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FEEDFORWARD COMPENSATION OF CONTROL SYSTEM WITH STATIC OUTPUT FEEDBACK FOR EXOGENOUS DISTURBANCE SUPPRESSION OF THE RUAV

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The problem of feedforward compensation of control system with static output feedback for exogenous disturbance suppression of the rotorcraft-based unmanned aerial vehicle in hover is considered. Results of the introduced algorithm are evaluated both analytically and with the help of simulation.

Розглянуто задачу пригнічення зовнішніх збурень, що діють на безпілотний вертоліт у режимі висіння, за допомогою статичного зворотного зв'язку за виходом системи та керування за збуренням. Результати реалізації запропонованого алгоритму оцінено аналітично, а також за допомогою моделювання.

Рассмотрена задача подавления внешних возмущений, действующих на беспилотный вертолет в режиме висения, с помощью статической обратной связи по выходу системы и управления по ошибке. Результаты реализации предложенного алгоритма оценены аналитически, а также с помощью моделирования.

Statement of purpose

Rotorcraft-based unmanned aerial vehicles (RUAV) development is one of the aircraft building priorities nowadays. Due to their versatile maneuverability such as vertical takeoff and landing, sideslip, hovering they can be widely used for numerous practical tasks realization without any risks for the crew in extreme and dangerous conditions at comparatively low costs for their maintenance and exploitation.

Suppression of atmospheric disturbance acting the RUAV (stochastic turbulent wind, discrete wind gusts, etc.) is extremely important to perform given tasks ordered by the ground-based command station via wireless communication with high quality and efficiency. In modern rotorcrafts this problem is usually solved with the help of stability and controllability augmentation system (SCAS) design [1–5]. One of the effective methods of robust control theory of SCAS synthesis by static output feedback (SOF) is the linear matrix inequality (LMI) method [1–4; 6–8].

However results of the research demonstrate more efficient suppression of bounded-input bounded-output (BIBO) exogenous disturbances

acting the RUAV in hover via feedforward controller application both with the feedback one. The idea of the corrective feedforward control action is to start disturbance suppression before it affects the output variable [9–12].

To implement feedforward compensation of the static output feedback for atmospheric disturbance suppression their indirect estimates obtained with the special disturbance estimator (DE) can be taken into account.

Statement of the problem

In this paper combined feedforward-feedback control (fig. 1) is applied to the Berkeley RUAV [13] stabilization in the hovering flight taking into account the actuators dynamics and accelerometers incorporation into the measurement unit of the flight control system.

The algorithm of the problem solution includes the following stages.

I. LMI-based synthesis of the stabilizing “minimal controller” K_1 including the procedure of the feedback matrix spectral norm restriction and H_2/H_∞ -optimization of the SOF [1–4; 6].

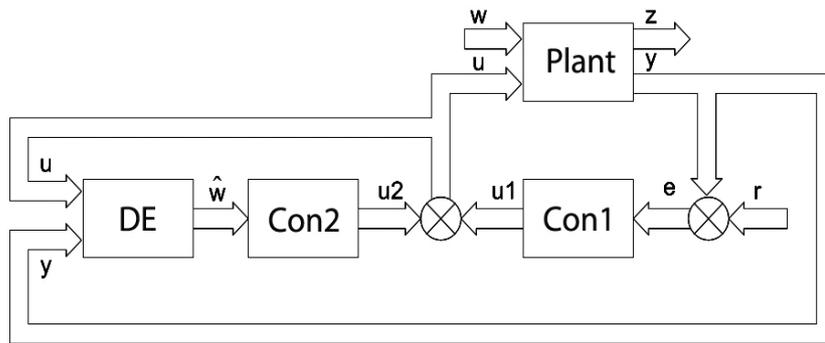


Fig. 1. Block diagram of the combined feedforward-feedback control system:

- Plant – control object;
- DE – disturbance estimator;
- Con2 – feedforward controller;
- Con1 – SOF controller;
- w – vector of exogenous disturbance;
- \hat{w} – disturbance estimate;
- u – control vector;
- z – output vector which is used to evaluate the closed-loop system performance;
- y – output vector which is used for SOF loop shaping;
- e – error;
- r – reference signal

II. Indirect estimate of exogenous actions via DE design on the basis of accelerometers output signals both with the control signals and the state vector x restoration.

