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STRUCTURED PARAMETRIC OPTIMIZATION OF MULTIVARIABLE ROBUST CONTROL BASED ON GENETIC ALGORITHMS

This paper is devoted to the parametric robust H_2/H_∞ - optimization of the LQG- controller designed for multivariable flight control system. At the 1st stage LQG- regulator is designed using the separation theorem. At the 2nd stage this controller was parametrically optimized on the basis of H_2/H_∞ -criterion using genetic algorithm to find the trade-off between the performance and robustness. The sensitivity theory is applied to reduce the number of parameters involved in the optimization procedure.

Розглянуто параметричні робастні H_2/H_∞ -оптимізації лінійно-квадратично-гаусового регулятора, синтезованого для багатовимірної системи керування польотом. На першому етапі лінійно-квадратично-гаусів регулятор визначається за допомогою теореми розділення. На другому етапі цей регулятор параметрично оптимізується на основі H_2/H_∞ -критерія з використанням генетичного алгоритму для знаходження компромісу між якістю та робастністю. Теорію чутливості застосовано для пониження кількості параметрів оптимізаційної процедури.

Introduction

The design of flight control system, which is capable to provide good aircraft handling qualities over a wide range of operating conditions, suppress different internal and external disturbances to enable effectively diverse flight missions has always been a challenge for designer [1–4].

In recent years, many methods of designing have been developed in the area, starting from simple structures to more complicated and using advanced control techniques [1; 3; 4; 5].

In this paper, the robust parametric multivariable optimal control is investigated; the method is divided into two stages. In the first stage consists in linear quadratic Gaussian (LQG) regulator design, in the second task an optimization procedure is used to “robustify” the optimal controller found at the first stage. This problem was solved in [4] for SISO systems.

The application of this approach for MIMO systems required usage additional efforts and some new approaches due to significant increases in dimension of optimization problem and necessity to apply more robust and advanced search procedures.

The H_2/H_∞ optimization of the sensitivity and complementary sensitivity function based on genetic algorithm has been adopted in this paper.

The use of the genetic algorithms optimization has been largely overlooked, particularly as an optimization technique for the processes that are difficult to solve.

Genetic algorithms (GAs) are stochastic global methods that mimic the process of natural evolution. GAs have been shown to be capable of locating high performance areas in complex domains without experiencing the difficulties associated with high dimensionality or local optima as may occur with traditional techniques [6–9]. The case study and simulation results devoted to stabilization of the longitudinal motion of the Aerosonde UAV have proved that the used method is very efficient for multivariable control from the viewpoint of its robustness and performance.

Synthesis of the linear quadratic Gaussian regulator based on the separation theorem

The state space model of the controlled plant is given by the quintuple of matrices $[A, B, C, D, G]$, where

$A \in R^{n\times n}$, $B \in R^{n\times q}$, $C \in R^{p\times n}$, $D \in R^{p\times q}$, $G \in R^{n\times l}$,

and is given as follows:

$$X = AX + BU + Gw$$
$$Y = CX + DU + v$$

the vector w represents the process disturbances, and here it is given by the wind turbulence described by the outputs of the Dryden filter [2–4].

In order to perform the design of the controller based on the separation theorem it would be necessary to include the model of the Dryden filter in the state space model of the plant and to form an extended model. Let the quadruple of matrices $[A_{dr}, B_{dr}, C_{dr}, D_{dr}]$ represents the state space model of the forming filter, where

$A_{dr} \in R^{r\times r}$, $B_{dr} \in R^{r\times 2}$, $C_{dr} \in R^{l\times r}$, $D_{dr} \in R^{l\times r}$.

Then the extended state space model of the overall model is described in the following way:

$$\left[\begin{array}{c|c}\mathbf{A}_{ex} & \mathbf{B}_{ex}\\ \hline \mathbf{C}_{ex} & \mathbf{D}_{ex}\end{array}\right]=\left[\begin{array}{cc|cc}\mathbf{A} & GC_{dr} & B & GB_{dr}\\ 0_{r\times n} & \mathbf{A}_{dr} & 0_{r\times q} & B_{dr}\\ \hline C & 0_{p\times r} & D & 0_{p\times 2}\end{array}\right]. \qquad (1)$$

