} \hat{s}_{t-1}(\omega) \circ g \right\rangle \rightarrow \\ \rightarrow \max_{\gamma};$$

$$(s) \int_{\Omega} \left\{ \left[ \gamma \wedge \sigma_{t-1}(\omega) \right] \vee \left[ \sigma_t(\omega) \wedge \sigma_{t-1}(\omega) \right] \right\} \circ \left\langle (s) \int_{\Omega} \hat{s}_{t-1}(\omega) \circ g \right\rangle \rightarrow \\ \rightarrow \max_{\gamma};$$

For further consideration it is necessary to determine the value of the unknown parameter  $\gamma \in [0,1]$ .

Based on the properties of the fuzzy Sugeno integral [2], the above mentioned relation can be transformed to the form:

$$\left\{ (s) \int_{\Omega} \sigma_{t-1}(\omega) \circ \left\langle (s) \int_{\Omega} \hat{s}_{t-1}(\omega) \circ g \right\rangle \wedge \gamma \right\} \vee \\ \vee \left\{ (s) \int_{\Omega} [\sigma_t(\omega) \wedge \sigma_{t-1}(\omega)] \circ \left\langle (s) \int_{\Omega} \hat{s}_{t-1}(\omega) \circ g \right\rangle \right\} \rightarrow \\ \rightarrow \max_{\gamma}$$

Designate:

$$(s) \int_{\Omega} \sigma_{t-1}(\omega) \circ \left\langle (s) \int_{\Omega} \hat{s}_{t-1}(\omega) \circ g \right\rangle = I_{t-1}$$

value of criterion (5) on moment  $t-1 \in T$ .

$$(s) \int_{\Omega} [\sigma_t(\omega) \wedge \sigma_{t-1}(\omega)] \circ \left\langle (s) \int_{\Omega} \hat{s}_{t-1}(\omega) \circ g \right\rangle = K_t.$$

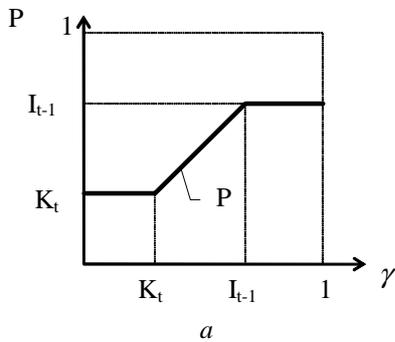
Function  $K_t$  plays the role of the correlation function of the observed fuzzy process. Based on the notation, the ratio for the transformed criterion becomes:

$$P = (I_{t-1} \wedge \gamma) \vee K_t \rightarrow \max_{\gamma} \quad (10)$$

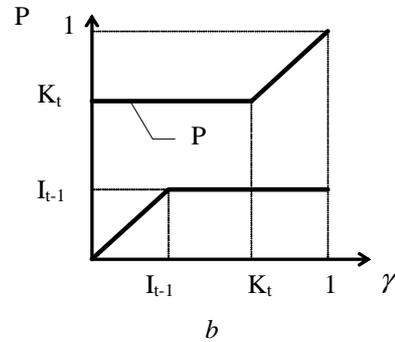
Lets analyze the relation (10). Fig. 2 shows dependences of the functions  $P$  on the parameter  $\gamma \in [0,1]$ .

As is evident from the graphs in fig. 2, to ensure the maximization of the criterion  $P$  value  $\gamma \in [0,1]$  it is advisable to choose from following condition:

$$\alpha \geq K_t = (s) \int_{\Omega} [\sigma_t(\omega) \wedge \sigma_{t-1}(\omega)] \circ \left\langle (s) \int_{\Omega} \hat{s}_{t-1}(\omega) \circ g \right\rangle.$$



a



b

Fig. 2. The  $P$  function value for the case:

a -  $K_t < I_{t-1}$ ;

b -  $K_t > I_{t-1}$ .

