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**Volodymyr T. Chemerys**, Candidate of Engineering, assoc. Prof.
**Iren O. Borodiy**, asst.

## DIFFUSION OF THE PULSED ELECTROMAGNETIC FIELD INTO THE MULTI-LAYER CORE OF INDUCTOR AT PULSED DEVICES

*The problem of the pulsed magnetic field distribution in the cross section of the inductor core at the induction accelerator of electron beam is under consideration in this paper. Owing to multi-layer structure of the core package it has the magnetic and electric anisotropy with different speed of the field diffusion along the sheets of magnetic and across the sheets. At the pulse duration less than one microsecond the essential non-uniformity of the field along both axes of the core cross section can be found. This effect reduces the efficiency of the ferromagnetic material using with corresponding loss of the accelerator efficiency. The main conclusion of the paper consists of the necessity to check the field diffusion characteristics in the process of inductor design to be sure that the pulsed field is able to fill the cross section of the core during the pulse switching. The magnetic characteristics of the anisotropic core have been investigated in the paper by one-dimensional and two-dimensional simulation in the quasi-stationary approximation using the traditional equation of the field diffusion.*

*Розглянуто питання про розподіл імпульсного магнітного поля в перерізі осердя індуктора індукційного прискорювача електронних пучків. Наслідком багатошарової структури шихтованого пакета осердя є магнітна та електрична анізотропія з різними значеннями швидкості проникнення поля вздовж листів феромагнетику та поперек листів. Коли тривалість імпульсу менша за одну мікросекунду, може виявитися істотна неоднорідність поля по осях перерізу осердя. Цей ефект здатний применшити ефективність використання феромагнітного матеріалу осердя та знизити ККД прискорювача. Під час конструювання індуктора необхідно досліджувати характеристики дифузії поля, щоб впевнитися в тому, що імпульсне поле здатне заповнити переріз осердя за час тривалості імпульсу. Магнітні характеристики анізотропного осердя досліджувалися в роботі шляхом одновимірного та двовимірного моделювання у квазістаціонарному наближенні з використанням традиційного рівняння дифузії поля.*

### Introduction

Technology of the power electronic beams acceleration needs a generation of high electric fields. In the induction accelerators these fields exist owing to the super-high re-magnetization of the inductor’s core at the rate of the filed change near several Tesla per microsecond. The core workability can be provided at this condition due to a strip structure applied at the core manufacturing [1]. The amorphous magnetic material as the Metglas (Honeywell company) is very usable for this purpose in the form of the thin layer of ferromagnetic (12…22 μm) on the polymer base of 4…6 μm thickness [2]. Resulting package of core presents the multi-layer structure with interlacing of magnetic and non-magnetic layers. Its magnetic and electric anisotropy makes a need to check the field diffusion speed into the package to be sure that the field is able to fill the cross section of core during the time of pulse. It can be done using the mathematic simulation under condition that the equivalent electromagnetic parameters of the core medium can be defined previously. The procedure of the equivalent parameters finding is the special problem in this area and demands a detail consideration, but in this paper only simplest way to define the equivalent parameters has been used with a main attention devoted to the analysis of the field diffusion along the orthogonal axes of the core cross section when it presents a rectangle.

### Approach to the mathematic simulation of the electromagnetic process in the core

*Inductor of Accelerator as the Object under Consideration.* The inductor of accelerator presents the ring core with several parallel sections of the primary winding. The electronic beam plays the role of the secondary winding. The schematic design of inductor is shown in the fig. 1.

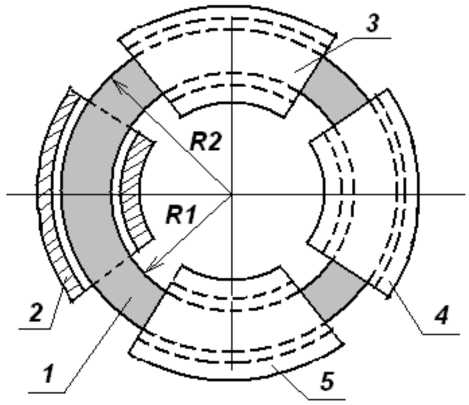



Fig. 1. Schematic design of the inductor:
*1* – core package;
*2–5* – sections of primary winding

The sectioning of primary winding is useful to provide more uniform distribution of the current density in the winding conductor and to improve the possibility of heat extraction from the core. The heat energy losses in the core depend on the re-magnetization rate and they have been investigated in the paper [3]. That is clear that all equivalent electromagnetic parameters of the sheet core medium are different along the line *A-A* (fig. 2) which is going in the radial direction (fig. 1) and along the line *B-B* (fig. 2) which is going parallel to the axis of symmetry in the fig. 1.

