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ALGORITHM FOR GROUND VEHICLE STABILIZER OPTIMAL SYNTHESIS

Approaches to creation of the ground vehicle stabilizer optimal synthesis are considered. Features of the computational procedure of the optimal synthesis for the studied system are defined. Simulation results are represented.

Розглянуто підходи до створення алгоритму оптимального синтезу стабілізатора наземного призначення. Визначено особливості обчислювальної процедури оптимального синтезу системи досліджуваного класу. Надано результати моделювання синтезованої системи.

ground vehicle, parametric robust optimization, robust control, stabilizer

Introduction

The classic approach to the synthesis of the optimum multiple-input–multiple-output (MIMO) control systems uses optimization methods on the basis of calculus of variations [1]. It is known, that application of optimum principles for designing of the MIMO systems of high order is characterized by the considerable difficulties in calculations. Other approach requires assigning in advance the dynamic properties of the closed-loop system taking into consideration limitations of the design parameters [2].
As a rule the complex multi-loop control systems have both global and local extremums, appearance of which is conditioned by limitations in space of the design parameters. Thus the necessity to repeat the optimization procedure may arise depending on the optimization results. Obviously, that it is necessary to use the simulation model which most full reproduces functioning of the real system for optimization results checking and obtaining.
The important problem of the modern control system designing is to create the high-accuracy system that guarantees acceptable performances in the presence of the parametric uncertainty and external disturbances. Many classic approaches of the synthesis of the control systems in general and stabilizers in particular are based on a fixed plant model with the given parameters. To obtain realistic results it is desirable to carry out synthesis of the system taking into account the possibility of these parameters change in some range. System is considered to be robust, if it is characterized by the sufficient level of stability and performances for some range of the plant variable parameters and external disturbances.

Analysis of researches and publications

Today a variety of ideas were developed in the area of the robust control. Basic approaches of the robust control are represented in [3]. A great amount of studies is devoted to creation of the robust aircraft control systems [4; 5]. Features of the parametric robust optimization for ground vehicle stabilizer are represented in the paper [6]. Some problems of the parametric robust optimization for ground vehicle stabilizer with the discrete regulator are described in the paper [7].
This paper is devoted to generalization of the approaches to creation of algorithms for the robust ground vehicle stabilizers optimal synthesis.

Algorithm for the ground vehicle stabilizer optimal synthesis

The ground vehicle stabilizers must be stable both to the external disturbances and parameter's changes in the sufficiently wide range. The synthesis of the stable to disturbances stabilizer includes the following stages:
− setting of the optimal synthesis problem;
− development of the complete mathematical description of the stabilizer taking into consideration all nonlinearities peculiar to the real systems;
− linearization of the obtained model and its representation in the state space;
− analysis of the system and obtaining of the minimal and balanced realizations in case of necessity;
− analysis of requirements given to the system and forming the corresponding objective and penalty functions;
− development of method for simulation of external disturbances based on features of motion of the ground vehicle, on which the studied stabilizer will be mounted;
− optimization method choice;

− development of algorithm for robust system synthesis using modern automated facilities;
− simulation and analysis of the obtained results;
− reiteration of the computing procedure depending on the optimization analysis results.

Synthesis of the stable to disturbance that is robust system may be based on minimization of the H_∞-norm of the matrix transfer function of the closed-loop system. There is also known approach to the system synthesis, which is based on minimization of the H_2-norm of the matrix transfer function of the closed-loop system. It is known that the H_2-norm characterizes accuracy of the robust system. From the point of view of computational algorithms development, H_∞-optimization is considerably more complicated than H_2-optimization because of necessity to use the search procedure.

Synthesis methods based on the H_2-norm minimization provide the high accuracy of the synthesized system, but the system remains sensible both to the external disturbances and parametric uncertainty. The H_∞-norm minimization allows providing the system's resistance to the external disturbances under condition of structural and non-structural parametric uncertainty.

Every of considered approaches of optimization have its advantages. But optimization by the mixed criterion allows combining these advantages. In this case the synthesized system will be characterized by the necessary accuracy in conditions of real exploitation taking into consideration disturbances influence. Features of H_2, H_∞ and mixed H_2/H_∞ optimization are represented in the paper [8].