Limiting case, when the parameter  $\gamma \in [0,1]$  satisfies the maximization criterion is a value equal to the conditional correlation function of the observed fuzzy process  $\gamma = K_t$ . In this case, the value criterion at the current time moment  $t \in T$  can be represented as follows:

$$(s) \int_{\Omega} \sigma_t(\omega) \circ \left\langle (s) \int_{\Omega} \hat{s}_t(\omega) \circ g \right\rangle = \\ = (s) \int_{\Omega} \sigma_t(\omega) \circ \left\langle (s) \int_{\Omega} \hat{s}_{t-1}(\omega) \circ g \right\rangle \vee K_t.$$

Since, in general, the condition for the maximization of the criterion is the condition  $\gamma \geq K_t$ , we can write:

$$(s) \int_{\Omega} \sigma_t(\omega) \circ \left\langle (s) \int_{\Omega} \hat{s}_t(\omega) \circ g \right\rangle = \\ = (s) \int_{\Omega} \sigma_t(\omega) \circ \left\langle (s) \int_{\Omega} \hat{s}_t(\omega) \circ g \right\rangle \vee K_t = \\ = (s) \int_{\Omega} [\sigma_t(\omega) \vee (\sigma_t(\omega) \wedge \sigma_{t-1}(\omega))] \circ \left\langle (s) \int_{\Omega} \hat{s}_{t-1}(\omega) \circ g \right\rangle = \\ = (s) \int_{\Omega} \sigma_t(\omega) \circ \left\langle (s) \int_{\Omega} \hat{s}_{t-1}(\omega) \circ g \right\rangle = R \in [0,1].$$

The above mentioned value  $R$  is a key parameter of filtering function  $\phi_R^t(\omega)$  over the states space. If the above ratio is satisfied the optimality criterion (5) takes maximum value and condition of optimality will be performed. Thus, using (3) we can define the parameters of the filter function  $\phi_R^t(\omega)$  satisfying the maximization criterion of optimality under the constraints (6).

In particular, we can write:

$$\begin{aligned} & (s) \int_{\Omega} \sigma_t(\omega) \circ \left\langle (s) \int_{\Omega} \hat{s}_t(\omega) \circ g \right\rangle = \\ & = (s) \int_{\Omega} \sigma_t(\omega) \wedge \left[ \sigma_t(\omega) \wedge \varphi_R^t(\omega) \right] \circ g(\cdot) = \\ & = (s) \int_{\Omega} \left[ \sigma_t(\omega) \wedge \varphi_R^t(\omega) \right] \circ g(\cdot) = R. \end{aligned}$$

Thus, the filtering function  $\varphi_R^t(\omega)$  of the state space based on the above given ratio, must be determined by the relations:

$$\begin{aligned} \varphi_R^t(\omega) &= \begin{cases} [R, 1], & \omega \in B_R(\omega), \quad R \in [0, 1]; \\ 0, & \omega \notin B_R(\omega); \end{cases} \\ R &= (s) \int_{\Omega} \sigma_t(\omega) \circ \left\langle (s) \int_{\Omega} \hat{s}_{t-1}(\omega) \circ g \right\rangle, \end{aligned} \quad (11)$$

The subset of  $B_R(\omega) \subseteq \Omega$  is determined in accordance with the graph.

We define the parameter filtering function over the space of time. Basing on the properties of the fuzzy integral to satisfy the equality:

$$(s) \int_{\Omega} \sigma_t(\omega) \circ \left\langle (s) \int_{\Omega} \hat{s}_t(\omega) \circ g \right\rangle = R.$$

Necessary and sufficient conditions:

$$\begin{aligned} \forall \omega \in B_R(\omega), \\ \hat{s}_t(\omega) \geq R. \end{aligned}$$

In accordance with expression (4) we have:

$$\hat{s}_t(\omega) = (1 - \alpha) \cdot \hat{s}_{t-1}(\omega) + \alpha \left[ \sigma_t(\omega) \wedge \varphi_R^t(\omega) \right].$$

Then, if the conditions are met:

$$\varphi_R^t(\omega) = \begin{cases} 1, & \omega \in B_R(\omega), \quad R \in [0, 1]; \\ 0, & \omega \notin B_R(\omega). \end{cases}$$

We must satisfy the following relationship:

$$\begin{aligned} \forall \omega \in B_R(\omega), \\ (1 - \alpha) \cdot \hat{s}_{t-1}(\omega) + \alpha \cdot \sigma_t(\omega) \geq R. \end{aligned}$$

It is necessary that the parameters satisfy the condition:

$$\begin{aligned} \forall \omega \in B_R(\omega), \\ \alpha \geq \frac{R - \hat{s}_{t-1}(\omega)}{\sigma_t(\omega) - \hat{s}_{t-1}(\omega)}. \end{aligned}$$

Or from the condition (8), as follows:

$$\alpha = \max_{\omega \in B_R(\omega)} \frac{R - \hat{s}_{t-1}(\omega)}{\sigma_t(\omega) - \hat{s}_{t-1}(\omega)}. \quad (12)$$

Thus, the relations for the parameters of the filter function (11), (12) determine the optimal two-dimensional filter function of the fuzzy discrete filter.