III. Feedforward controller K_2 design in agreement with the restriction

$$\|H_{zw}^C(s, K_1, K_2)\|_{\infty} < \gamma, \tag{1}$$

where $H_{zw}^C(s, K_1, K_2)$ – matrix of transfer functions (TF) which describes the relationship between the input exogenous disturbance w and output vector z of the closed-loop system;

$\|\cdot\|_{\infty}$ – H_{∞} -norm;

γ – scalar which represents the degree of exogenous disturbance suppression.

Feedback and feedforward controllers' spectral norms restriction allows to restrict coefficients of K_1 and K_2 that is very important to avoid or at less to diminish probability of the actuator saturation [14].

System description

In this paper linear time-invariant (LTI) multi-input multi-output (MIMO) model of Berkeley RUAV which is valid for hovering is considered [13].

A 6-degrees-of-freedom linear rigid body rotorcraft model is augmented with the first-order approximation of servomotor or Bell-Hiller Stabilizer (BHS) system dynamics [15] which modifies the RUAV dynamics significantly and has a pair of paddle-shaped blades that are connected to the main blades by a series of mechanical linkages. The BHS improves stability characteristics of the helicopter. The most important role of the servomotor is to slow down the roll and pitch response so that human pilot on the ground can control the small RUAV with the remote controller [13; 15].

The peculiarity of the LTI MIMO model of the RUAV is the absence of its separation on the model of longitudinal and lateral motion which is especially justified for hover.

The set of differential equations describing dynamics of the system in time-domain can be represented by:

$$\begin{cases} \dot{x} = Ax + B_u u + B_w w, \\ y = C_y x + D_{yu} u + D_{yw} w, \\ z = C_z x + D_{zu} u + D_{zw} w, \end{cases} \tag{2}$$

where $x \in R^{1 \times 1}$ – state vector;

$u \in R^{4 \times 1}$ – control vector;

$y \in R^{1 \times 1}$ – output vector;

$w \in R^{3 \times 1}$ – vector of atmospheric disturbance which affects the RUAV in horizontal and vertical plane (by three axes);

$z \in R^{3 \times 1}$ – output vector which is used to evaluate the closed-loop system performance;

$A \in R^{1 \times 11}$, $B_u \in R^{1 \times 4}$, $B_w \in R^{1 \times 3}$,
 $C_y \in R^{1 \times 11}$, $D_{yu} \in R^{1 \times 4}$, $D_{yw} \in R^{1 \times 3}$, $C_z \in R^{3 \times 11}$,
 $D_{zu} \in R^{3 \times 4}$, $D_{zw} \in R^{3 \times 3}$ – matrices which describe the RUAV state-space model.

Numerical values of the state $A \in R^{1 \times 11}$ and control $B_u \in R^{4 \times 11}$ matrices of the RUAV MIMO model (2) are given in [13]. Another state-space matrices which describe the RUAV state-space model (2) are given in the example below.

State vector of the RUAV includes the following components [13]:

$$x = [u \ v \ p \ q \ \varphi \ \theta \ a_s \ b_s \ w \ r \ r_{fb}]^T,$$

where u, v, w – body-fixed linear longitudinal, lateral and vertical velocity respectively;

θ, φ – pitch and roll angle respectively;

q, p, r – pitch, roll and yaw rate respectively;

a_s, b_s – BHS flapping angles;

r_{fb} – feedback gyro sensor state.

Control vector $u \in R^{4 \times 1}$ consists of four components [13]:

$$u = [u_{as} \ u_{bs} \ u_{\theta} \ u_{r_{fb}}]^T,$$

where u_{as}, u_{bs} – main rotor and flybar cyclic inputs;

u_{θ} – main rotor collective input;

$u_{r_{fb}}$ – tail rotor collective input.

Output vector which is used to evaluate the closed-loop system performance is:

$$z = [a_x \ a_y \ a_z]^T, \quad (3)$$

where $a_x = \frac{du}{dt}$; $a_y = \frac{dv}{dt}$; $a_z = \frac{dw}{dt}$ – longitudinal,

lateral and vertical acceleration respectively.