Robust parametric and structural-parametric synthesis requires creation of the appropriate software package. Features of this package programs are defined by the aim of the synthesis stage. To achieve the research goal it is necessary to use the linearized state space model for optimization and the complete nonlinear model for its results checking and analysis. Both type models must have the basic properties of the system to be studied.

Creating the stabilizer model it is necessary to take into consideration that the plant in this system is in the elastic combination with the engine [9]. The stabilizer to be studied consists of the control unit, which carries out functions of signals processing and control laws forming, the pulse-width modulator, the voltage amplifier. Usually the control unit includes the high and low frequencies filters and the bandwidth filter. All these units include nonlinear elements and the pulse-width modulator is the essentially nonlinear unit. The main feature of stabilizer control laws is application of the so called complex control, when together with the control by error is used the control by disturbance [10]. To provide such complex control in this system the signals proportional to the current and voltage of the engine rotor circuit are used. As measuring device in the researched stabilizer the rate gyro may be used. The software package may be developed by means of the system MatLab which is considered to be one of the best tools for control systems simulation. Taking into consideration features of the stabilizer separate units it is possible to create the stabilizer model using Simulink toolbox for creation of the complete model and Control System toolbox for creation of the linearized model in the state space. It worth noting, that the model of the stabilizer to be studied consists of the vertical and horizontal channels, which are fully independent.

The complete mathematical description of the stabilizer is represented in the paper [9]. Such model was developed taking into consideration friction, unbalanced, inertia torques and presence of the elastic balanced system widely used in the studied stabilizers. Such approach provides maximum possible correspondence between the model and the real system. To achieve such correspondence the rigidity and gap of the driving gear were taking into consideration.

To create the optimal synthesis procedures it is possible to use the special toolboxes, for example, Control System and Robust Control toolboxes, which include functions assigned for automation of the necessary for the optimal synthesis calculations. One of such toolboxes advantages is provision of the possibility to design the digital optimal regulators. This provides solution one of the most important problems in the modern device building taking into consideration the drastic progress of the modern computing facilities.

To calculate quality indexes of the robust stabilizer the H_2-norm transfer function of the closed-loop system may be used. As a measure of robustness that is the system's resistance both to external disturbances and parametric uncertainty the H_∞-norm may be used. In other words, the H_∞-norm may characterize system reaction by the external disturbances under condition of uncertainty in the mathematical description of the system. In the robustness theory the H_2 - norm is considered to be characteristic of the sensitiveness function $S(s)$. The H_∞-norm is characteristic of the complementary function of sensitiveness $T(s)$. These functions are correlated by the expression $S(s)+T(s)=1$ [3], which allows achieving the compromise between accuracy and robustness of the system to be synthesized. That is why for the synthesis of the studied stabilizer it is expedient to use a mixed criterion, which includes the H_2 and H_∞-norms with some weighting coefficients, by change of which it is possible to achieve compromise between accuracy and robustness of the system. So far as robustness is measure of parametric uncertainty of the system, it is necessary to include the H_2 and H_∞-norms of nominal and parametrically disturbed models to this criterion. As to the H_2-norm, the mixed criterion of optimal synthesis procedure should take into account the corresponding norms of both deterministic and stochastic systems. Such approach requires using the so called expanded model, which includes the forming filters that allows taking into consideration influence of the external disturbances differing from the white noise. Then for the researched stabilizer the mixed criterion can be describing by the expression

$$J=\lambda_2^{nom} H_2^{nom} + \lambda_\infty^{nom} H_\infty^{nom} + \lambda_2^{dist} H_2^{dist} + \sum_{i=1}^{n} \lambda_{2_i}^{par} H_{2_i}^{par} + \sum_{i=1}^{n} \lambda_{\infty_i}^{par} H_{\infty_i}^{par}$$

where

$\lambda_2^{nom}, \lambda_\infty^{nom}, \lambda_2^{dist}, \lambda_{2_i}^{par}, \lambda_{\infty_i}^{par}$ are the weighting coefficients of the above mentioned norms for nominal, disturbed and *n*-parametrically disturbed models.