Synthesized fuzzy discrete filter due to the use of the fuzzy fuzzy-integral dependences for the fuzzy estimation process has robust properties, and allows obtain stable estimates in a non-stationary and non-additive perturbation. We carried out a study of the synthesized fuzzy discrete filter.

### Investigation of the synthesized optimal discrete fuzzy filter

*Structure of Investigation. Assumptions and limitations of Investigation.* Earlier in [1] we were carried out investigation that showed workability and the convergence of the fuzzy discrete filter. To synthesize the optimal fuzzy discrete filter the investigation will be in two steps:

1. In the first stage, we investigate the optimal parameters obtained by two-dimensional filter function. For this purpose, we use the model signals that cluttered by nonstationary nonadditive noise, and we assess the quality of filtering for different values of the two-dimensional filter function.

2. In the second stage to confirm the robustness of the synthesized optimal fuzzy discrete filter, we will study UAV flight filtering signal information in the longitudinal channel. For example, to consider the height of the sensor signal in the case when the control loop is synthesized with optimal Kalman filter-Byusi and with optimal fuzzy filter.

While the effectiveness study of the synthesized optimal fuzzy discrete filter, we made the following assumptions:

a. While the effectiveness study of discrete optimal fuzzy filter the ‘‘Aerozond’’ UAV dynamics model is used in the longitudinal channel, implemented in software package MathLab [5]. Formation of UAV control is based on the use of LQR controller.

b. To investigate the estimation quality of process is considered a one-dimensional fuzzy process describing the change in UAV flight parameters, that is the state space  $\omega \in \Omega$  displays possible states set of one of the flight parameters, in particular, heights.

c. Nonadditive and nonstationary noises effect on the measurement with unknown parameters, due to wind disturbances, noise sensors.

d. Model changes for the selected parameter of flight is a fuzzy dynamical system [2], whose state is determined by the fuzzy process. The model indicated fuzzy process is described by fuzzy-integral equation:

$$s_M(\omega) = (s) \int_T h(\omega, t) \circ \tilde{g}_{f_T(\omega)}(\cdot),$$

where  $h(\omega, t)$  is fuzzy model relation (transfer characteristic of fuzzy process);

$\tilde{g}_{f_T(\omega)}(\cdot)$  is extended fuzzy measure on a time process  $f_T(\omega)$  [2].

e. For investigation we assume that the space states and the measurement space are identical  $\Omega' \equiv \Omega$ .

For a discrete space  $\Omega$  conditional fuzzy measure that relates the data space is defined as the identity matrix  $R_p(\cdot|s) = I$ . Thus, for measuring the input we get the relation:

$$R_p(p_t^A(\omega) | s) = p_t^A(\omega),$$

where  $p_K^A(\omega)$  is fuzzy image [2] of the input noisy sensor signal.

To obtain a fuzzy image signals distribution measures on the state space in fig. 3 are used.

f. Defuzzification procedure resulting fuzzy evaluation process  $\hat{s}(\omega)$  used by the operation of

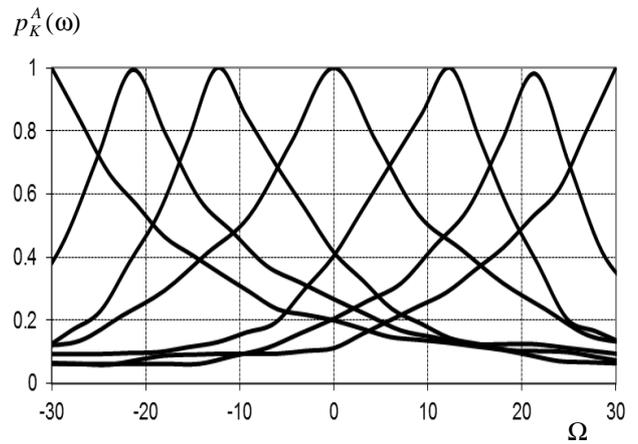


Fig. 3. Variables distribution of fuzzy measures for fuzzy image signals

the “mass center” definition of a fuzzy distribution [6] with the properties of the median.

As quality criteria for filtering the percentage taken from the standard error of the true and noisy signal, in the case that in the absence of filtering error is 100%.

### Optimality Investigation of the Two-Dimensional Filter Function

To investigate the optimality of the synthesized fuzzy discrete filter were selected model true input signals noisy by nonstationary noise with unknown algorithm parameters. Version of the true input signal and the signal with noise at a discrete time space  $T=[590,600]$ ,  $\Delta t=0,02$  s; is shown in fig. 4.

