PHYSICO-MATHEMATICAL SCIENCES

Viktor A. Borodin, Candidate of Engineering, assoc. Prof.

A QUICK METHOD OF SEGMENT AND LINE DRAWING ON THE SCREEN BASED ON CONTINUOUS FRACTION ALGORITHM

In the paper a quick method for segment and line drawing on the screen is proposed. This method is proposed for using in interactive aviation complexes. The method is grounded on the algorithm of continuous fraction and uses half as many operations comparing to classical methods.

Запропоновано швидкий метод відображення на екрані прямої та відрізку для використання в інтерактивних авіаційних комплексах. Метод ґрунтується на алгоритмі неперервного дробу та використовує вдвічі меншу кількість операцій порівняно з класичними методами.

algorithm, continuous fraction, drawing on a screen, line, segment

Introduction

Many computer applications and interactive complexes, that require graphic programming to visualize moving objects and pictures (computer games, computer animation, geoinformation complexes etc) involve the subproblem of line or segment drawing on the screen. The amount of such lines (segments) can be great, therefore arises the question of finding the most optimal algorithm for the quickest execution of these drawings [1–4].

Particularly, for the realization of dynamic movement of raster image, the image can be represented as the union of the set of segments, each of them parallel to the horizontal axis. Thus, to turn this image at the angle φ we should perform the turn of all those segments at the same angle φ [3; 4]. This way we execute the visualization of the series of parallel segments with coefficient of turn equal to $\mathrm{tg}\,\varphi$ and some displacement on horizontal axis.

Previous research analysis

One of the most famous methods for solution of this problem is Bresanhame method [1] for lines and segments drawing and its modifications [4].

This method is based on the fact that line $y = ax + b$ is visualized on a screen as "footsteps", i.e. in dependence with the sign of the number $|a| - 1$ either each row or each column of a screen pixels consists of the segments of our line colour with the length of these segments resembling no more than 1 unit. Therefore to draw this line it is enough to calculate the lengths of these segments k_i with the help of one operation of floating point division and then either for each row y_i, or for each column x_i execute the cycle of painting points with coordinates $(x, y_i)..(x + k_i, y_i)$

(or $(x_i, y)..(x_i, y + k_i)$) over [2].

For this method of line (segment) drawing the main task is the task of calculating numbers k_i.

In the classic variant of Bresenhame algorithm the number of operations for its execution is proportional to the number of points in the given segment. Lets denote the number of points in the segment as l. If we use economical method [3], which for calculation of the coefficients of the turn uses not the sinus or cosine, but tangent or cotangent of the given angle, that gives a certain economy in numbers of operations. In this method we get the number of operations proportional to $l \min\{\mathrm{tg}\varphi, \mathrm{ctg}\varphi\}$, where these operations are operations of division and comparison, that are performed quicker than operations of division.

The method of image turn using segment drawing method

The particularity of the economic method of quick image turn is the keeping in already prepared table, that is stored in computer memory, the values of tangents for all possible angles of turn. There is a fixed resolution for the screen images and, thus, for all possible angles, that's why it is a finite set. The resolution of the screen is defined by equalities *dx1:=h1·*tg(φ)*; dy1:=h2·*ctg(φ)*.*

For the calculated values of x' and y' new values, that represent the image after drawing are calculated by the following algorithm. This algorithm is applied to the matrix of image $S[i, j]$ in raster format with sizes of matrix equal to $X \times Y$

x1:=x'-dx2; y1:=y'-dy2;

For i=0 to X do begin

j:=0; x:=x+dx1; y1:=y1+1;

if int{x}=int{x-dx1} then x1:=x1+1;

x2:=x1;y2:=y1;

S[i,j]:=S[x1,y1];

Fort j:=1 to Y do begin

x2:=x2+1; y:=y+dy1;

if int{y2}=int{y2-dy1} then y2:=y2+1;

S'[x2,y2]:=S[i,j];

end;

end;

The algorithm of Bresenhame is represented here in the inner cycle.

It is obvious that this method of the quick turn with the help of Bresenhame algorithm uses fewer operations for the reason of performing only one operation of sum for each point, while classic algorithms of turn use at least two operations [2; 4]. The other operations of addition in the body of the cycle are executed only for integer numbers, which are executed four times quicker then (if to count that real numbers are represented by 16 digits). Therefore, the time of execution should drop by 25 % comparing to the famous quick turn methods [3].

It can be also noticed that in the body of the cycle we can use any other modifications of Bresenhame algorithm, including those which use only one operation of real numbers division, and then only two operations of integer numbers addition. This way, the number of operations for image turn consists of $Y+19$ operations of real numbers division and $2X(Y+1)$ operations of integer numbers addition. It creates increase in the speed of turn comparing to the methods mentioned in literature.

The goal of the article

In the present paper the modification of economical algorithm is presented, that uses the representation of tangent of the line with the help of continuous fraction analogue. The algorithm of segment and line drawing based on this method uses half as many operations as the classical algorithms.