The optimal synthesis problem setting is represented in the fig. 1.

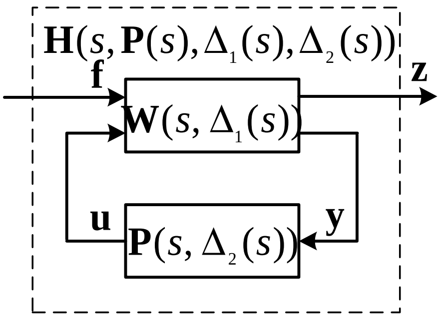

Fig. 1. The optimal synthesis problem setting

In the fig.1 the plant transfer function $\mathbf{W}(s,\Delta_1(s))$, the regulator transfer function $\mathbf{P}(s,\Delta_2(s))$ and the transfer function of the closed-loop system $H(s,\mathbf{P}(s),\Delta_1(s),\Delta_2(s)$ are shown, here Δ_1,Δ_2 represent the plant and regulator uncertainties.

In accordance with the fig. 1 the formalized optimal synthesis setting in the mathematical form becomes

$$J(\mathbf{H}(s,\mathbf{P}(s),\Delta_1(s),\Delta_2(s))) \to \inf_{\mathbf{P}(s)\in\Omega},$$

where

Ω is the set of the transfer functions whose elements are proper fractional rational functions for which the characteristic polynomial satisfies the Hurwitz criterion.

The synthesis methods for the stochastic stabilizer need consideration of specific external disturbances that act on it during exploitation. For the ground stabilizers the basic external disturbances are predetermined by roughness of road. There are deterministic and statistical approaches of consideration of disturbances that act on the ground vehicle on which the stabilizers are mounted [11]. At the first approach the road cutting is considered to be described by the harmonic function. At the second approach the disturbances, conditioned by roughness of road, are considered to be random variables. The consideration of random disturbances allows creating procedures of the stochastic system synthesis for the studied stabilizers. The detailed research of disturbed motion features for ground vehicles is represented in the paper [11], but it is necessary to analyze this information and choose characteristics of disturbances typical for exploitation of the studied stabilizers.

For the ground vehicle the external disturbances are defined by road roughness which depends on the road relief. To simulate such disturbances it is necessary to use the so-called road cutting that is crossing of relief in direction of the ground vehicle motion.

In many cases it is more convenient to use the so called micro-cutting for disturbances simulation. It differs from cutting due to absence of the lowest frequencies. The main advantage of such approach is the possibility to consider micro-cutting as the time-invariant random function that provides its use in problems of the optimal synthesis and analysis for the ground vehicle stabilizer. The transformation from cutting $h(t)$ to the micro-cutting $q(t)$ may be carried out by means of fractional-rational transfer function H_q. As characteristics of cutting and micro-cutting are random variables, more preferably to use such their characteristics as the spectral density. Analytical expressions of the spectral densities for the different type's roads and countries are represented in the paper [11].

Analytical representation of disturbance for the ground stabilizer looks like [11]

$$K_q(\omega) = |H_c(\omega)|^2 |H_q(\omega)|^2 K_h(\omega)$$

for micro-cutting and

$$K_q(\omega) = |H_c(\omega)|^2 K_h(\omega)$$

for cutting, where $H_c(\omega)$ is the transfer function of averaging by the contact area; $H_q(\omega)$ is the transfer function corresponding to transformation from the cutting to the micro-cutting; $K_h(\omega)$ is the spectral density of the road cutting.

To create the optimal synthesis algorithm for the ground vehicle stabilizer it is necessary to choose an optimization method, to determine limitations to the design parameters and the objective function (optimization criterion), as the solution of optimization problem lies in minimization of the objective function as function of design parameters. So, to solve the optimal control problem is important to choose an effective optimization method. Hence, it is necessary to take into account possibility of its practical application for the calculation procedure of the optimal synthesis. That is why preferably to consider optimization methods having corresponding software, for example, in the system MatLab, which nowadays is one of the most widespread tools for creation of the modern control and stabilization systems.

