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STATEMENT OF OPTIMAL SYNTHESIS PROBLEM FOR AUTONOMOUS HIGH-ACCURACY RESISTANT TO DISTURBANCE STABILIZATION AND COURSE SYSTEM

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Approaches to statement of the optimal synthesis problem for the control systems of the wide kind vehicle are analysed and the formalized statement of the optimal synthesis problem for autonomous high-accuracy resistant to disturbance stabilization and course system is formulated. The optimum criterion for the formulated problem is defined. The structural and non-structural uncertainties of the stabilization systems are analysed.

Проаналізовано підходи до постановки задач оптимального синтезу систем управління рухомими об'єктами широкого класу. Сформульовано формалізовану постановку задачі оптимального синтезу стійкої до збурень автономної високоточної системи стабілізації та визначення курсу. Визначено критерій оптимальності сформульованої задачі. Проаналізовано структуровані та неструктуровані невизначеності системи.

Проанализированы подходы к постановке задач оптимального синтеза систем управления подвижными объектами широкого класса. Сформулирована формализованная постановка задачи оптимального синтеза устойчивой к возмущениям автономной высокоточной системы стабилизации и определения курса. Определен критерий оптимальности сформулированной задачи. Проанализированы структурированные и неструктурированные неопределенности системы.

Statement of purpose

Today the strapdown stabilization and course systems are the most widespread. This situation is caused by presence of the satisfactory accurate gyroscopic devices based on the new operation principles (laser, fiber-optic, micro-electro-mechanical ones), computing devices of the high speed and the GPS correction possibilities. Now principles of the inertial stabilization platforms are used only for the autonomous high-accurate stabilization and course systems. Such systems include the measuring unit mounted at the platform in the three-frame gimbals and the computing unit. In turn, the measuring unit consists of three accelerometers with the measuring axes directed along the platform axes and the gyroscopic devices, which provide determination of the vehicle complete attitude. For the domestic instrument engineering the most actual is creation of the autonomous high-accurate stabilization and course systems intended for exploitation at the marine vehicles.

Now the matrix norms of the multi-dimensional closed system transfer functions are the widespread measure of the quality for the control processes in general and the stabilization processes in particular [1]. Values of the matrix transfer function norms allow to estimate the output signals for the definite class input signals.

If the external disturbances are considered to be such signals, the stabilization processes quality will improve with growth of these signals suppression strength by the system.

The quantitative estimation of the stabilization processes may be carried out based on the norms of the closed system matrix transfer function.

So, the problem of the optimal stabilization and course system creation may be formulated as a problem of the system parametrical and structural-parametrical optimization. At that the minimum of the closed system matrix transfer function certain norm will be achieved.

Analysis of the last researches and publications

The general approach to the formalized statement of the resistant to disturbance stabilization systems synthesis problem is presented in the paper [1].

The detail review of approaches to creation of the wide class stabilization systems and comparative analysis of the H_2-optimiaztion, H_∞-optimization and mixed H_2/H_∞-optimization is given in the paper [2]. Features of the optimal synthesis of the systems for control by motion of the aircraft and approach to creation of the complex quality criterion “accuracy-robustness” are presented in the paper [3]. The optimal synthesis formalization for marine vehicles resistant to disturbance stabilization and course systems is problem of today.

The concepts of the robust quality and the robust stability are considered in the paper [4]. The analysis of the typical for control systems uncertainties including multiplicative, additive, inverse multiplicative and division uncertainties is given in the paper [5].

The goal of this paper is to formulate statement of the optimal synthesis problem for the marine vehicle resistant to disturbance stabilization and course system.

The formalized statement of the optimal synthesis problem for the stabilization and course system

To formalize statement of the studied problem in accordance with the approach represented in the paper [1], the linear time-invariant stabilization system, the structure chart of which is represented in fig. 1, may be used.

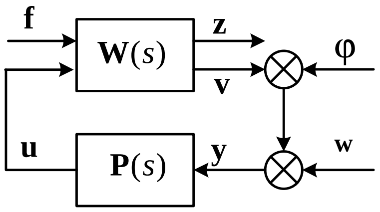

Fig. 1. The structure chart of the stabilization system

At the represented chart:

$\mathbf{W}(s)$ is the transfer function of the stabilization object;

$\mathbf{P}(s)$ is the regulator transfer function;

$\mathbf{w}$ is the vector of reference signals of dimension k_1, which in this case represents the vehicle angular rate, that is the relative angular rate of the object, at which the studied system is mounted;

$\mathbf{u}$ is the vector of controls of dimension m;

$\mathbf{f}$ is the vector of disturbances of dimension k_2;

$\mathbf{z}$ is the vector of the system output signals, which are used for observation of dimension k_3;

$\mathbf{v}$ is the vector of the system output signals, which are used for stabilization of dimension k_4;

$\mathbf{y}$ is the vector of the measured output signals of dimension k_5;

φ is the vector of measurement noise of dimension k_6.

