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Inna M. Konchenko, student
Felix J. Yanovsky, D. E., Prof.

INFLUENCE OF MULTILATERATION SURVEILLANCE SYSTEM ARRANGEMENT ON THE TARGET LOCALIZATION

In this article accuracy of target localization in multilateration surveillance system is estimated with the help of two approaches. The first approach is based on using data processing algorithm for computing target position. The second one is done with the help of Cramer−Rao Lower Bound. Results of these two approaches at the same initial data are compared and analysis is done. The results are relevant for development and future implementation of prospective multilateration systems in Ukraine.

Оцінено точність локалізації цілі в мультилатераційній системі спостереження за допомогою двох підходів. Перший підхід базується на використанні алгоритму обробки даних для обчислення координат цілі. Другий підхід виконано за допомогою нерівності Крамера−Рао. Зроблено порівняння та аналіз підходів за однаково вихідних умовах. Результати є корисними для розробки та майбутнього впровадження мультилатераційних систем в Україні.

accuracy, Cramer−Rao Lower Bound, error ellipse, multilateration, Remote Unit, Time Difference of Arrival

Introduction

Multilateration (MLAT) is an attractive new surveillance technique, which is used for air traffic control and management. This system operates on the basis of on the triangulation principle. Set of ground stations receive emitted aircraft signal. Travel time of received target signal at each station is compared by the central processor and aircraft's position is derived. Each Time Difference of Arrival measurement result represents hyperbolic curve, along which the emitter may be located. The repetition of these measurements, and the accumulation of results on the same signal received on several pairs of stations, leads to the target position through the intersection of several hyperboloids. At least four receivers are needed to determine 3D position of the target. Additional stations can be added in order to improve accuracy of target localization either to overcome line-of-site restrictions or to increase the overall surveillance volume. The accuracy in MLAT system depends on the positional relationship of the Remote Units and a target [1]. There are different algorithms for emitter localization by means of Time of Arrival estimation proposed [2−4]. The most important factor influencing the accuracy of the system is correct receivers arrangement. In this work we consider two algorithms for target localization, described in [3; 4], in the context of optimization of the system components arrangement.

Analysis of the latest research and publications

The latest deployment and implementation plans of multilateration system in airports through over the world are described in works [5−7]. In works [4; 8] accuracy of object localization in multilateration system is analysed on the basis of Cramer-Rao inequality.

But the main complexity of multilateration system implementation is to find optimal location of Remote Units in order to provide required accuracy of target localization in controlled zone.

The aim of this work is to propose the solution of optimal sensors placement in the multilateration surveillance system.

Concept of data processing algorithm

Calculation algorithm [3] is based on the equation for defining distance between two points:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}\ .$$

In this paper we name this approach as hyperbolic method. Distance between emitter and receiver is calculated by measuring time, which is taken by emitted signal to reach the target. Multiplying this time by the speed of light c=3*10^8 m/s, we obtain distance d.

System of three equations is solved to define three coordinates (x, y and z) of the target (where location of receivers *1*, *2* and *3* is known):

$$ct_1 = R_1 = \sqrt{(x_1 - x)^2 + (y_1 - y)^2 + (z_1 - z)^2}\ ,$$

$$ct_2 = R_2 = \sqrt{(x_2 - x)^2 + (y_2 - y)^2 + (z_2 - z)^2}\ ,$$

$$ct_3 = R_3 = \sqrt{(x_3 - x)^2 + (y_3 - y)^2 + (z_3 - z)^2}\ ,$$

where

x_1, y_1, z_1, x_2, y_2, z_2,
x_3, y_3, z_3 are coordinates of 1st, 2nd and 3rd receivers correspondingly.