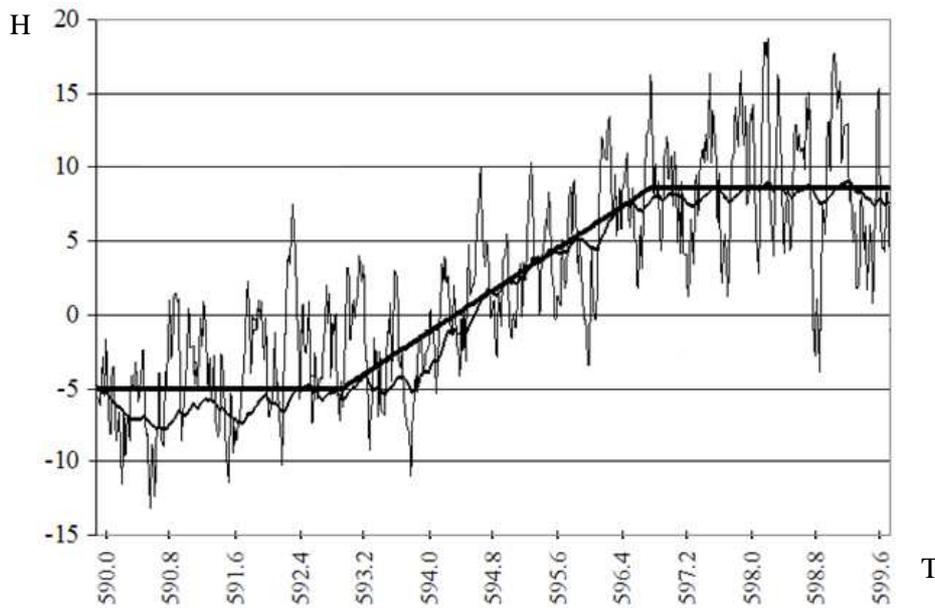


Fig. 4. True, noisy and filtered signal, at the optimum parameters of the filter function

As a result of the experiment at selected the original data in case of forming a filter with the parameters defined by equations (11) and (12) filtered signal is shown in fig. 5. In this case, the optimal parameters have the values:

$$\alpha_{opt} = 0,071;$$

$$R_{opt} = 0,3085.$$

Fig. 5 shows that the error signal is filtered by various parameters filtering function.

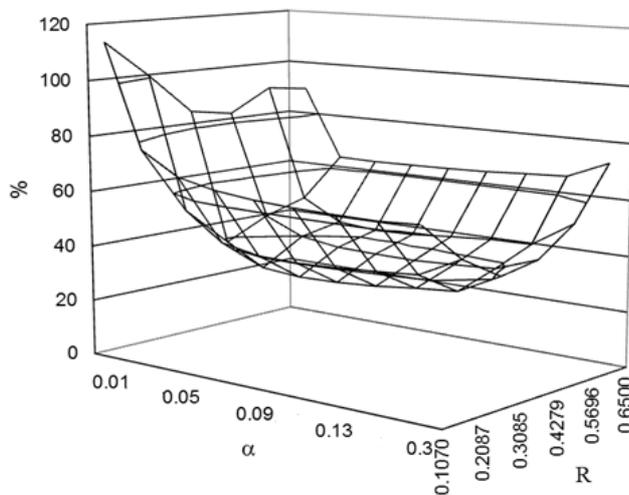


Fig. 5. Dependence of the filtering quality on parameters of the function

As it is seen from the investigation results, obtained in accordance with relations (1), (2) the parameters of the fuzzy discrete filter provides optimal filtration of the signal, reducing noise by 70%.

### Robustness investigation of the synthesized optimal fuzzy discrete filter

To investigate the robustness of the synthesized filter we will use the longitudinal control channel of UAV [5]. Investigation will consider signal measuring the height of the UAV.

In the ordinary case in the UAV control loop to operate the LQR controller is used Kalman-Bucy filter. In the case of the influence of steady external disturbances, the characteristics of which are known under the synthesis parameters of the filter, the signal measurement ensure to be sustainable and optimal. Fig. 6 shows the results of filtering the sensor signal height at work Kalman-Bucy in stationary noise conditions.