The mathematical model of the stabilization object in the state space may be represented in the following form:

$$\begin{aligned} &\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{u} + \mathbf{F}\mathbf{f}\ ; \\ &\mathbf{v} = \mathbf{C}_1\mathbf{x} + \mathbf{D}_1\mathbf{u}\ ; \\ &\mathbf{z} = \mathbf{C}_2\mathbf{x} + \mathbf{D}_2\mathbf{u}\ , \end{aligned} \qquad (1)$$

where $\mathbf{A}, \mathbf{B}, \mathbf{F}, \mathbf{C}_1, \mathbf{D}_1, \mathbf{C}_2, \mathbf{D}_2$ are the matrices, which describe features of the system, controls, disturbances and measuring system;

$\mathbf{x}$ is the vector of the system state of dimension n.

Under zero initial conditions the stabilization object model in the state space (1) may be represented by means of the matrix transfer function

$$\begin{bmatrix} \mathbf{v} \\ \mathbf{z} \end{bmatrix} = \mathbf{W}(s) \begin{bmatrix} \mathbf{f} \\ \mathbf{u} \end{bmatrix},$$

where

$$\mathbf{W}(s) = \begin{bmatrix} \mathbf{C}_1 \\ \mathbf{C}_2 \end{bmatrix} (\mathbf{I}s - \mathbf{A})^{-1} [\mathbf{F} \quad \mathbf{B}] + \begin{bmatrix} \mathbf{0} & \mathbf{D}_1 \\ \mathbf{0} & \mathbf{D}_2 \end{bmatrix},$$

here $\mathbf{I}$ is the identity matrix.

To formalize the statement of the optimal synthesis problem the structure chart represented in fig. 1 may be given in the more generalized form as it is represented in fig. 2.

The vector $\mathbf{d}=[\mathbf{f} \quad \varphi \quad \mathbf{w}]^{\mathrm{T}}$ in fig. 2 represents the formalized vector of the input signals which act on the system.

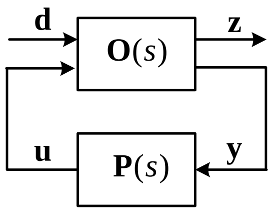

Fig. 2. The generalized structure chart of the stabilization system

Taking into consideration the model (1) and the equation $\mathbf{y}=\mathbf{v}+\varphi+\mathbf{g}$ the stabilization object model in the state space may be represented in the following form:

$$\dot{\mathbf{x}}=\mathbf{A}\mathbf{x}+\mathbf{B}\mathbf{u}+\mathbf{F}\mathbf{f}\,;$$
$$\mathbf{y}=\mathbf{C}_1\mathbf{x}+\mathbf{D}_1\mathbf{u}+\varphi+\mathbf{g}\,; \qquad (2)$$
$$\mathbf{z}=\mathbf{C}_2\mathbf{x}+\mathbf{D}_2\mathbf{u}\,.$$

Based on the expression (2) the stabilization object model becomes:

$$\begin{bmatrix}\mathbf{y}\\ \mathbf{z}\end{bmatrix}=\mathbf{O}(s)\begin{bmatrix}\mathbf{f}\\ \varphi\\ \mathbf{g}\\ \mathbf{u}\end{bmatrix},$$

where

$$\mathbf{O}(s)=\begin{bmatrix}\mathbf{C}_1\\ \mathbf{C}_2\end{bmatrix}(\mathbf{I}s-\mathbf{A})^{-1}\begin{bmatrix}\mathbf{F} & \mathbf{0} & \mathbf{0} & \mathbf{B}\end{bmatrix}+$$
$$+\begin{bmatrix}\mathbf{0} & \mathbf{I}_{k_6\times k_6} & \mathbf{I}_{k_1\times k_1} & \mathbf{D}_1\\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{D}_2\end{bmatrix}$$

For the formalized vector of the input signals the matrix transfer function may be represented in the following form:

$$\begin{bmatrix}\mathbf{y}\\ \mathbf{z}\end{bmatrix}=\begin{bmatrix}\mathbf{O}_{11}(s) & \mathbf{O}_{12}(s)\\ \mathbf{O}_{21}(s) & \mathbf{O}_{22}(s)\end{bmatrix}\begin{bmatrix}\mathbf{d}\\ \mathbf{u}\end{bmatrix}. \qquad (3)$$

If to supplement the stabilization object equations (3) by the regulator equation

$$\mathbf{u}=\mathbf{P}(s)\mathbf{y}\,, \qquad (4)$$

it is possible to pass on to the generalized system, the structure chart of which is represented in fig. 3.