At this stage we are ready to apply the separation theorem [10], which states, as it is known, that optimal stochastic controller for the plant (1) consists of optimal stochastic observer – Kalman filter to restore the full state vector of (1) and optimal deterministic controller – state feedback. The optimal Kalman filter is defined as:

$$\dot{\tilde{X}} = A_{ex}\tilde{X}_{ex} + B_{ex}U + L\left(Y - C_{ex}\tilde{X}_{ex} - D_{ex}U\right);$$

$$\begin{bmatrix}\tilde{Y}\\ \tilde{X}\end{bmatrix}=\begin{bmatrix}C_{ex}\\ I\end{bmatrix}\tilde{X}_{ex}+\begin{bmatrix}D_{ex}\\ 0\end{bmatrix}U,$$

where

L is the Kalman gain matrix given by the following expression:

$$L = PC_{ex}^{T}R_{N}^{-1};$$

P is the unique positive-definite solution to the following Algebraic Riccati Equation (ARE):

$$A_{ex}P + PA_{ex}^{T} + B_{ex}Q_{N}B_{ex}^{T} - PC_{ex}^{T}R_{N}^{-1}C_{ex}P = 0;$$

Q_N and R_N are the covariance matrices associated with the measurement and process noises respectively.

The state feedback K is given in the following expression:

$$K = R^{-1}B_{ex}^{T}S, \qquad (2)$$

where

S is the unique positive definite matrix of *(ARE)* associated with the optimal feedback problem:

$$A_{ex}^{T}S + SA_{ex} - SB_{ex}R^{-1}B_{ex}^{T}S + Q = 0$$

and the optimal control law minimizing the performance index, is as follows:

$$U = -K\tilde{X}_{ex}.$$

The state space model of the closed loop system shown in fig.1 is given by the following equations:

$$\begin{bmatrix}X\\ \tilde{X}\end{bmatrix}=\begin{bmatrix}A_{ex} & -B_{ex}K\\ LC_{ex} & A_{ex}-LC_{ex}-B_{ex}K\end{bmatrix}\begin{bmatrix}X\\ \tilde{X}\end{bmatrix}.$$

Parameterization and robustization of the linear quadratic Gaussian controller

The robustness in flight control system design is the most crucial property one should care about [1–4]. During flight various uncertainties occur due to the parameters change.

These uncertainties could be external and/or internal, structural and/or unstructured, which produce certain deviation from the nominal behavior to perturbed one.

The main task of the robust control is to allow the control of any perturbed plant with a single controller designed for nominal plant. Many methods were proposed in the literature to recover the robustness of the closed loop system [11]. In this paper multi-objectives optimization procedure is proposed to ''robustize'' (to increase robustness) of the optimal controller designed in the first section [1; 3–6; 11].

The nonlinear model of the UAV is linearized for some N operating conditions inside the flight envelope, and N models associated with certain operation modes were found [2–4]. The problem is to find the same control law for the N linear models that assures the stability and the expected performances.

The solution to this problem can be achieved by the multi-objective optimization procedure [6], where several objectives should be taken into account. In this case, one can seek the compromise between the performances and the robustness objectives of the overall system.

A composite performance index is formed from the estimation of the performances and the robustness for the N models based on the H_2/H_∞ - norms computed for the different transfer functions of the block diagram depicted in fig.1, with corresponding LaGrange factors weighting the contribution of each estimated term.

As known in control system theory [6–8; 11], the performance can be estimated with H_2-norm and the robustness – with H_∞-norm. Therefore the performances are estimated for both stochastic and deterministic transfer functions, for stochastic case the transfer function is computed from η to Z and deterministic case is computed from U to Z; these computations are repeated for N models in order to find the trade-off between the performances in deterministic and stochastic cases for the nominal and perturbed models [3–6; 12].

Eventually we have:

for deterministic case:

$$\left\|H_{UZ}\right\|_{2}^{dn} = \sqrt{\int_{0}^{\infty}\left(X^{T}QX + U^{T}RU\right)dt},$$