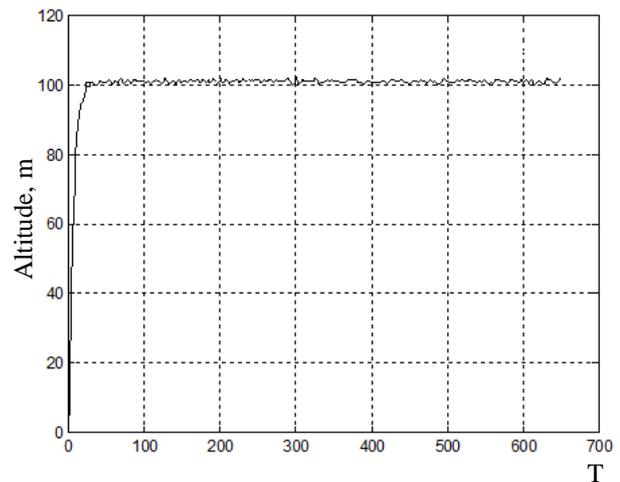


Fig. 6. Measurement of the UAV height signal using Kalman-Bucy filter with a known stationary noise

As we see in fig. 6 Classic filter provides good filtering results of a noisy signal. However, investigations have shown that in the case of nonstationary perturbation, which are quite frequent in real UAV flight conditions Kalman-Bucy filter can become unstable and break up. In particular, as shown in fig. 7, with occurrence of disturbances such as shift wind, followed by progressive impulse, in the range of real possible atmospheric turbulence leads to a loss of Kalman-Bucy filter stability (fig. 8).

Using the synthesized optimal fuzzy filter, even in conditions, when Kalman-Bucy filter diverges, yields to a robust estimate of the measured sensor height signal and provide a performance of UAV control system flight.

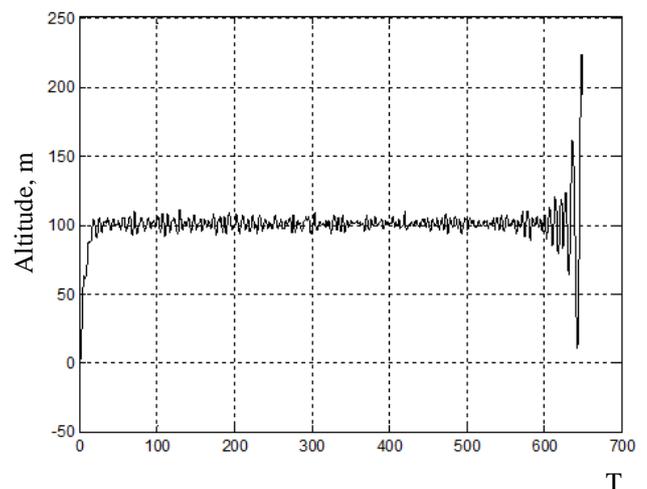


Fig. 7. The UAVs wind disturbance in the longitudinal channel

The results of filtering the signal at the time from 590 to 600 seconds, when the Kalman-Bucy filter loses its stability are presented in fig. 8.

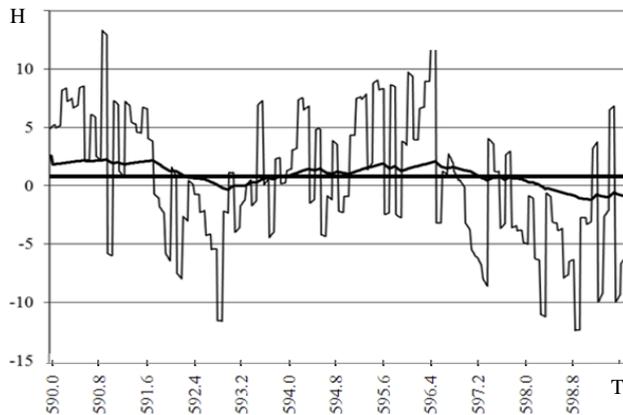


Fig. 8. Filtering of sensor height signal on the basis of discrete optimal fuzzy filter with a nonstationary wind disturbance

Error of filtered height signal is 29,75%.

The signal from the filter does not diverge. This confirms the effectiveness of the optimal fuzzy filter and its robust properties in nonstationary noise with unknown characteristics.

Thus, the optimal fuzzy discrete filter synthesized in this paper, allows to obtain stable estimates of UAV flight signal parameters and thereby increase management efficiency UAVs in real flight conditions.

### Conclusions

Thus, a robust fuzzy digital filter synthesized in this paper allows to obtain robust estimates of the UAV parameters signals flight and there by to improve the management of UAVs in real flight conditions. In further studies it is reasonable to define optimal parameters for two-dimensional filtering function, taking into account the possibility of considering the depth of the time window more than two, that will enhance the estimation quality of the useful signal.

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