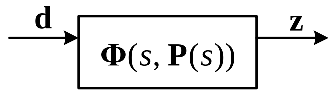

Fig. 3. The structure chart for the formalized statement of the stabilization system synthesis problem

Based on the expression (3) the closed system equations become

$$\mathbf{y}=\mathbf{O}_{11}(s)\mathbf{d}+\mathbf{O}_{12}(s)\mathbf{u};$$
$$\mathbf{z}=\mathbf{O}_{21}(s)\mathbf{d}+\mathbf{O}_{22}(s)\mathbf{u}. \qquad (5)$$

Based on the first equation of the set (5) and the relationship (4) it is possible to write

$$\mathbf{y}=\mathbf{O}_{11}(s)\mathbf{d}+\mathbf{O}_{12}(s)\mathbf{P}(s)\mathbf{y}\,.$$

Hence, taking into consideration the equation (1.4) it is possible to write the expression for the vector of controls

$$\mathbf{u}=\mathbf{P}(s)[\mathbf{I}-\mathbf{O}_{12}\mathbf{P}(s)]^{-1}\mathbf{O}_{11}(s)\mathbf{d}\,.$$

Now the second equation of the set (5) may be represented in the form

$$\mathbf{z}=\mathbf{O}_{21}(s)\mathbf{d}+\mathbf{O}_{22}(s)\mathbf{P}(s)[\mathbf{I}-\mathbf{Q}_{12}\mathbf{P}(s)]^{-1}\mathbf{O}_{11}(s)\mathbf{d}\,.$$

The obtained expressions may be used for representation of the transfer function $\mathbf{T}(s,\mathbf{P}(s))$, which corresponds to the structure chart shown in fig. 3, in the following form

$$\mathbf{T}(s,\mathbf{P}(s))=\mathbf{O}_{21}(s)+\mathbf{O}_{22}(s)\mathbf{P}(s)\times$$
$$\times[\mathbf{I}-\mathbf{Q}_{12}\mathbf{P}(s)]^{-1}\mathbf{O}_{11}(s).$$

This transfer function connects the disturbance vector with the system output vector. At that the goal of decrease of disturbance action on the system may be achieved due to decrease of the gain of the transfer function $\mathbf{T}(s,\mathbf{P}(s))$. To solve this task it is convenient using the matrix norm concept.

So, the formalized statement of the optimal synthesis problem for the stabilization system resistant to the external disturbances may be represented in the following form:

$$\mathbf{P}^* = \arg \inf_{\mathbf{K}(j\omega)\in D} ||\mathbf{\Phi}(s,\mathbf{P}(s))||, \qquad (6)$$

where D is the set of the transfer functions with the fractional rational components, for which the closed system characteristic polynomial satisfies the Hurwitz criterion.

The formalized problem statement must take into consideration the specific features of the system to be studied. For the marine vehicles stabilization and course system the coefficient of the accelerated setting to the meridian is very important. If to denote this coefficient k, the formalized problem statement may be represented in the following form:

$$\mathbf{P}^* = \arg \inf_{\mathbf{K}(j\omega)\in D,\, k\geq k_{per}} ||\mathbf{\Phi}(s,\mathbf{P}(s))||, \qquad (7)$$

where k_{per} is the permissible coefficient.

Optimization of the linear control systems may be based on the H_2 and H_∞-norms of the Hardy space, that is the space of the function of the complex variable analytical in the left half-plane of this variable.

There are different types of the optimization tasks [1] depending on the concrete norm choice in the problem statements (6) or (7):

1) the H_2-optimal synthesis, when the H_2-norm of the closed system transfer function $\mathbf{\Phi}(s,\mathbf{P}(s))$ is minimized;

2) the H_∞- optimal synthesis, when the H_∞-norm of the closed system transfer function $\mathbf{\Phi}(s,\mathbf{P}(s))$ is minimized;

3) the mixed H_2/H_∞ optimization.

Problem of the H_2-optimization lies in determination of the regulator belonging to the permissible set, for which the H_2-norm of the closed system transfer function achieves minimum. It is known that the squared H_2-norm under the definite conditions is equivalent to the quality functional of the LQG-problem. It is worth noting that the computational procedure of the system synthesis based on the H_2-norm may be more simplified in comparison with the similar procedure of the LQG-synthesis, as approaches to their organization are different.

In the first case the theory of the Lebesgue space is used. In the second case the probabilistic approach is considered.

The requirements to the computational procedure in the first case include finiteness of the H_2-norm of the matrix transfer function for the closed system. In the second case it is necessary to take into consideration the requirements to the disturbance $\mathbf{f}(t)$, which represents the white noise with the covariance matrix

$$\mathrm{M}[\mathbf{f}(t_1)\mathbf{f}(t_2)^{\mathrm{T}}] = \mathbf{V}(t_1)\delta(t_2 - t_1),$$

where M is the symbol of the mathematical expectation;

$\mathbf{V}(t_1)$ is the matrix of the white noise intensity.

Use of the H_∞-norm of the transfer function $\mathbf{W}$ as the optimality criterion is possible because it represents the accurate upper bound of the square root of the gain between H_2-norm of the input signals $\mathbf{u}$ and H_2-norm of the output signals $\mathbf{y}$, that is [2]

$$\|\mathbf{W}\|_\infty = \sup\{\|\mathbf{y}\|_2 = \|\mathbf{Wu}\|_2 : \mathbf{u} \in l_2[0,\infty), \|\mathbf{u}\|_2 \leq 1\}.$$

Therefore in the physical interpretation the H_∞-norm of the transfer function is the square root of the energy of the output signals under condition that the disturbance with the unit energy enters to the input. So, minimization of this norm means minimization of the error energy for the worst case of the studied class input disturbances [2].