The strict definition of the equivalent parameters must be done taking into attention the capacitive currents in the layers of electric insulation between the ferromagnetic layers and their shunt influence on the induced currents in the ferromagnetic sheets [4]. At the relatively low frequencies the magnetic permeability of sheet core can be defined via the surface effect of each sheet and via filling coefficient of core with no influence of sheets for each other. At the high frequencies and at the small thickness of insulation sheets the core must be considered as the complex body with a complex magnetic permeability. Meanwhile for the microsecond range of the pulse duration we believe as possible to consider the equivalent parameters of the core medium neglecting the displacement currents in the insulator layers.

*The Field Penetration Equations into the Medium of Core.* The most full analysis of the field penetration into the medium of core considered as the anisotropic solid body can be realized on the base of the induction equation which takes into account not only displacement currents but also the possible vibration of sheets, when their velocity $V \neq 0$:

$$\Delta\vec{H} = \frac{\partial}{\partial t}(\mu\sigma\vec{H}) + rot\left[(\mu\sigma)\vec{V}\times\vec{H}\right] + \frac{\partial^2}{\partial t^2}\left(\mu\varepsilon\vec{H}\right).$$

The further analysis will be limited only by consideration of the classic diffusion according to the cut equation

$$\Delta\vec{B} = \frac{\partial}{\partial t}\left(\mu\sigma\vec{B}\right) \qquad (1)$$

neglecting the wave processes and assuming $V = 0$. With respect to the wave processes they can be analyzed in the first approximation in the core as in the ideal magneto-dielectric, what is the topic of separate investigation [5]. Equation (1) is correct for quasi-stationary processes and is used often for the analysis of the pulsed fields in the technical systems [6].

*The Components of Diffusion Coefficient for the Field Diffusion into the Cross-Section of Core.* The first step of preparing to simulation of the field diffusion into the anisotropic medium implies to establish correspondence between directions of penetration and components of electromagnetic parameters. It is possible to fulfill following to the electromagnetic field equations:

$$rot\vec{H} = \vec{j},$$

$$\Delta\vec{B} = \mu\sigma\frac{\partial\vec{B}}{\partial t}.$$

The plane picture of the field implies
$\vec{H} = (0, H_y, 0)$,
then induced currents are

$$j_x = \sigma_x E_x = \frac{\partial H_y}{\partial z},$$

$$j_z = \sigma_z E_z = -\frac{\partial H_y}{\partial x}$$

and in the equation of diffusion we must use the next components:
for diffusion along the $x$-axis

$$\frac{\partial^2 B_y}{\partial x^2} = \mu_y\sigma_z\frac{\partial B_y}{\partial t}$$

$$\frac{\partial^2 B_y}{\partial z^2} = \mu_y\sigma_x\frac{\partial B_y}{\partial t}.$$

So, for the both direction of the field diffusion (lines *A-A*, *B-B* in the fig. 2 corresponds to axes $x$, $z$, respectively) the only components of equivalent parameters $\mu_y$, $\sigma_x$ and $\sigma_z$ are of interest.

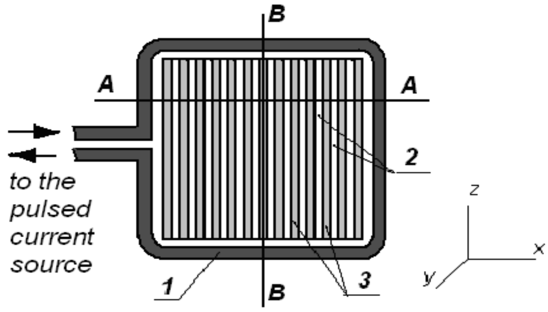
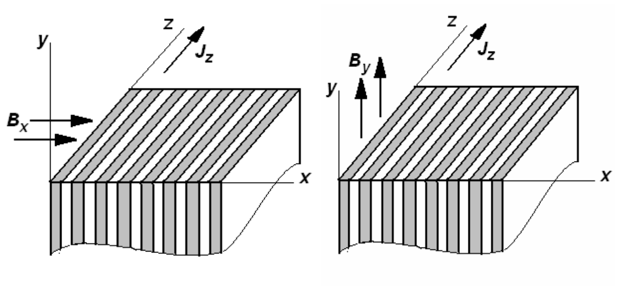



Fig. 2. The cross section of inductor with primary winding:
*1* – conductor of the primary winding;
*2* – layers of the insulator strip;
*3*– layers of ferromagnetics

From the fig. 3 it is possible to see that $\mu_z = \mu_y$ .