LMI-based Synthesis of the Stabilizing “Minimal Controller” and SOF Loop Shaping

On this stage control law for the system (2) can be represented with the equation:

$$u = K_1 y, \quad (4)$$

where $K_1 \in R^{4 \times 11}$ – matrix of gain coefficients of the controller Con1.

Taking into account accelerometers data a_x, a_y, a_z the output vector is:

$$y = [a_x \ a_y \ p \ q \ \varphi \ \theta \ a_s \ b_s \ a_z \ r \ r_{fb}]^T \quad (5)$$

Controller Con1 design and its gain matrix K_1 determination is implemented by LMI method and includes three main stages [1; 3]:

1. Linear-quadratic (LQ) problem solution and “minimal controller” K_1 synthesis which satisfies the control law (4) and guarantees the constraint $\|H_{zw}^c(s, K_1)\|_{\infty} < \gamma$ [1–4; 6–7]. In general this problem is reduced to the standard LMI Eigenvalue problem [6] and a set of inequalities solution [1; 3; 6]. To solve this problem in MATLAB environment the procedure *gevp* is used at a given value of γ (1).

2. Inverse LQ problem solution for K_1 and determination of weighting matrices Q, R, N of the quadratic functional [1–4].

3. Optimization of the system by the SOF loop shaping. Output vector is (5). On this stage it is necessary to introduce some scalar μ which provides stability of the system:

$$A_{\mu} = A + \mu I, \quad \text{Re}(\lambda(A_{\mu})) < 0,$$

where λ – eigenvalues of the state matrix A_{μ} .

Scalar μ is also used as an additional optimization parameter together with the matrix K_1 [1; 3].

Disturbance Estimator Design

On this stage it is necessary to estimate indirectly exogenous disturbance w acting the RUAV in hover on the basis of output accelerations with the help of DE.

In this case DE includes Luenberger filter (LF) [16].

Matrices A , B_u and C_{y1} are used as initial data for LF synthesis.

Vector of available measurements in this research includes the next components:

$$y_1 = [p \quad q \quad \varphi \quad \theta \quad a_s \quad b_s \quad r \quad r_{fb}]^T.$$

The model of LF in time domain can be represented by the set of equations:

$$\begin{cases} \dot{x}_{est} = A_{est} x_{est} + B_{est} u_{est}, \\ y_{est} = C_{est} x_{est} + D_{est} u_{est}, \end{cases} \quad (6)$$

where $x_{est} \in R^{3 \times 1}$,

$$u_{est} = [y_1 \quad u]^T \in R^{12 \times 1},$$

$$y_{est} \in R^{3 \times 1},$$

$$A_{est} \in R^{3 \times 3},$$

$$B_{est} \in R^{3 \times 12},$$

$$C_{est} \in R^{3 \times 3},$$

$$D_{est} \in R^{3 \times 12}.$$

To evaluate exogenous disturbance acting the RUAV the next relation is used:

$$B_w w = \dot{x}_1 - A_{est} x_{est} - B_u u, \quad (7)$$

$$\dot{x}_1 = [\dot{x}(1,:) \quad \dot{x}(2,:) \quad \dot{x}(9,:)]^T = [a_x \quad a_y \quad a_z]^T,$$

$$x_{est} = [\tilde{u} \quad \tilde{v} \quad \tilde{w}]^T - \text{restored state variables.}$$

Thus the output of the designed DE is exogenous disturbance estimate:

$$\hat{w} = B_w^{-1} w. \quad (8)$$

Feedforward Controller Design

On the last stage it is necessary to design feedforward controller [9–12; 17] by means of its gain coefficients matrix K_2 determination on the basis of disturbance estimate \hat{w} (8) evaluated on the previous stage. These gain coefficients have to provide the constraint (1) at a spectral norm $\|K_2\|_s$ restriction.

Control law for the SCAS represented on fig. 1 is:

$$u = K_1 y + K_2 \hat{w}. \quad (9)$$

As far as the matrix of gain coefficients of SOF K_1 was determined before the problem of H_∞ -optimization of the combined feedforward-feedback control system at the spectral norm $\|K_2\|_s$ restriction can be represented as determination of K_2^* which provides:

$$K_2^* = \arg \min_{K_2} \left\{ \|H_{zw}^C(s, K_1, K_2)\|_\infty + \eta \|K_2\|_s \right\} \quad (10)$$

where $\|K_2\|_s$ – spectral norm of K_2 ;

η – weighting coefficient.