There is no simple solution of this system because of the presence of square root terms. The solution can be simplified by adding *4*th receiver for additional equation. So, the system of four equations, which indicates time difference of signal arrival (expressed by distances R_{12}, R_{13}, R_{32}, R_{34}), is solved:

$$R_1-R_2=R_{12}=\sqrt{(x_1-x)^2+(y_1-y)^2+(z_1-z)^2}-\sqrt{(x_2-x)^2+(y_2-y)^2+(z_2-z)^2},$$

$$R_1-R_3=R_{13}=\sqrt{(x_1-x)^2+(y_1-y)^2+(z_1-z)^2}-\sqrt{(x_3-x)^2+(y_3-y)^2+(z_3-z)^2},$$

$$R_3-R_2=R_{32}=\sqrt{(x_3-x)^2+(y_3-y)^2+(z_3-z)^2}-\sqrt{(x_2-x)^2+(y_2-y)^2+(z_2-z)^2},$$

$$R_3-R_4=R_{34}=\sqrt{(x_3-x)^2+(y_3-y)^2+(z_3-z)^2}-\sqrt{(x_4-x)^2+(y_4-y)^2+(z_4-z)^2}.$$

In accordance with this algorithm, we can estimate the coordinates of a target and then the accuracy of target localization, choosing four receivers with fixed and defined location. Distance between each pair of Remote Units is precisely known.

Cramer-Rao Lower Bound concept

The Cramer-Rao Lower Bound (CRLB) represents the lower-bound on the covariance matrix of the parameters of interest of any unbiased estimator when the wanted signal is corrupted by additive Gaussian noise. In our case the parameters of interest are: the unknown carrier frequency f_e and the emitter coordinates (x_e, y_e, z_e) [4]. So, we estimate the parameter vector $\mathbf{x}_e = [x_e, y_e, z_e, f_e]$. For the case of two receivers with (x_1, y_1, z_1, Vx_1, Vy_1, Vz_1) and (x_2, y_2, z_2, Vx_2, Vy_2, Vz_2) TDOA and Frequency Difference of Arrival are given by:

$$s_1(x_e, y_e, z_e) = \tau_{12} = \frac{R_1 - R_2}{c},$$

$$s_2(x_e, y_e, z_e, f_e) = V_{12} = \frac{f_e}{c}\frac{d}{dt}(R_1 - R_2),$$

where
c is the speed of light,
R_i is the range between receiver i and the emitter.
The CRLB matrix is obtained by taking the inverse of the Fisher Information Matrix:

$$C_{CRLB}(x_e)=[H^T C^{-1} H]^{-1},$$

where
H is the Jacobian matrix (in case of two pairs of receivers):

$$H = \frac{\partial s(x_e)}{\partial x_e} = \begin{bmatrix} \frac{\partial s_1(x_e)}{\partial x_e} & \frac{\partial s_1(x_e)}{\partial y_e} & \frac{\partial s_1(x_e)}{\partial z_e} & \frac{\partial s_1(x_e)}{\partial f_e} \\ \frac{\partial s_2(x_e)}{\partial x_e} & \frac{\partial s_2(x_e)}{\partial y_e} & \frac{\partial s_2(x_e)}{\partial z_e} & \frac{\partial s_2(x_e)}{\partial f_e} \\ \frac{\partial s_3(x_e)}{\partial x_e} & \frac{\partial s_3(x_e)}{\partial y_e} & \frac{\partial s_3(x_e)}{\partial z_e} & \frac{\partial s_3(x_e)}{\partial f_e} \\ \frac{\partial s_4(x_e)}{\partial x_e} & \frac{\partial s_4(x_e)}{\partial y_e} & \frac{\partial s_4(x_e)}{\partial z_e} & \frac{\partial s_4(x_e)}{\partial f_e} \end{bmatrix};$$

C is the covariance matrix of zero-mean Gaussian vector:

$$C = \begin{bmatrix} C_1 & 0 & 0 & 0 \\ 0 & C_2 & 0 & 0 \\ 0 & 0 & C_3 & 0 \\ 0 & 0 & 0 & C_4 \end{bmatrix}.$$

Error ellipse (in two-dimensional case), derived from CRLB variance-covariance matrix is the smallest error ellipse that an unbiased estimator can achieve. As this ellipse is a statistical measure of the positional error at a given point we use its square as an objective function. In three-dimensional case error ellipsoid is derived from CRLB matrix, the shape of which is confidence region (three-dimensional space that contains a certain percentage of the total probability distribution) and exact for a Gaussian probability density function. The size of such ellipsoid indicates the relative magnitude of the error.