*a* *b*

Fig. 3. Orientation of the induced currents for the penetration of the $B_x$ and $B_y$ components of the field into the lateral side of core (scheme *a* or *b* respectively) and for diffusion along the *z*-axis

*Definition of the Equivalent Parameters of Medium by the Equivalent Flows Method.* The equality of the magnetic flux and the current density flow has been used as the conditions for the electromagnetic parameters calculation for solid anisotropic medium which can be equivalent to the multi-layer medium. For the equivalent magnetic permeability along axes *x* or *z* the next expression can be used:

$$\mu_z = \mu_y = k_f \, \mu_f ,$$

where coefficient of the package filling $k_f$ contains the thickness of the ferromagnetic layer $a_f$ and thickness of the insulator layer $a_{ins}$:

$$k_f = \frac{a_f}{a_f + a_{ins}} = \left(1 + \frac{a_{ins}}{a_f}\right)^{-1} .$$

For the equivalent magnetic permeability relatively the flux passage along the *x*-axis the next expression is true:

$$\frac{\mu_x}{\mu_0} \approx \frac{1}{1 - k_f} .$$

This magnitude does not serve for our calculations directly but it must be entered to the Quick Field software as needed characteristics of medium band in direction transversal with respect to chosen for calculation.

The expression for the components of equivalent electrical conductivity includes the same geometric characteristics of layers ($k_f$, $a_f$, $a_{ins}$) and electrical conductivity of each layer ($\sigma_f$ is a conductivity of the ferromagnetic layer, $\sigma_{ins}$ is a conductivity of the insulation layer):

$$\sigma_z = \sigma_y = k_f \, \sigma_f \left(1 + \frac{a_{ins}}{a_f} \frac{\sigma_{ins}}{\sigma_f}\right).$$

The coefficient of diffusion for the field penetration along the axis *x*, i.e. perpendicular to the plane of sheets, is

$$D_x = \frac{1}{\mu_y \, \sigma_z} = \frac{D_f}{k_f^2 \left(1 + \frac{a_{ins}}{a_f} \frac{\sigma_{ins}}{\sigma_f}\right)} = K_X D_f ,$$

where

$$K_X = \left(1 + \frac{a_{ins}}{a_f}\right)^2 \Bigg/ \left(1 + \frac{a_{ins}}{a_f} \frac{\sigma_{ins}}{\sigma_f}\right).$$

The coefficient of diffusion for the field penetration along the axis *z*, i.e. along the plane of sheets, is

$$D_z = K_{z0} \, D_{ins} = K_{zf} \, D_f ,$$

hereby

$$K_{z0} = \frac{1 - k_f}{k_f} \frac{\mu_{ins}}{\mu_f} \left(1 + \frac{a_{ins}}{a_f} \frac{\sigma_{ins}}{\sigma_f}\right);$$

$$K_{Zf} = \frac{1 - k_f}{k_f} \frac{\sigma_f}{\sigma_{ins}} \left(1 + \frac{a_{ins}}{a_f} \frac{\sigma_{ins}}{\sigma_f}\right);$$

$$D_{ins} = \left(\mu_{ins} \, \sigma_{ins}\right)^{-1} ,$$

$$D_f = \left(\mu_f \, \sigma_f\right)^{-1}$$

are the initial coefficients of diffusion for the field penetration into the monolithic material of insulator or ferromagnetic, respectively. For the amorphous material Metglas 2605CO the own parameters of ferromagnetic in dynamic mode are the next [2]:

$$\mu_f = (0.5 \cdot 10^2 ... 1.0 \cdot 10^5) \cdot \mu_0 ;$$

$$\rho_f = \sigma_f^{-1} = 123 \cdot 10^{-6} \, \text{Ohms} \cdot \text{cm} ;$$

$$\sigma_f \approx 0.8 \cdot 10^6 \ \text{S/m}.$$

## One-dimensional simulation of electromagnetic process

To study the magnetic field distribution in the process of the field penetration the pulse duration has been taken equal to 100 ns. The first step of simulation was devoted to one-dimensional diffusion in the band of cross section at the width *dx* (for diffusion along the axis *z*, i.e. along the plane of sheets) or at the width *dz* (for diffusion along the axis *x*, i.e. perpendicular to the plane of sheets). The results of 1D simulation are illustrated below in the fig. 4 – fig. 13 for the cross section of core dimensions 0.1m x 0.1m.

Each example has been obtained in the end of time interval 100 ns at the same difference of vector magnetic potential ΔA = 0.2 Wb/m at the edges of the space interval of simulation. In the fig.4 a ratio of the layers thickness is

$$\frac{a_{ins}}{a_f} = 0.25,$$

it means

$$k_f = 0.8;$$

$$\sigma_z = 0.8\sigma_f = 0.64 \cdot 10^6 \text{ S/m};$$

$$\frac{\mu_y}{\mu_0} = 0.8\mu_f;$$

$$\frac{\mu_x}{\mu_0} = 5 \cdot$$

The simulation program "Quick Field" [7] and its Russian analogue "Elcut" have been used at this stage of research. Simulation has been perform in the linear approximation ($\mu_f$= const). The magnetic permeability of the ferromagnetic layers was variable in the limits since 400 $\mu_0$ up to 40,000 $\mu_0$, nevertheless the picture $B_y(x)$ was the same, as it is shown in the fig. 4.