Optimization by the mixed H_2/H_∞-criterion unites advantages of the different approaches. From this point of view it is possible to believe that the mixed H_2/H_∞-approach may be used for the synthesis of the optimal quality system under condition of its capacity for work under conditions of the worst disturbances. So, the best approach to the synthesis of the studied system is the mixed H_2/H_∞-approach.

Specific features of the system to be studied are the significant variations of some its parameters and variable conditions of its exploitation. Therefore it is convenient to carry out the synthesis of the studied system from the point of view of its robustness provision. Such approach allows to keep the certain performances of the system in conditions of the disturbance action. Taking into consideration the features of the studied system the optimal synthesis problem may be solved as the problem of the robust system optimization

$$\mathbf{P}^* = \arg \inf_{\mathbf{K}(j\omega)\in D,\, k\geq k_{per}} ||\,\mathbf{\Phi}(s,\mathbf{P}(s),\Delta)\,||,$$

where Δ is the system uncertainty.

Concepts of the robust stability and the robust quality are given in the paper [4]. For these concepts analysis the studied system was believed to correspond to the structure chart represented in fig. 4, *a*.

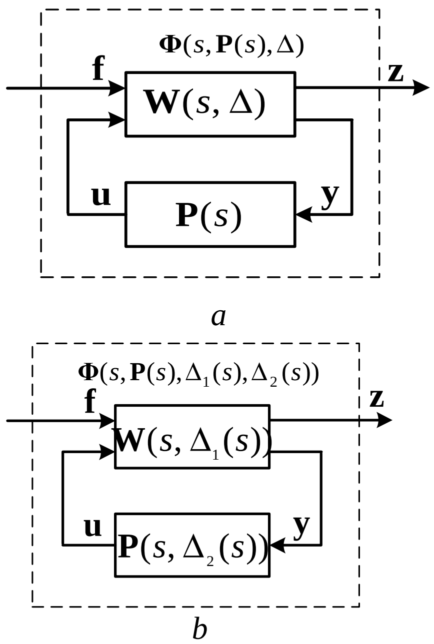

Fig. 4. The structure chart for analysis (*a*) and for definition (*b*) of the robust stability and the robust quality

For the represented structure chart $\mathbf{W}(s,\Delta)$, $\mathbf{P}(s)$, $\mathbf{\Phi}(s,\mathbf{P}(s),\Delta)$ are the transfer functions of the stabilization object, the regulator and the closed system.

In accordance with this structure chart the system may be described by the equation

$$\mathbf{z} = \mathbf{\Phi}(s,\mathbf{P}(s),\Delta)\mathbf{f}\,,$$

where Δ is an uncertainty in the stabilization object representation.

The single limitation is that the uncertainty belongs to some given set $\Delta \in D$.

To define the robust stability concept it is necessary to consider the closed system characteristic polynomial $\delta(s,\mathbf{P}(s),\Delta)$, which depends on the kind of the regulator and the uncertainty.

Roots of the stable system

$$\delta_i = \delta_i(\mathbf{P}(s),\Delta)\,,\ i=\overline{1,n}\,,$$

when n is degree of polynomial must lie in the left half-plane of the complex variable C^-.

The closed system is the robust stable relative to the uncertainty $\Delta \in D$, if the conditions

$$\delta_i = \delta_i(\mathbf{P}(s),\Delta) \in C^-,\ i=\overline{1,n}$$

are satisfied.

The closed system has some robust quality, if it is robust stable relative to the uncertainty Δ and the condition $F \in R$ is satisfied, where F is the quality functional; R is the set of the permissible quality functionals for the studied system. The regulator $\mathbf{u} = \mathbf{P}(s)\mathbf{y}$ is believed to provide the robust quality of the closed system.

Such definition of the robust stability and the robust quality may be widened due to presence of the different kind uncertainties in the mathematical description of both the stabilization object and the regulator.

Uncertainties of the time-invariant systems are divided in the structured (parametrical) and non-structured (non-simulated dynamics). Correspondingly, to take into consideration these uncertainties it is possible to use the variations of the system matrix or some additional linear fractional link as the feedback of the system.

For many practical applications the optimal synthesis of the control system for the regulator with the given structure is the actual problem. At that the structured and non-structured uncertainties of the transfer functions of both the stabilization object and the regulator take place.

The structure chart of the closed system which may be used for definition of the robust stability and the robust quality and takes into consideration the different kind uncertainties of both the stabilization object and the regulator is represented in fig. 4, *b*.

In the represented scheme $\Delta_1(s), \Delta_2(s)$ are the structured uncertainties of the stabilization object model and the regulator in the general form. Then the robust stability and the robust quality concepts may be defined in the following way [4].