Results of simulation

All of simulations were performed in MATLAB. We considered system of four Remote Units and target, distributed on the area of size 6000×10000 m (fig. 1). Initial target coordinates are:

x = 3232,5 m, y = 646,5 m, z = 4 m.

We changed x-coordinate of the target from 3232,5 m to 9697,5 m. Such change in x-coordinate corresponds to situation when target is moved along the runway, for example. Fig. 2 shows dependence of absolute error of target coordinates measurement (by using algorithm, described in [3]) on changing x-coordinate of the target. Changes are also observed in error ellipse size and orientation.

Fig. 3 show the projections of error ellipsoids (subject to changes in target x-coordinate) on X-Y, X-Z and Y-Z planes correspondingly.

As it is seen from fig.2, fig.3 general pictures of two approaches differ. There is a decreasing character in changing square of error ellipses. At the same time from fig.2 we can see increasing of absolute error in measuring target coordinates. But from all of these pictures (fig.2, fig.3, a, b) it is seen that about a point x≈5000 m error of measuring target coordinate is minimum.

Now let us consider how the changes in Remote Unit placement influence the accuracy in defining target location. We changed y-coordinate of RU#1 from 1293 m to 3555.75 m (fig. 4).

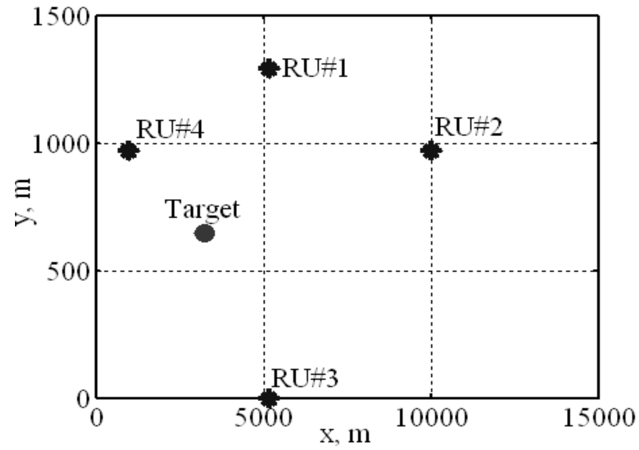
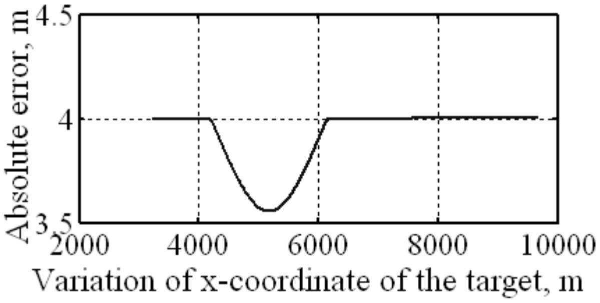
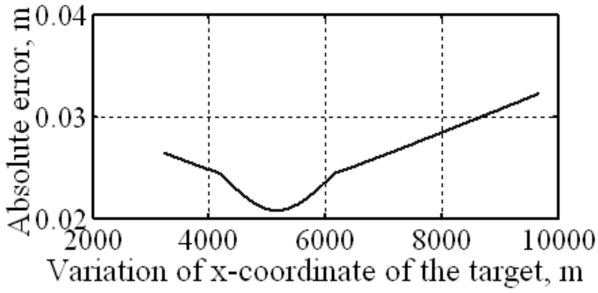