Case Study

Efficiency of the introduced algorithm of feedforward compensation of the control system with SOF for BIBO exogenous disturbance suppression is demonstrated for the Berkeley RUAV [13] stabilization in the hovering flight taking into account the actuators dynamics and accelerometers incorporation into the measurement unit of the flight control system.

Numerical values of the state matrix $A \in R^{11 \times 11}$ and control matrix $B_u \in R^{4 \times 11}$ of the RUAV MIMO model (2) are given in [13].

Matrices which describe the RUAV state-space model (2) are:

$$C_y = \begin{bmatrix} A(1,:) \\ A(2,:) \\ O_{6 \times 2}, I_{6 \times 6}, O_{6 \times 3} \\ A(9,:) \\ O_{2 \times 9}, I_{2 \times 2} \end{bmatrix};$$

$$\begin{aligned} B_w &= [-A(:,1) \quad -A(:,2) \quad -A(:,9)]; \\ D_{yw} &= [B_w(1,:); B_w(2,:); O_{6 \times 3}; B_w(9,:); O_{2 \times 3}]; \\ C_z &= [A(1,:); A(2,:); A(9,:)]; \\ D_{zw} &= [B_w(1,:); B_w(2,:); B_w(9,:)]; \end{aligned}$$

where $X(m,:)$ – row of a matrix X ;

m – ordinal number of the row;

$X(:,n)$ – column of a matrix X ;

n – ordinal number of the column;

$O_{i \times j}$, $I_{r \times r}$ – zeros and eye matrix of dimension $i \times j$ and $r \times r$ respectively.

Scalar γ (1) which shows the degree of atmospheric disturbance suppression is set: $\gamma = 1,5$.

As the result of the first problem solution H_2 -optimal stabilizing SOF controller is designed. Its gain matrix $K_1 \in R^{4 \times 11}$ provides $\|H_{zw}^C(s, K_1)\|_\infty = 1,3550 < \gamma$ and has a spectral norm which equals $\|K_1\|_s = 0,0221$.

As the result of the DE design via LF (6) synthesis and equation (7) solution estimate of exogenous disturbance (8) is obtained.

Optimal gain coefficients of the feedforward controller K_2 which satisfies the control law (9) and optimization problem (10) is determined at $\eta = 1$.

Spectral norm of the feedforward controller $\|K_2(s)\|_s = 0,1907$.

H_∞ -norm of the matrix of transfer functions between the exogenous disturbance $w \in R^{3 \times 1}$ and output vector $z \in R^{3 \times 1}$ (3) of the combined feedforward-feedback control system (fig. 1) which includes SOF controller $K_1 \in R^{4 \times 11}$ and feedforward controller $K_2 \in R^{4 \times 3}$:

$$\|H_{zw}^C(s, K_1, K_2)\|_\infty = 0,2699.$$

It should be noted that 5-times decrement of H_∞ -norm of the matrix of transfer functions between the exogenous disturbance $w \in R^{3 \times 1}$

and output vector $z \in R^{3 \times 1}$ (3) in the combined feedforward-feedback control system indicates higher robustness of the system and more efficient disturbance suppression in comparison with the system where only SOF controller $K_1 \in R^{4 \times 11}$ is implemented.

Simulation of the Designed SCAS

Simulation of the designed feedforward-feedback SCAS (fig. 1) was fulfilled in Simulink environment at atmospheric disturbances which affect control system in real conditions. Standard Discrete Wind Gust Model (Aerospace Blockset, Simulink) was used to simulate discrete wind gusts acting the RUAV in hover in horizontal and vertical plane accordingly to the USA standard MIL-F-8785C.

Numerical values which characterize simulated wind gusts and degree of their suppression are the next:

1) along the longitudinal axis: $a_{x_in} = 0,47 \text{ m/s}^2$; relation between the maximal values of input and output acceleration: $\Delta(a_x) = 6,5321$;

2) along the lateral axis: $a_{y_in} = 0,47 \text{ m/s}^2$; relation between the maximal values of input and output acceleration: $\Delta(a_y) = 5,7163$;

3) along the vertical axis: $a_{z_in} = 0,2 \text{ m/s}^2$; relation between the maximal values of input and output acceleration: $\Delta(a_z) = 1,5236$.