The closed stabilization system described by the equation

$$\mathbf{z} = \mathbf{\Phi}(s, \mathbf{P}(s), \Delta_1(s), \Delta_2(s))\mathbf{f}$$

is the robust stable relative to the uncertainties

$\Delta_1(s) \in D_1$, $\Delta_2(s) \in D_2$,

where D_1, D_2 are the sets of the permissible transfer functions, if the condition

$$\delta_i = \delta_i(\mathbf{P}(s), \Delta_1(s), \Delta_2(s)) \in C^{-1}, \ i = \overline{1,n}$$

is satisfied.

At that the set of regulators

$$\mathbf{u} = \mathbf{P}(s, \Delta_2(s))\mathbf{y}$$

is considered to provide the robust stability of the closed system.

The closed stabilization system has certain robust quality, if it is robust stable relative to the uncertainties $\Delta_1(s), \Delta_2(s)$ and the condition $F \in R$ takes place, where F is the quality functional; R is the set of the permissible functionals.

The regulator

$$\mathbf{u} = \mathbf{P}(s)\mathbf{y}$$

chosen from the set of regulators $\mathbf{P}(s, \Delta_2(s))$ is believed to provide the robust quality of the closed system.

Now it is possible to define the formalized statement of the studied system optimal synthesis problem finally. As stated above, it is convenient to solve this problem from the point of view of the mixed H_2 / H_∞ approach. At that it is necessary to take into consideration that transfer functions of the stabilization object and the regulator must belong to the spaces RH_2, RH_∞.

It is known that RH_2 is the space of the strongly proper fractional rational functions which have not peculiarities in the left half-plane and at the imaginary axis. RH_∞ is the space of the proper fractional rational functions which have not peculiarities in the left half-plane and at the imaginary axis. This implies that $RH_\infty \supset RH_2$.

It worth noting, that in accordance with the definition these spaces can not include some enough widespread control systems such as the astatic systems.

To carry out synthesis of such systems it is necessary to implement some transformations, that is pass on to the systems, which have not the above stated peculiarities but keep in full the basic performances of the studied system.

So, the procedures of the robust system optimal synthesis may use norms of the transformed closed system transfer function

$$\mathbf{\Phi}_F(s, \mathbf{P}(s), \Delta_1(s), \Delta_2(s)) =$$
$$= F[\mathbf{\Phi}(s, \mathbf{P}(s), \Delta_1(s), \Delta_2(s))].$$

Uncertainties of the synthesized system mathematical description have many sources such as [5]:

1) the errors in determination of the linear model parameters;

2) the unsuspected non-linearities and changes in operation conditions;

3) the unsuspected time delays and energy dissipation processes;

4) imperfection of the measuring instruments;

5) use of the reduced models, that is models of the reduced order for the synthesis procedure simplification;

6) the unsuspected features of the model dynamics at the high frequencies;

7) reduction of the regulator order and imperfection of its implementation.

Above stated sources of the mathematical description uncertainties may be divided into three groups [5]:

1) the parametrical or structured uncertainties, when parameters of the model with the determined structure and order change in some given space of parameters;

2) the uncertainties of the unsuspected and non-simulated dynamics;

3) the lumped or non-structured uncertainties, caused by the parametrical uncertainty and non-simulated dynamics united in the single lumped disturbance of the preassigned structure.

It worth noting, that in the scientific and technical literature the second and the third groups are often united into the single group of the unstructured disturbances.

The parametrical uncertainty is defined for the set of parameters bounded by the some bounds $[p_{\min}, p_{\max}]$. The set of parameters may be described by the expression [5]

$$p = p_{av}(1 + r\Delta),$$

$$r = \frac{p_{\max} - p_{\min}}{p_{\max} + p_{\min}}; \ |\Delta| \le 1,$$

where p_{av} is the average value of the parameter.

For $\Delta = 1$ value of the parameter p will be maximal, and for $\Delta = -1$ – minimal.

The mathematical description of the parametrical uncertainties for the real systems is significantly complicated due to large quantity of the undetermined parameters. To create such description it is necessary to have the model with the well-defined structure. Usually the non-simulated dynamics is not taken into consideration in such models. Therefore it is convenient to carry out estimation of such system robustness taking into consideration the unstructured uncertainties after parametric optimization termination.

The unsuspected and non-simulated dynamics is more complex for the mathematical representation and usually for this it is necessary to use the frequency domain. At that the disturbance Δ is believed to be normalized by the H_∞-norm, that is the condition $\|\Delta\|_\infty \le 1$ will be satisfied, where $\|\Delta\|_\infty = \sup_\omega |\Delta(j\omega)|$.