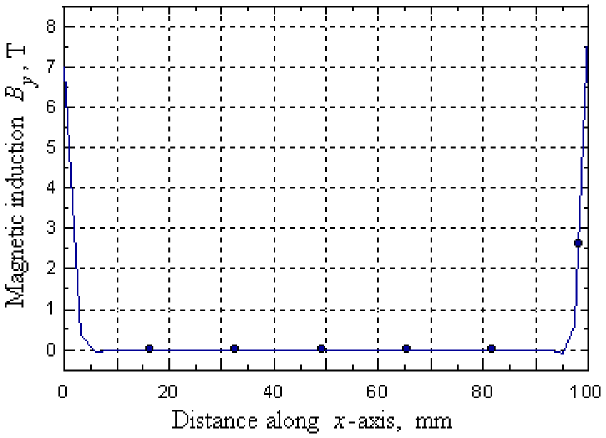



Fig. 4. Magnetic induction distribution along $x$-axis: $\frac{a_{ins}}{a_f} = 0.25$; $\frac{\mu_z}{\mu_0} = 400 \ldots 40{,}000$

The strong concentration of the magnetic flux near the borders to the end of pulse says about a low speed of the field diffusion along the axis $x$, i.e. across the planes of the sheets.

Fig. 5 – fig. 7 show the field distribution along the $z$-axis, i.e. along the sheets plane, when the thickness ratio is

$$\frac{a_{ins}}{a_f} = 0.1,$$

what means

$$k_f = 0.9,$$

$$\frac{\mu_y}{\mu_0} = 0.9\mu_f,$$

$$\frac{\mu_x}{\mu_0} = 1.1,$$

$$\sigma_z = 0{,}09 \text{ S/m}.$$

At $\frac{\mu_y}{\mu_0} = 450$

a drop of induction in the center of interval is less than 1% (fig. 5).

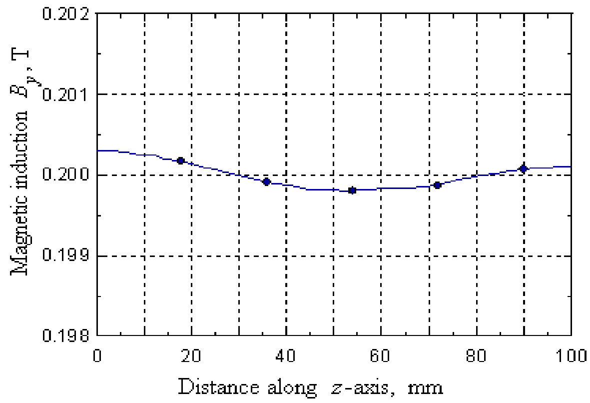



Fig. 5. Magnetic induction distribution along $z$-axis: $\frac{a_{ins}}{a_f} = 0.1$; $\frac{\mu_y}{\mu_0} = 450$

Increase of magnetic permeability $\mu_y$ up to 4500 $\mu_0$ leads to drop of induction in the center of interval in 20 times (fig. 6).

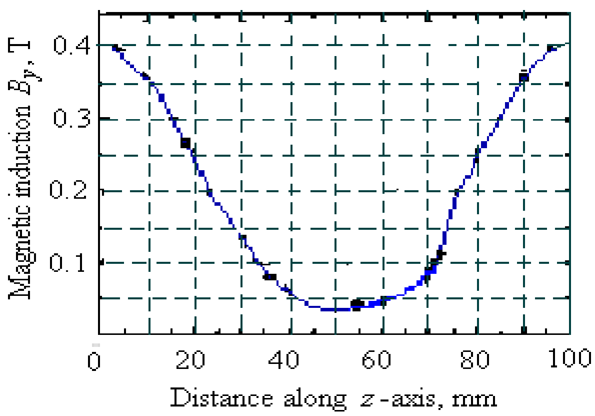



Fig. 6. Magnetic induction distribution along $z$-axis: $\frac{a_{ins}}{a_f} = 0.1$; $\frac{\mu_y}{\mu_0} = 4500$

With a new increase of $\mu_y$ up to 45,000 $\mu_0$ the picture of the field distribution fig. 7 transforms into the picture similar to fig. 4, with a drop of induction up to zero in the central part of the length interval.

The next step of simulation presents the estimation of the ratio of layers thickness influence on the drop of induction. The fig. 8 – fig. 10 show the field distribution along the *z*-axis for $\frac{a_{ins}}{a_f} = 0.15$.

It corresponds to meanings

$k_f = 0.87$,

$\frac{\mu_x}{\mu_0} = 1.18$,

$\sigma_z = 0.07$ S/m,

$\frac{\mu_y}{\mu_0} = 0.87\mu_f$.