The most widespread methods of optimization which have software implementation in MatLab are such search methods as the golden section method, the method of the quadratic approximation, the Nelder-Mead method, the fast descent method, the Newton method, the method of the gradient transforms, the method of the "model tempering", the genetic algorithm. Analysis of the known optimization methods enables to choose a method most suitable for calculation procedure for the ground stabilizer optimal synthesis. It is worth noting that the great amount of optimization methods has built-in MatLab functions that makes easier their use in calculation procedures and improves their application importance.

For the optimization problems with the limitations in the space of the design parameters the method of penalty functions is widely used. This method is implemented in two stages. At the first stage the new objective function is formed. This function must include the components (penalties) which are formed after analysis of the special conditions connected with the optimized system properties. If such condition is carried out, the penalty is equal to zero. If the condition is not carried out – the penalty becomes a great number.

At the second stage the new objective function is minimized by means of the chosen optimization method, which may be used for the solution of optimization problem without limitations. The gradient optimization methods in this situation can not be used. The Nelder-Mead method is the most preferable for this problem solution.

Procedures of the parametric and structural-parametric optimal synthesis for the aircraft robust control systems on the basis of the mixed H_2/H_∞ Nelder-Mead are represented in the papers [4; 5]. The development of the corresponding methods for the ground stabilizers remains the actual problem.

The synthesis of the MIMO system requires many calculations and matrix transformations. It is convenient to use for optimal synthesis calculation procedures the Control System and Robust Control toolboxes, which include the large set of procedures that may be useful for the optimal synthesis of the control and stabilization systems.

Such toolboxes allow designing digital optimal regulators for the continuous system that is one of main tasks for the ground stabilizers development. The advantage of such toolboxes is the possibility of robust system creation that is actual task for the synthesis of the systems to be studied.

Control System toolbox may be applied for designing, analysis and modeling of the control systems in general and the ground stabilizers in particular. This toolbox uses both traditional methods on the basis of transfer functions, and the modern theory control methods based on representation of the models in the state space. This toolbox may be used for calculation procedure for optimal synthesis of both continuous and discrete systems. It allows to create optimal feed-back in the control system, has developed resources for LQG-synthesis, that do it the powerful tool for the structural synthesis of the control systems. The Control System toolbox includes a large quantity of built-in functions for analysis and synthesis of the control systems. In addition, it has adjustable programming environment, that allows to use original algorithms of own development. Among the traditional methods it is possible to mention the possibility of the gain factor and damping constant determination and analysis of the singular points.

Direct estimates of the stabilization transients may be obtained on the basis of the transient characteristics. The overshoot and its time are the most important characteristics for the ground stabilizers. The overshoot characterizes a difference between the maximal value of transient and its stable value. The overshoot time allows estimating the transient duration. The quality of stabilization depends on distribution of the system zeros and poles. Pole location is determined by parameters η, ξ and $\mu = \mathrm{tg}\psi$. [10] The parameter η represents a distance from the imaginary axis to the nearest root. The parameter ξ characterizes a distance from the imaginary axis to the outermost root. The parameter μ characterizes variability. The parameters η, ξ, μ may be used for the penalty function forming in the calculation procedures for the ground stabilizer optimal synthesis.

Results of the optimal synthesis for the ground stabilizer are represented in the fig. 2–4.