There are some forms of the unstructured uncertainties such as multiplicative, additive, inverse multiplicative and division. Correspondingly they may be described in the following way [5]:

1) multiplicative:

$$G_m(s): G_{db}(s) = G_{nom}(s)[1 + w_m(s)\Delta_m(s)],$$
$$\|\Delta_m\|_\infty \le 1,$$

where $G_m(s)$ is the set of permissible linear time-invariant models with the multiplicative uncertainties;

$G_m(s)$ is the stabilization object transfer function;

$G_{db}(s)$ the set of the disturbed models of the stabilization object;

$G_{nom}(s)$ is the nominal model of the stabilization object;

$w_m(s)$ is the weighting function;

$\Delta_m(s)$ is the multiplicative unstructured uncertainty;

2) additive:

$$G_a(s): G_{db}(s) = G_{nom}(s) + w_a(s)\Delta_a(s) \ (),$$
$$\|\Delta_a\|_\infty \le 1,$$

where $G_a(s)$ is the set of permissible linear time-invariant models with the additive uncertainties;

$w_a(s)$ is the weighting function;

$\Delta_a(s)$ is the additive unstructured uncertainty;

3) inverse multiplicative:

$$G_{im}(s): G_{db}(s) = G_{nom}(s)[1 + w_{im}(s)\Delta_{im}(s)]^{-1},$$
$$\|\Delta_{im}\|_\infty \le 1,$$

where $G_{im}(s)$ is the set of permissible linear time-invariant models with the inverse multiplicative uncertainties;

$w_{im}(s)$ is the weighting function;

$\Delta_{im}(s)$ is the inverse multiplicative unstructured uncertainty;

4) division:

$$G_d(s): G_{db}(s) = G_{nom}(s)[1 + w_d(s)\Delta_d(s)G_{nom}]^{-1},$$
$$\|\Delta_d\|_p \le 1,$$

where $G_d(s)$ is the set of permissible linear time-invariant models with the division uncertainties;

$w_d(s)$ is the weighting function;

$\Delta_d(s)$ is the division unstructured uncertainty.

All the listed uncertainties are represented by the stable transfer functions with the magnitude less 1 at all the frequencies.

Obviously, the unstructured uncertainties of the regulators will correspond to the same qualification.

The multiplicative and additive uncertainties are equivalent if the condition

$$|w_m(j\omega)| = \frac{|w_a(j\omega)|}{|G(j\omega)|}$$

is satisfied at all frequencies.

The conditions are known, for which the system is stable for all the disturbances which belong to the above stated set. These conditions define the robust stability and may be described by the following expressions [5]:

1) the additive unstructured uncertainty:

$$\|w_a(s)P(s)S(s)\|_\infty < 1,$$

where $P(s)$ is the transfer function of the regulator;

$S(s)$ the sensitivity function;

2) the multiplicative unstructured uncertainty:

$$\|w_m(s)T(s)\|_\infty < 1,$$

where $T(s)$ is the function of the complementary sensitivity;

3) the inverse multiplicative unstructured uncertainty:

$$\|w_{im}(s)S(s)\|_\infty < 1;$$

4) the division unstructured uncertainty:

$$\|w_\text{p}(s)K(s)S(s)\|_\infty < 1,$$

$$K(s) = W(s)P(s)$$

where $W(s)$ is the transfer function of the stabilization object.

The sensitivity function is defined by the expression [6]

$$S(s) = \frac{1}{1+W(s)P(s)}.$$

The complementary sensitivity may be defined by the relationship [6]

$$T(s) = \frac{W(s)P(s)}{1+W(s)P(s)}.$$

The quality and the robustness of a system are connected by the known relationship

$$S(s) + T(s) = 1.$$

Based on the sensitivity function the condition of fulfillment the requirements to the system quality may be defined in the following way [6]

$$\|w_q(s)S(s)\|_\infty < 1,$$

where w_q is the weighting coefficient, which depends on the system performances.

Taking into consideration the sensitivity function and the Nyquist criterion the condition of the system nominal quality may be defined as the inequality

$$|w_q(j\omega)| < |1 + K(j\omega)|, \ \forall\omega.$$

Based on this condition the robust quality may be defined by the same condition with its fulfillment for all the disturbed systems transfer functions $K_{db}(s)$. Then the above stated condition becomes

$$|w_q(j\omega)| < |1 + K_{db}(j\omega)|, \ \forall K_{зб}(j\omega), \ \forall\omega.$$

In the simplest case of the multiplicative disturbance the condition of the robust quality looks like

$$\big\| |w_q(j\omega)S(j\omega)| + |w_{db}(j\omega)T(j\omega)| \big\|_\infty < 1.$$

From this condition follows that to provide the robust quality it is necessary to decrease the function of the complementary sensitivity $T(j\omega)$.

To achieve the system nominal quality it is necessary to decrease the sensitivity function $S(j\omega)$. So, problems of the robust systems synthesis are characterized by the conflicting objectives and require to use the complex quality index taking into consideration both aspects.