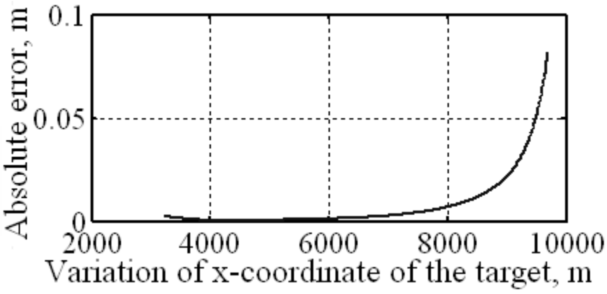
Fig. 1. Initial location of Remote Units and target



a

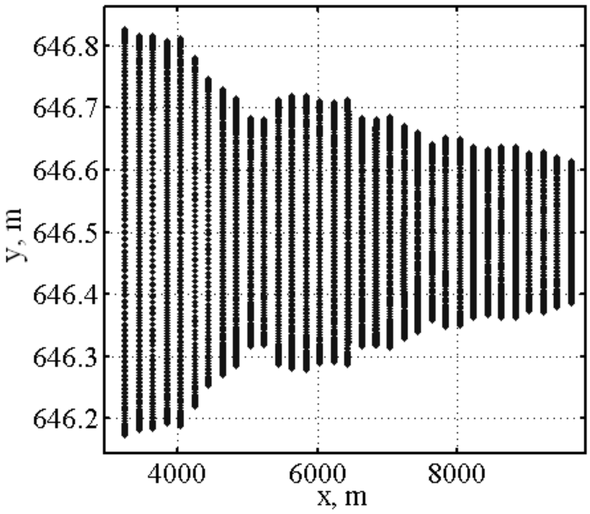


b

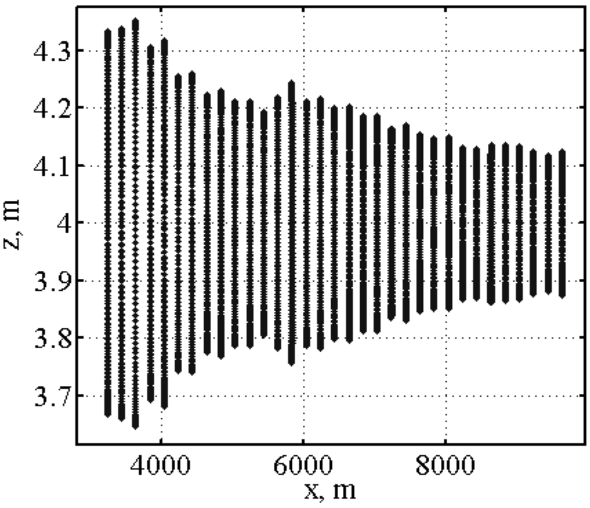


c

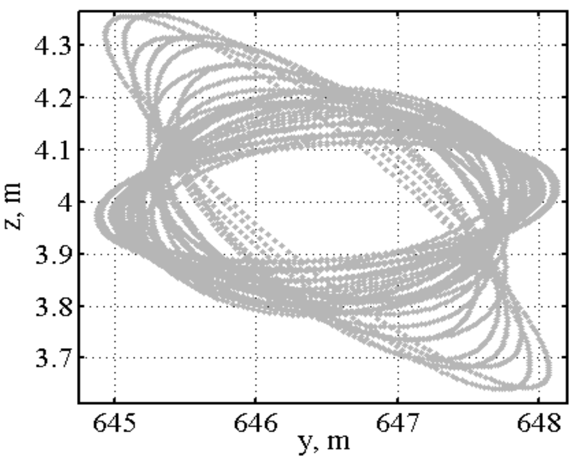
Fig. 2. Dependence of absolute error in measuring target's *z*-coordinate (*a*), *y*-coordinate (*b*) and *x*-coordinate (*c*) on changing target location along *x*-axis



a



b



c

Fig. 3. Projection of error ellipsoids on X-Y plane (*a*), X-Z plane (*b*), Y-Z plane (*c*)