At the low magnetic permeability of material (fig. 8) non-uniformity in the field distribution along the *z*-axis not more than 0.005%, what is much better than in the fig. 5. For the bigger value $\frac{\mu_y}{\mu_0} = 4350$ the drop of induction in the center reaches 7 times (fig. 9), while at the next value $\frac{\mu_y}{\mu_0} = 43{,}500$ the picture of the field distribution fig. 10 becomes similar to shown in the fig.7.

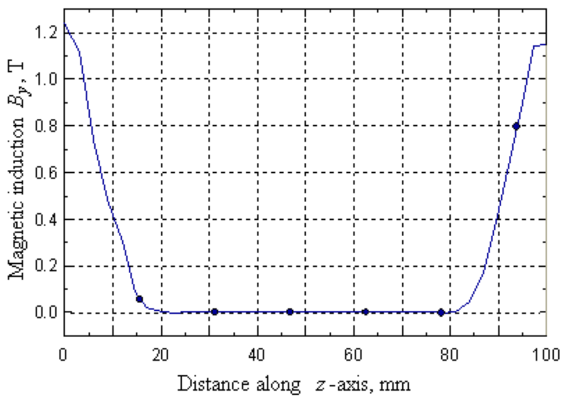



Fig. 7. Magnetic induction distribution along *z*-axis: $\frac{a_{ins}}{a_f} = 0.1$; $\frac{\mu_y}{\mu_0} = 45{,}000$

The last examples of 1D simulation which are shown in the fig. 11 – fig. 13 have been obtained for the same ratio of the layers thickness $\frac{a_{ins}}{a_f} = 0.25$, as it was used above in the fig. 4. First example (fig. 11) demonstrates that for relatively small value of magnetic permeability ($\frac{\mu_y}{\mu_0} = 400$) a distribution of magnetic induction is absolutely uniform along *z*-axis. Increased value of permeability $\frac{\mu_y}{\mu_0} = 4000$ leads to drop of induction in the center in 2.35 times (fig. 12) what is not so big as for $\frac{a_{ins}}{a_f} = 0.15$ (fig. 9). Last example (fig. 13) confirms that for any ratio of the layers thickness at the high magnetic permeability (here $\frac{\mu_y}{\mu_0} = 40{,}000$) magnetic induction always reaches zero in the central area of core, thus magnetic flux is concentrated in the border areas of near 15 mm width what is in sum only 1/3 of the core cross dimension.

**Two-dimensional simulation of the field penetration into the core**

*Numerical Analysis of Two-dimensional Diffusion of Magnetic Field into Equivalent Medium.* The most decisive results concerning of penetration of the pulsed field into the cross section of the inductor core can be obtained by way of 2D simulation in the standard finite-element software for the transient electromagnetic processes (for example, FemLab or Comsol).

The same geometry of the core cross section has been taken for this procedure (0.1 m × 0.1 m). It was supposed that the sheets of ferromagnetics have been situated in the planes parallel to *yz*-plane. The next meaning of diffusion coefficient for the field penetration along the axes *x* and *z* have been specified: $D_x = 0.132$ m$^2$/s, $D_z = 0.132 \cdot 10^{-4}$ m$^2$/s.

It can be realized in the model by two methods:

1) due to choice of anisotropic electrical conductivity of equivalent medium in the matrix form

$$[\sigma_{eff}] = \begin{vmatrix} \sigma_x & 0 \\ 0 & \sigma_z \end{vmatrix}$$

with $\sigma_y = 6.0 \cdot 10^3$ (Ohm·m)$^{-1}$, $\sigma_z = 6.0 \cdot 10^7$ (Ohm·m)$^{-1}$ at the isotropic value of magnetic permeability $\mu_{xy} = \mu_x = \mu_y = 1000\mu_0$;

2) due to choice of anisotropic magnetic permeability in the matrix form

$$[\mu_{eff}] = \begin{vmatrix} \mu_x & 0 \\ 0 & \mu_z \end{vmatrix}$$

with isotropic electrical conductivity of equivalent medium $\sigma_{xz} = \sigma_x = \sigma_z$ (it the software enables this combination).

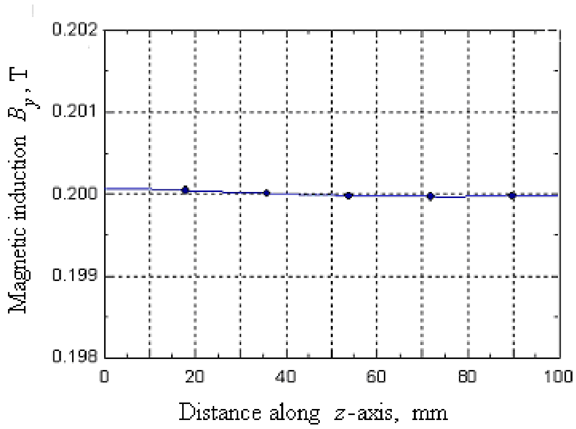



Fig. 8. Magnetic induction distribution along *z*-axis:

$$\frac{a_{ins}}{a_f} = 0.15; \ \frac{\mu_y}{\mu_0} = 435$$

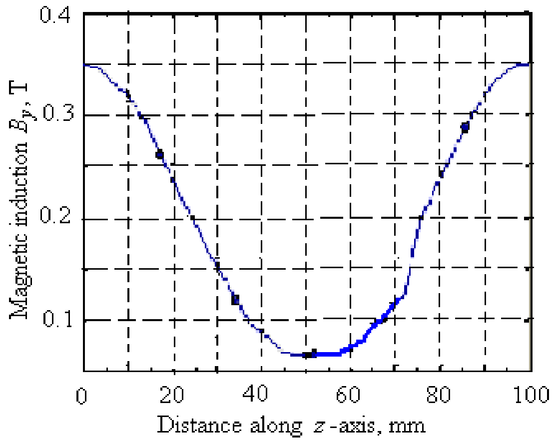



Fig. 9. Magnetic induction distribution along *z*-axis:

$$\frac{a_{ins}}{a_f} = 0.15; \ \frac{\mu_y}{\mu_0} = 4350$$

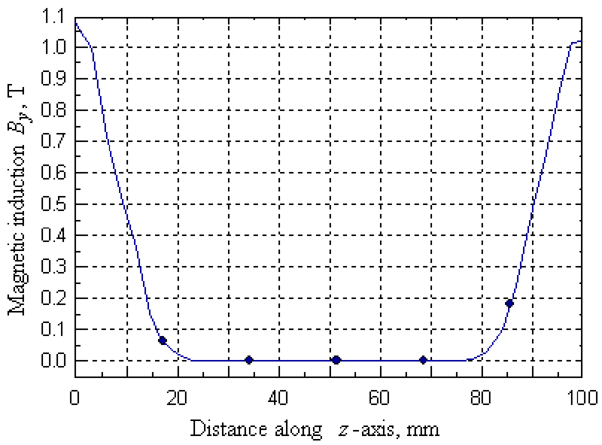



Fig. 10. Magnetic induction distribution along *z*-axis:

$$\frac{a_{ins}}{a_f} = 0.15; \ \frac{\mu_y}{\mu_0} = 43{,}500$$

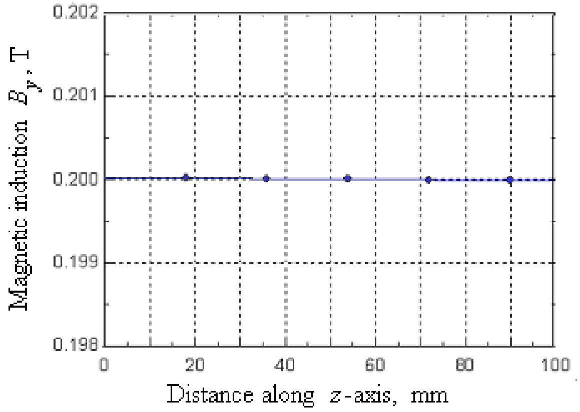



Fig. 11. Magnetic induction distribution along *z*-axis:

$$\frac{a_{ins}}{a_f} = 0.25; \ \frac{\mu_y}{\mu_0} = 400$$

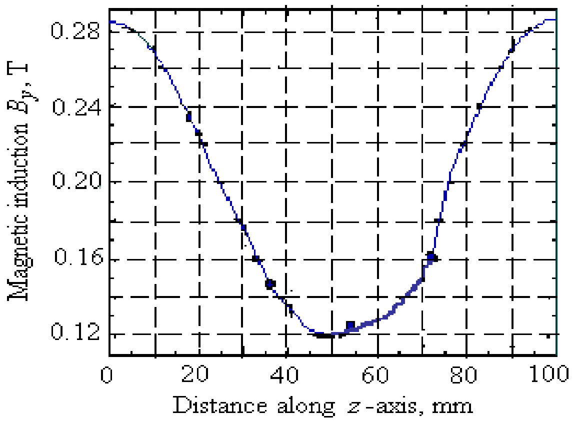



Fig. 12. Magnetic induction distribution along *z*-axis:

$$\frac{a_{ins}}{a_f} = 0.25; \ \frac{\mu_y}{\mu_0} = 4000$$

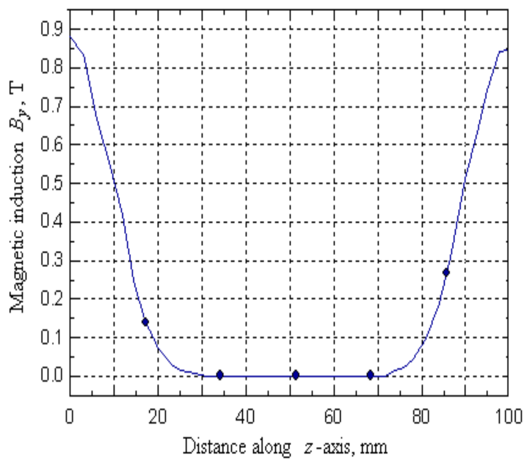



Fig. 13. Magnetic induction distribution along *z*-axis:

$$\frac{a_{ins}}{a_f} = 0.25; \ \frac{\mu_y}{\mu_0} = 40{,}000$$

Main goal is to reach the real anisotropy of diffusion coefficient as

$$[D]=\mu_{xy}^{-1}\cdot[\sigma_{eff}]^{-1}=\sigma_{xz}^{-1}\cdot[\mu_{eff}]^{-1}.$$

Results of the transient process simulation are shown in fig. 14 – fig. 16. There are presented the pictures of the field excitation in the anisotropic core at the rectangular time-form of the exciting pulse at induction 1.5 T on the external border of the core cross section. Both graphs have the lines of the field distribution drawn via time interval equal to 0.1*T*, where *T* is the pulse duration. How it is seen in the fig. 14 the field distribution reflects the anisotropic properties of the core medium.

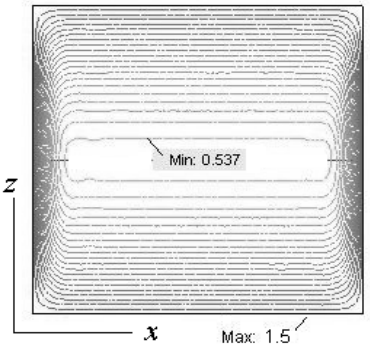



Fig. 14. The constant levels of magnetic induction at the end of pulse duration in 2D finite elements model

Distribution of the magnetic induction along two orthogonal lines which are the axes of cross section symmetry is drawn in the fig. 15 (following to the central line parallel to axis *x*) and in the fig. 16 (following to the central line parallel to axis *z*). Influence of transversal diffusion (along z) allows to reach the elevation of the field up to the value 0.537 T in the central part of core with quasi-uniform distribution along the *x*-axis (fig. 15) what differs from the 1D results. Nevertheless the strong concentration of the field near the edges is seen in the fig. 15.

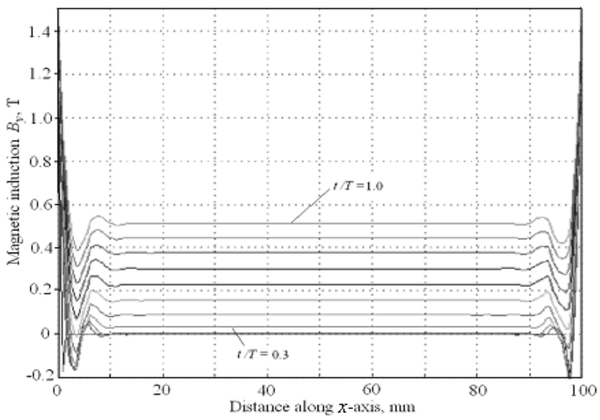



Fig. 15. Magnetic induction distribution along the *x*-axis growing in time

Distribution of induction along *z*-axis (fig. 16) demonstrates that penetration of field into ferromagnetic is going mainly along the plane of sheet but has not a time to reach a stable level corresponding to the border condition.

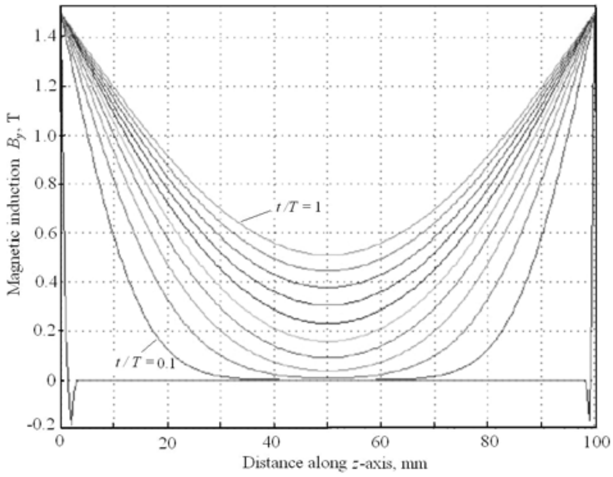



Fig. 16. Distribution of the magnetic induction along *z*-axis (evolution in time)

*Field Diffusion in the Package of Ferromagnetic Sheets*. All consideration described above presents in fact the efforts to reduce the complex problem of the field diffusion into multi-layer medium to the simplified problem of 1D of 2D diffusion into the equivalent solid medium. The greater interest consists of the pulsed field analysis in the real multi-layer structure with real interlacing of magnetic and non-magnetic sheets. Such trial simulation has been realized in the striated model drawn in the fig. 17 with following implementation in the finite-element program.