It worth noting, that the synthesized regulator must belong to the class of the permissible regulators that is regulators which provide the internal stabilization of the object which corresponds to the condition

$$\boldsymbol{\Phi}_F(s,\mathbf{P}(s),\Delta_1(s),\Delta_2(s)) \in RH_\infty .$$

In accordance with approach represented in the paper [2] and fig. 4, *a* the matrix transfer function of the stabilization object may be divided into four block matrices

$$\mathbf{W}(s)=\begin{bmatrix}\mathbf{W}_{11}(s) & \mathbf{W}_{12}(s)\\ \mathbf{W}_{21}(s) & \mathbf{W}_{22}(s)\end{bmatrix},$$

where $\mathbf{W}_{11}$ is the transfer function from the vector of signals $\mathbf{f}$ to the vector of signals $\mathbf{z}$;

$\mathbf{W}_{12}$ is the transfer function from the vector $\mathbf{u}$ to the vector $\mathbf{z}$;

$\mathbf{W}_{21}$ is the transfer function from the vector $\mathbf{f}$ to the vector $\mathbf{v}$;

$\mathbf{W}_{22}$ is the transfer function from the vector $\mathbf{u}$ to the vector $\mathbf{v}$.

According to fig. 4, *a* the output signals of the stabilization object and the regulator on the basis of the above stated transfer functions may be defined in the following way

$$\mathbf{z}=\mathbf{W}_{11}(s)\mathbf{f}+\mathbf{W}_{12}(s)\mathbf{u}\,; \quad (8)$$

$$\mathbf{v}=\mathbf{W}_{21}(s)\mathbf{f}+\mathbf{W}_{22}(s)\mathbf{u}\,; \quad (9)$$

$$\mathbf{u}=\mathbf{P}(s)(\mathbf{v}+\varphi+\mathbf{g})\,. \quad (10)$$

After substitution of the expression (10) into the relationship (9) it is possible to obtain

$$\mathbf{v}=\mathbf{W}_{21}(s)\mathbf{f}+\mathbf{W}_{22}(s)\mathbf{P}(s)(\mathbf{v}+\varphi+\mathbf{g})$$

or after transformations

$$\mathbf{v}=[\mathbf{I}-\mathbf{W}_{22}\mathbf{P}(s)]^{-1}[\mathbf{W}_{21}(s)\mathbf{f}+\mathbf{W}_{22}(s)\mathbf{P}(s)\mathbf{g}+$$
$$+\mathbf{W}_{22}(s)\mathbf{P}(s)\varphi]. \quad (11)$$

After substitution of the expression (10) into the relationship (8) it is possible to find

$$\mathbf{z}=\mathbf{W}_{11}(s)\mathbf{f}+\mathbf{W}_{12}(s)\mathbf{P}(s)(\mathbf{v}+\varphi+\mathbf{g})\,. \quad (12)$$

Substituting the relationship (11) in the obtained expression (12) it is possible to define

$$\mathbf{z}=\mathbf{W}_{11}(s)\mathbf{f}+\mathbf{W}_{12}(s)\mathbf{P}(s)\mathbf{g}+\mathbf{W}_{12}(s)\mathbf{P}(s)\varphi+$$
$$+\mathbf{W}_{12}(s)\mathbf{P}(s)[\mathbf{I}-\mathbf{W}_{22}\mathbf{P}(s)]^{-1}[\mathbf{W}_{21}(s)\mathbf{f}+$$
$$+\mathbf{W}_{22}(s)\mathbf{P}(s)\mathbf{g}+\mathbf{W}_{22}(s)\mathbf{P}(s)\varphi]$$

or after transformations

$$\mathrm{z}=\{\mathrm{W}_{11}(s)+\mathrm{W}_{12}(s)\mathrm{P}(s)[\mathrm{I}-\mathrm{W}_{22}\mathrm{P}(s)]^{-1}\times$$
$$\times\mathrm{W}_{21}(s)\}\mathrm{f}+\{\mathrm{W}_{12}(s)\mathrm{P}(s)+$$
$$+\mathrm{W}_{12}(s)\mathrm{P}(s)[\mathrm{I}-\mathrm{W}_{22}\mathrm{P}(s)]^{-1}\mathrm{W}_{22}(s)\mathrm{P}(s)\}\mathrm{g}+$$
$$+\{\mathrm{W}_{12}(s)\mathrm{P}(s)+\mathrm{W}_{12}(s)\mathrm{P}(s)\times$$
$$\times[\mathrm{I}-\mathrm{W}_{22}\mathrm{P}(s)]^{-1}\mathrm{W}_{22}(s)\mathrm{P}(s)\}\varphi. \quad (13)$$