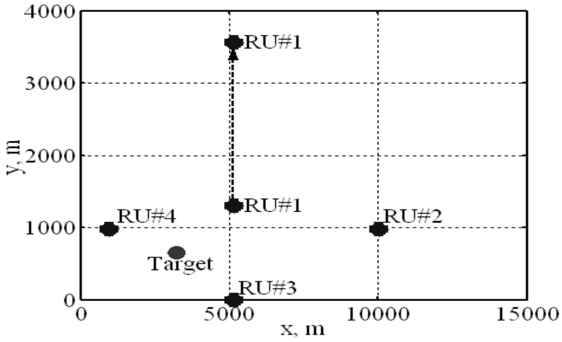


Fig. 4. Location of Remote Units and target (changing *y*-coordinate of RU#1)

Dependence of error ellipse size, orientation (Cramer-Rao Lower Bound approach) and accuracy of target localization (algorithm [3]) on changing *y*-coordinate of RU#1 is shown in fig. 5. Fig. 6 (zoomed picture of fig. 5) represents measured target coordinates, when we moved RU#1 along *y*-axis. Triangle on fig. 6 represents measurement of target location at initial placement of RU#1, and the square corresponds to the computing target coordinates when *y*-coordinate of RU#1 is equal to 3555,75 m. So, it is seen that moving RU#1 along *y*-axis (increasing value of *y*-coordinate) leads to more accurate target localization.

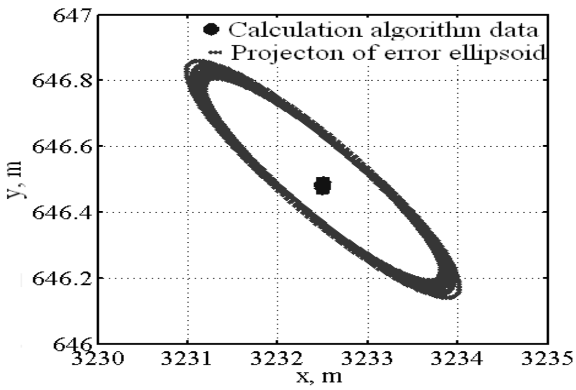


Fig. 5. Dependence of measurement *x* and *y* target's coordinates and error ellipse (X-Y plane) size and orientation on changing y-coordinate of RU#1

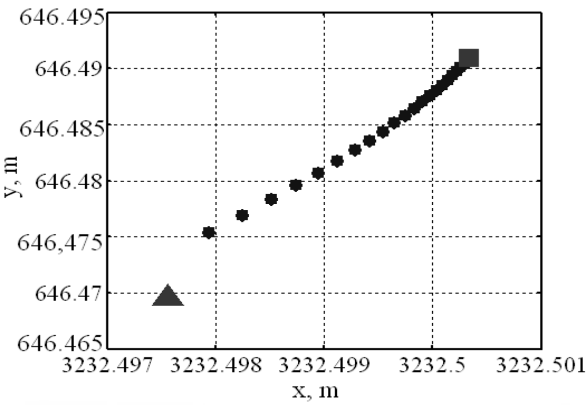


Fig. 6. Dependence of measurement *x* and *y* target's coordinates on changing *y*-coordinate of RU#1

Conclusion

Obtained results allow us to predict accuracy of target localization. The task was solved by two approaches:

− based on simple hyperbolic algorithm;

− based on Cramer-Rao inequality. In the first case, the accuracy has been estimated depending on data processing algorithm, and in the second case it has been estimated regardless of it.

Data of two algorithms may be combined in order to predict "the final" accuracy in target localization. Cramer-Rao Lower Bound makes it possible to achieve potential accuracy of emitter localization, taking into account only possible Gaussian noises. So, data derived from CRLB and different calculation algorithms may be combined to predict general level of accuracy. Value of error ellipse can be used as an objective function in order to find optimal system configuration in defined surveillance area.

Our further work will be directed at finding optimal configuration of multilateration surveillance system and number of sensors with regard to required accuracy of emitter localization in defined control area.

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