Simulation results of the designed feedforward-feedback SCAS are introduced on fig. 2. They demonstrate efficient suppression of external disturbances of the RUAV in the hovering flight via combined control implementation.

Conclusion

Results of the combined feedforward-feedback SCAS design and simulation demonstrate more efficient suppression of BIBO exogenous disturbances of the RUAV in hover via feedforward controller application both with the feedback one.

Small values of spectral norms of the feedforward K_2 and feedback K_1 controllers unavailable the actuators saturation.

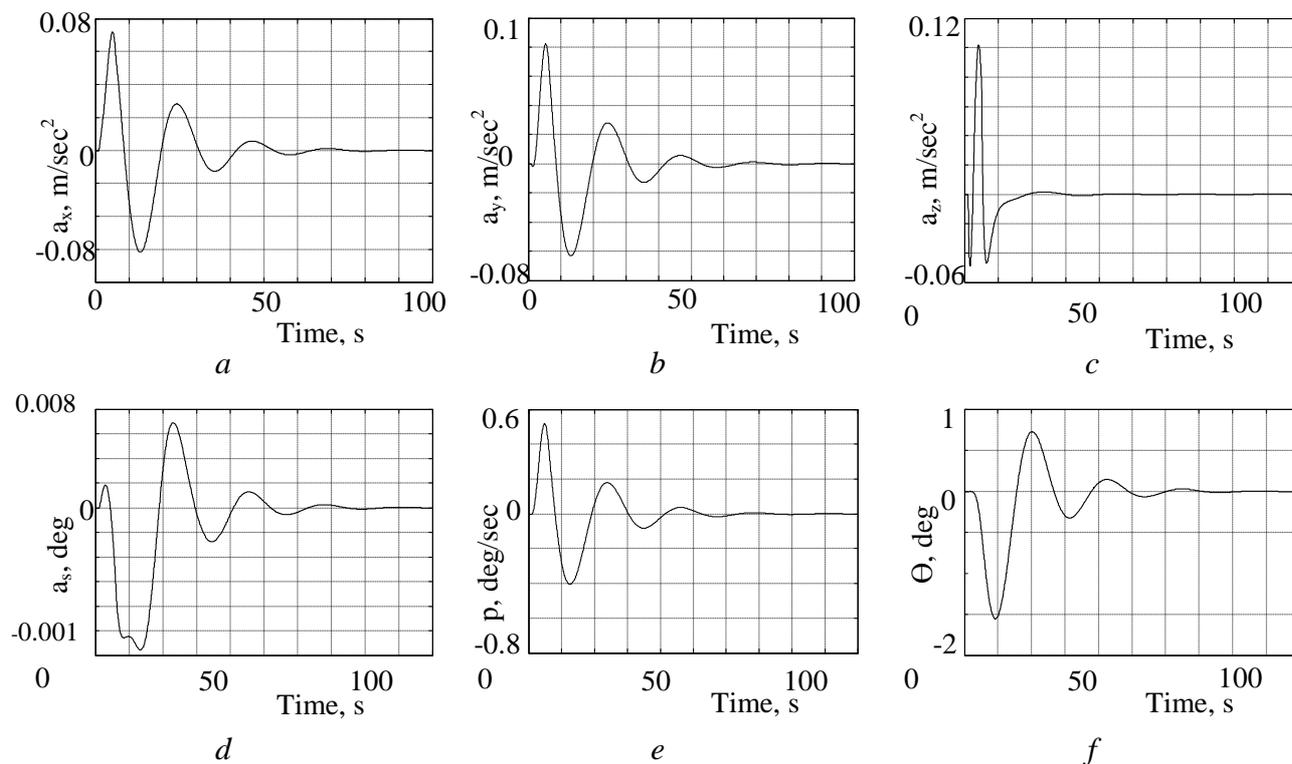


Fig. 2. Simulation results of the designed SCAS at the discrete wind gust action:

- a* – longitudinal acceleration;
- b* – lateral acceleration;
- c* – vertical acceleration;
- d* – flapping angle;
- e* – roll rate;
- f* – pitch angle

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