Introducing the generalized vectors of the output

$$\mathbf{x}=[\mathbf{z}\ \mathbf{v}]^{\mathrm{T}}$$

and the input

$$\mathbf{d}=[\mathbf{f}\ \mathbf{g}\ \varphi]^{\mathrm{T}}$$

and the generalized matrix transfer function $\boldsymbol{\Phi}$, it is possible to represent the studied system in the following form

$$\mathbf{x}=\boldsymbol{\Phi}\mathbf{d}$$

or

$$\begin{bmatrix}\mathbf{z}\\ \mathbf{v}\end{bmatrix}=\begin{bmatrix}\boldsymbol{\Phi}_{11} & \boldsymbol{\Phi}_{12} & \boldsymbol{\Phi}_{13}\\ \boldsymbol{\Phi}_{21} & \boldsymbol{\Phi}_{22} & \boldsymbol{\Phi}_{23}\end{bmatrix}\begin{bmatrix}\mathbf{f}\\ \mathbf{g}\\ \varphi\end{bmatrix},$$

where the matrix $\boldsymbol{\Phi}$ components according to the expressions (11), (13) may be defined in the following way

$$\boldsymbol{\Phi}_{11}=\mathbf{W}_{11}(s)+\mathbf{W}_{12}(s)\mathbf{P}(s)[\mathbf{I}-\mathbf{W}_{22}\mathbf{P}(s)]^{-1}\mathbf{W}_{21}(s)\,;$$

$$\boldsymbol{\Phi}_{12}=\mathbf{W}_{12}(s)\mathbf{P}(s)+\mathbf{W}_{12}(s)\mathbf{P}(s)\times$$
$$\times[\mathbf{I}-\mathbf{W}_{22}\mathbf{P}(s)]^{-1}\mathbf{W}_{22}(s)\mathbf{P}(s);$$

$$\boldsymbol{\Phi}_{13}=\mathbf{W}_{12}(s)\mathbf{P}(s)+\mathbf{W}_{12}(s)\mathbf{P}(s)[\mathbf{I}-\mathbf{W}_{22}\mathbf{P}(s)]^{-1}\times$$
$$\times\mathbf{W}_{22}(s)\mathbf{P}(s);$$

$$\mathbf{\Phi}_{21} = [\mathbf{I} - \mathbf{W}_{22}\mathbf{P}(s)]^{-1}\mathbf{W}_{21}(s)\,;$$

$$\mathbf{\Phi}_{22} = [\mathbf{I} - \mathbf{W}_{22}\mathbf{P}(s)]^{-1}\mathbf{W}_{22}(s)\mathbf{P}(s)\,;$$

$$\mathbf{\Phi}_{23} = [\mathbf{I} - \mathbf{W}_{22}\mathbf{P}(s)]^{-1}\mathbf{W}_{22}(s)\mathbf{P}(s)\,.$$

The system is inherently stable, if all the components of its matrix transfer function are stable

$$\mathbf{\Phi}_{ij}(s) : i,j = \overline{1,3}\,,\ \mathbf{\Phi}(s) \in RH_{\infty}\,.$$

To calculate the quality indexes of the robust control systems the H_2-norm of the closed system transfer function is used. The H_{∞}-norm is a measure of robustness that is stability to both the external and parametrical disturbances.

In other words, the H_{∞}-norm is the effective characteristic of the system reaction to the external disturbances of the different kind in conditions of uncertainties in the system mathematical description. H_2-norm is the characteristic of the system sensitivity function. H_{∞}-norm is the characteristic of the complementary sensitivity function. The above stated relationship allows to achieve compromise between the quality and the robustness of the system. Therefore to synthesize the studied system it is convenient to choose the complex criterion which includes the H_2 and the H_{∞}-norms with the weighting coefficients change of which allows to achieve compromise between the quality and the robustness of the system. As the robustness is a measure of the system parametric uncertainty, it is convenient to use the H_{∞}-norms of the nominal and the parametrically disturbed models as components of this criterion. With respect to the H_2-norm the corresponding norms of the deterministic and stochastic systems it is necessary to use as components of this complex criterion. At that it is necessary to take into consideration disturbances, which are the most important and specific for the synthesized system. Then the complex criterion may be described by the expression [3]:

$$J = \lambda_2^{nom} H_2^{nom} + \lambda_{\infty}^{nom} H_{\infty}^{nom} + \lambda_2^{db} H_2^{db} + \\ + \sum_{i=1}^{n} \lambda_{2_i}^{par} H_{2_i}^{par} + \sum_{i=1}^{n} \lambda_{\infty_i}^{par} H_{\infty_i}^{par}\,,$$

where $\lambda_2^{nom}, \lambda_{\infty}^{nom}, \lambda_2^{db}, \lambda_{2_i}^{par}, \lambda_{\infty_i}^{par}$ are the weighting coefficients for the corresponding norms of the nominal, disturbed and n parametrically disturbed models of the system.

Taking into account the above stated considerations the formalized statement of the studied system synthesis problem becomes:

$$J^* = \arg \inf_{\mathbf{\Phi}_F \in PH_{\infty}, k \geq k_{per}} J(\mathbf{\Phi}_F(s, \mathbf{P}(s), \Delta_1(s), \Delta_2(s)))\,.$$

This approach to stabilization and course system synthesis allows to achieve compromise between such conblict objectives as accuracy and resistance to disturbances.

Conclusion

The formalized statement of optimal synthesis problem for the marine vehicle resistant to disturbances stabilization and course system is defined.

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