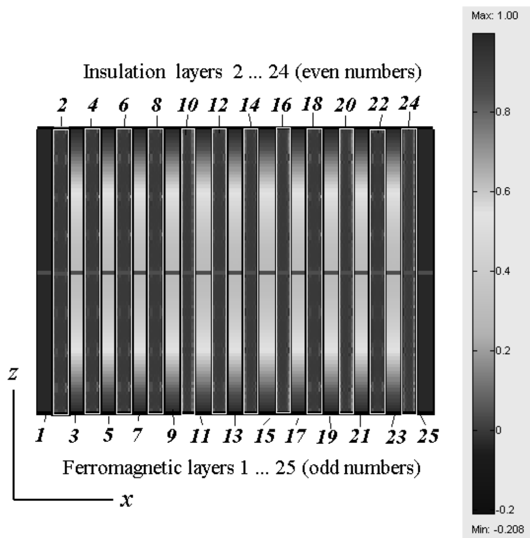



Fig. 17. Cross section of the multi-layer model with grey color using for the field solution in magnetics

If one has the powerful solver it is possible to draw not only 25 sheets as it is shown in the fig. 17 but greatly bigger number of sheets. To relieve the study of the main peculiarities of electromagnetic process the number of sheets has been limited 25 and ratio of thickness has been taken equal to $\frac{a_{ins}}{a_f} = 1.0$. Initial condition along the contour of the core cross section has been specified as $B_y\big|_{border}$ = 1Tesla at $T > t > 0$ (here $T$ = 100 ns is the pulse duration). The own coefficient of the field diffusion for each kind of matter ($D_f$ for ferromagnetic, $D_{ins}$ for insulator) has been entered into the model, with correlation $\frac{D_f}{D_{ins}} = 10^{-7}$. Only switch in has been calculated for the field in the cross section, switch off was out of consideration. As it is possible to see in the fig. 18, the penetration of field along the ferromagnetic sheet is going absolutely likely to predicted in the 1D and 2D model for equivalent anisotropic medium.

The graph in the fig. 19 confirms that field penetrates into core exclusively along the ferromagnetic material, i.e. along the axis $z$ in the fig. 17, due to unsaturated state of magnetics.

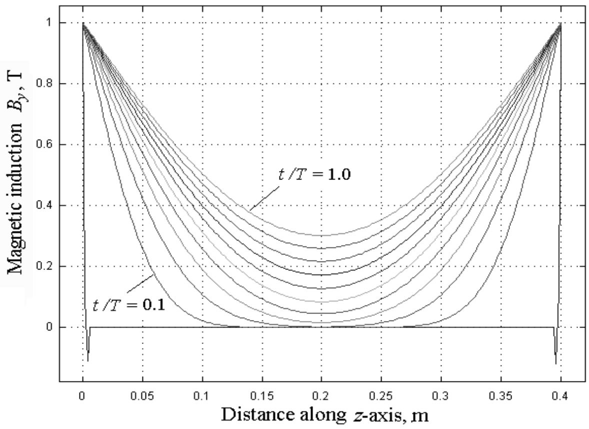



Fig. 18. Magnetic induction distribution for set in along $z$-axis in the mid plane of ferromagnetic sheet

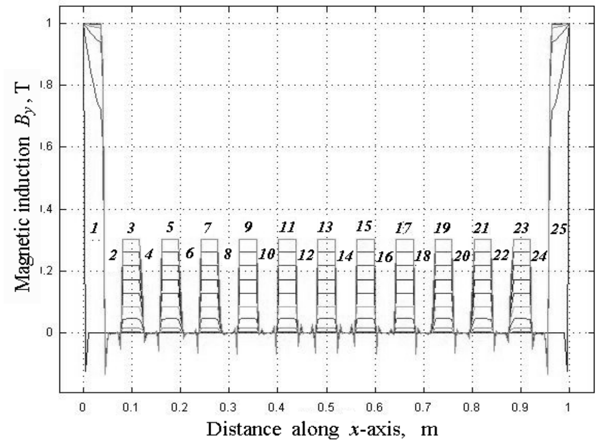



Fig. 19. Set in of the magnetic induction in the striated model along $x$-axis (central line)

## Conclusion

Numerical analysis of the pulsed magnetic field diffusion into the cross section of the multi-layer core of inductor shows that the non-uniform distribution of induction as result of the finite speed of the field diffusion is a serious engineering problem which must be taken into consideration at the powerful electron accelerators design development. Both 1D and 2D approaches to the transient field simulation with equivalent anisotropic medium are productive for the study of diffusion specifics. They display the results which are similar to results obtained in the model which uses the real striated structure of core.

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