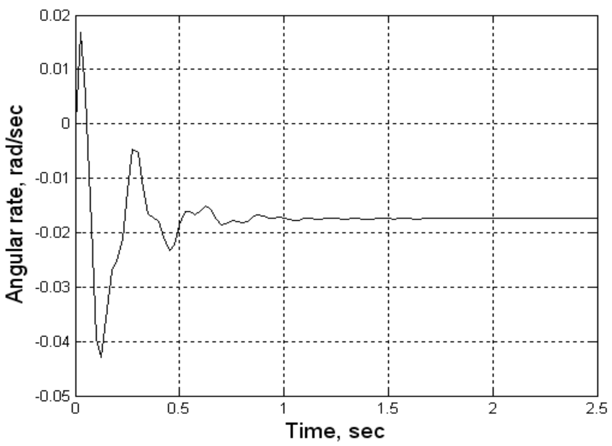

Fig. 2. The ground stabilizer transient

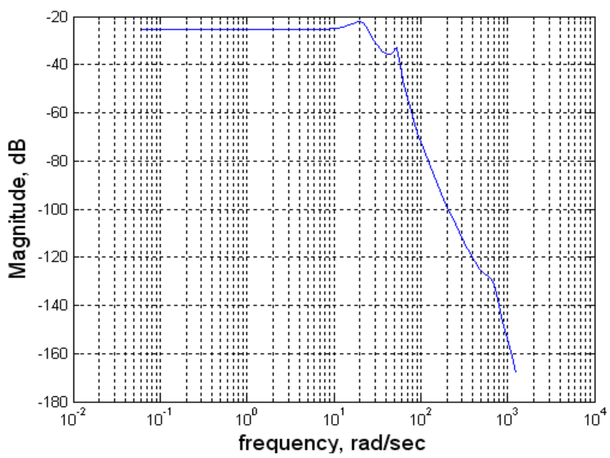

Fig. 3. The logarithmic gain characteristic of the open-loop system

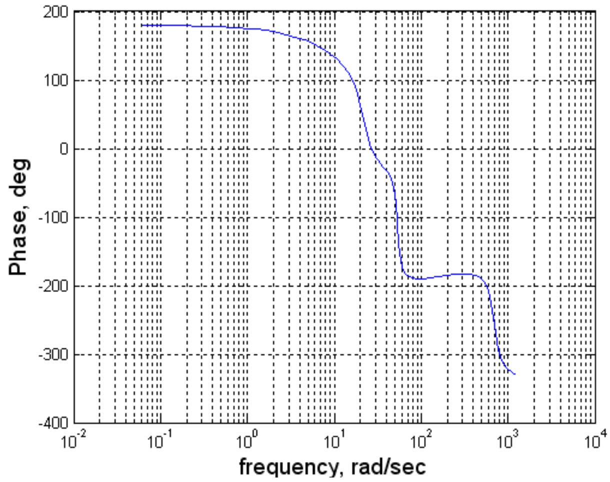

Fig. 4. The logarithmic frequency characteristic of the open-loop system

So, the algorithm for the ground stabilizer parametric optimization consists of the following stages.

1. Development of the mathematical model of the joint “plant – engine” system as single device united by the elastic connection taking into account rigidity of the reducing gear and gap of the driving gear.
2. Choice of the regulator structure for the ground stabilizer on the basis of experimental researches and theoretical approaches to designing of such type systems.
3. Development of the stabilizer mathematical model taking into account the joint “plant-engine” model, measuring device, regulator and necessary additional devices (the voltage amplifier, the pulse-width modulator and other) and all nonlinearities, inherent to the real systems (the signal limitations, the hysteresis, the dead zone).
4. Linearization and development of the mathematical model in the state space.
5. Obtaining of the open-loop and closed-loop transfer functions.
6. Determination of the minimum and balanced realizations of the model.
7. Scaling of the model on the basis of balanced realization algorithm.
8. Setting of initial values and execution of the Nelder-Mead optimization algorithm, with the cyclic execution of such steps:

– calculation of H_2 and H_∞- norms for the synthesized system;
– calculation of poles, analysis of their location on the complex plane and determination of the corresponding penalty function;
– determination of the new objective function taking into consideration the penalty function.

9. Analysis of the synthesized system, including such steps:
– calculation of the H_2, H_∞ -norms and obtaining of logarithmic gain-frequency characteristic with magnitude and phase margins;
– analysis of transients using the complete model with all nonlinearities;
– checking of system rigidity by moment using the complete model with all nonlinearities.
10. Conclusion about completion of parametric optimization or its reiteration with the new initial conditions or the new weighting coefficients.

The synthesized system has norms $H_2 = 0{,}3526$, $H_\infty = 0{,}122$. The obtained norms correspond to high level of accuracy and robustness.

Conclusion

The algorithm of optimal synthesis for robust ground stabilizer is created. The features of the computational procedure for the ground stabilizer parametric optimization are considered.

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