The proposed analogue of continuous fraction

Lets draw on the screen the part of the line with coefficient of turn equal to $\mathrm{tg}\varphi$.

Lets define $\alpha = \min\{\mathrm{tg}\varphi, \mathrm{ctg}\varphi\}$.

It is known that for any number α a series can be built that approximates this number as the sum of rational numbers the best possible (for approximation) way

$$\alpha = a_0 + \frac{t_1}{a_1} + \frac{t_2}{a_2} + \ldots + \frac{t_n}{a_n} + \ldots,$$

where

$t_i \in Z$, $a_i \in N$ for $i \in N$ and $a_0 \in Z$.

These best rational approximations are described by the sequence of fractions of a kind p_k/q_k, that are called proper and can be found from the number expansion with the help of continuous fraction [5; 6]. Therefore, we can write

$$t_n = p_n q_{n+1} - p_{n+1} q_n,$$

and $a_n = q_n q_{n+1}$.

According to the famous properties of continuous fractions [5,6], it can be easily deduced, that

$$t_n = (-1)^n t_1 \ (n \geq 2), \text{ and } a_n \geq 2^{2^n}.$$

Algorithm of this row creation can be described in the following way.

If $\{\alpha\} \leq 0.5$, then $a_0 = [\alpha]$, $t_1 = 1$, and the next elements can be found for the number $\alpha_1 = \{\alpha\}$.

If $\{\alpha\} > 0.5$, then $a_0 = [\alpha]+1$, $t_1 = -1$, and the next elements are found for the number $\alpha_1 = 1 - \{\alpha\}$.

From the definition of α it can be said that we always use the first variant, because $\min\{\mathrm{tg}\varphi, \mathrm{ctg}\varphi\} \leq 1/\sqrt{2} < 1/2$.

Then, the denominator of the fraction $\frac{t_i}{a_i}$ is defined by those of the numbers $([1/\alpha_i], [1/\alpha_i]+1)$, for which

$$\left|\alpha_i - 1/a_i\right| < \frac{1}{2a_i^2}.$$

In the first case $t_{i+1} = 1$, and in the second case $t_{i+1} = -1$. Then we should continue this algorithm for the number $\alpha_{i+1} = \left|\alpha_i - 1/a_i\right|$.

The proposed method description

Lets note that for the segment drawing on the screen it is required to find just the rational approximation of the segment, because only integer number of the pixels are shown on the screen. Moreover, the denominator of this fraction shouldn't exceed $l \min\{\mathrm{tg}\varphi, \mathrm{ctg}\varphi\}$.

Thus, the algorithm of segment drawing looks as follows. The approximation of the number $\alpha = \min\{\operatorname{tg}\varphi, \operatorname{ctg}\varphi\}$ is built while $a_n \le l\alpha$.

Lets define by n the greatest number that satisfies this condition. According to previous properties of our approximation, the number n is not less than $\log_2 \log_2(l\alpha)$).

Its not difficult to note that a_0 is equal to the most popular coefficient k_i.

Then, the values of k_i (deviation relive to a_0) are calculated by the following algorithm.

For $i = 1$ to n do

$s := 1$;

While $a_i s < l\alpha$ do $k[a_i s] = k[a_i s] + t_i$;

$s := s + 1$;

end;

end.

This algorithm uses $\sum_{i=1}^{n} l\alpha / a_i \le l\alpha / 2$ of operations of addition.

In such a way we can obtain the values of lengths of the segments k_i, that are representing our segment (line). Therefore, it can be possible to execute algorithm of segment drawing by painting the points with coordinates $(x, y_i)..(x + k_i, y_i)$ (or $(x_i, y)..(x_i, y + k_i)$) over, i.e. on each step for each row (column) the addition of row (column) is performed for each of the pixel.

Then the turn of the complex symbol can be executed as follows.

With the help of calculated values of k_i the turn of horizontal segment that intersects point O (the geometric centre of the image) of the image is performed. For this action we can use the same algorithm with the values k_i for the row. After that, each of the received pixels is chosen as the beginning of the vertical segment turned. Each of the new segments should be drawn as described above.

So, by including this algorithm in the algorithm of economic turn, we can speed up the execution of image turn. This method can be applied in air traffic control systems and other computer complexes that perform operations of turn, drawing of moving images so on.

Estimation of the method's speed

The number of operations which are required for this method is equal to $l\alpha/2$ operations of addition and $\log_2 \log_2(l\alpha)$ operations of division.

Therefore, the proposed algorithm uses half as many operations comparing to other methods. Moreover, instead of operations of division we can use operations of addition, that are performed more quickly on most computers.

Conclusion

In the article the method of quick and economic image turn is represented. The method is grounded on the representing image as a set of parallel segments.

For implementation of the turn the economic algorithm is proposed. For the drawing of segments after turn the method of segment and line drawing is described. This method is based on the analogue of continuous fractions approximation and uses half as many operations as the most optimal famous ones.

The described method is proposed to use in air traffic complexes and other computer systems that require graphic representations of images and lines.

References

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