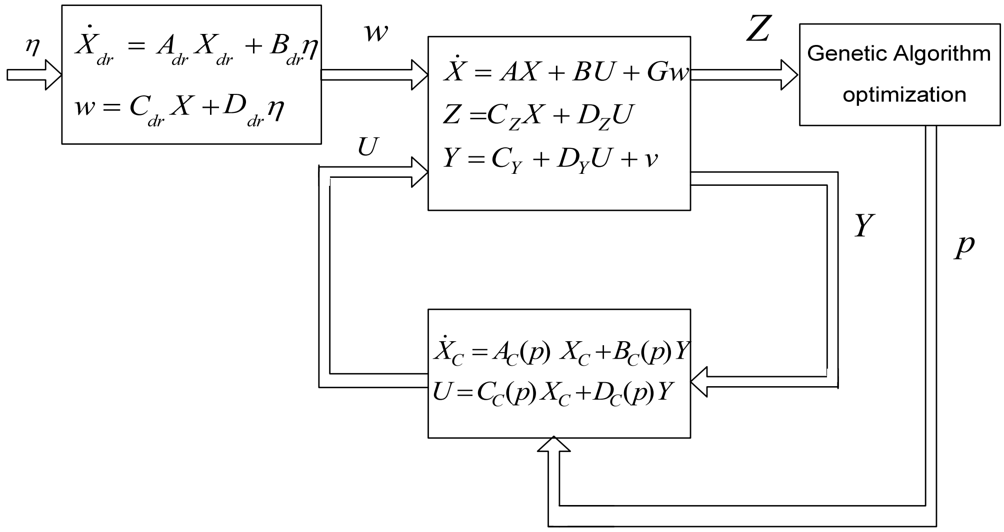

Fig. 1. Closed loop system (p is a vector of adjustable parameters of controller)

where Q and R are diagonal matrices weighting each state and control input respectively, and for stochastic case:

$$\|H_{UZ}\|_2^{sn} = \sqrt{E\int_0^{\infty}\left(X^TQX + U^TRU\right)dt}\,,$$

where E stands for expected value.

The H_∞ will be computed for the matrix transfer function from w to Z that is to estimate the robustness of the control law relative to any noises input and is given by the following expression:

$$\|T_{wZ}\|_\infty = \sup\overline{\sigma}\left(T_{wZ}(j\omega)\right)$$

where

$\overline{\sigma}\left(T_{wZ}(j\omega)\right)$ is the maximal singular value of the matrix $T_{wZ}(j\omega)$ at the current frequency ω: $0 \le \omega \le \omega_N$;

ω_N is the Nyquist frequency.

Computation of these norms could be performed on the basis of the following quadruple of matrices for deterministic nominal model:

$$\left[\begin{array}{c|c}\mathbf{A}_{UZ}^{no} & \mathbf{B}_{UZ}^{no}\\ \hline \mathbf{C}_{UZ}^{no} & \mathbf{D}_{UZ}^{no}\end{array}\right] = \left[\begin{array}{cc|c} A^{no} & -B^{no}K & B^{no}\\ LC_y^{no} & A^{no} - LC_y^{no} - B^{no}K & 0_{p1\times q}\\ \hline C_Z^{no} & 0_{p1\times n} & D^{no}\end{array}\right]. \quad (3)$$

Quadruples of matrices for parametrically disturbed models can be derived from (3) by replacement of matrices $\left[A_{UZ}^{no}, B_{UZ}^{no}, C_{UZ}^{no}, D_{UZ}^{no}\right]$ with matrices $\left[A_{UZ}^{pk}, B_{UZ}^{pk}, C_{UZ}^{pk}, D_{UZ}^{pk}\right]$,

where k is a number of perturbed model: k=1,…,N.

Then the state space model of closed loop system for the stochastic case can be defined as series connection of the closed loop system "plant + controller" and the Dryden filter:

$$\left[\begin{array}{c|c}\mathbf{A}_{UZ}^{st} & \mathbf{B}_{UZ}^{st}\\ \hline \mathbf{C}_{UZ}^{st} & \mathbf{D}_{UZ}^{st}\end{array}\right] = \left[\begin{array}{ccc|cc} \mathbf{A}_{dr} & \mathbf{0}_{r\times n} & \mathbf{0}_{r\times q} & \mathbf{B}_{dr} & \mathbf{0}_{r\times q}\\ G\mathbf{C}_{dr} & \mathbf{A} & -\mathbf{B}K & G\mathbf{D}_{dr} & \mathbf{B}\\ \mathbf{0}_{p\times r} & L\mathbf{C} & \mathbf{A}-L\mathbf{C}-\mathbf{B}K & \mathbf{0}_{r\times 2} & \mathbf{0}_{r\times q}\\ \hline \mathbf{0}_{p\times r} & C_z & \mathbf{0}_{p\times n} & \mathbf{0}_{r\times 2} & \mathbf{D}\end{array}\right]. (4)$$

The same closed loop model is computed for all N perturbed models. At this point we are ready to formulate the cost function to be optimized using genetic algorithm. The performance indices for each model of closed loop system are computed using H_2 norm of the system matrices (3), (4) using the controllability Gramian [12]. The H_∞ – norm is also performed using the system matrices (4) [12]. As is shown the above block matrices the closed loop systems are dependent on the Kalman gain matrix L and controller gain matrix K, so are the H_2 – norm and H_∞ – norm.

The multi-objective functional is given in the following expression

$$J = \lambda_{dn}\|H_{UZ}\|_2^{dn} + \lambda_{sn}\|H_{UZ}\|_2^{sn} + \lambda_{\infty n}\|T_{wZ}\|_\infty^{n} +$$
$$+ \sum_{k=1}^{N-1}\lambda_{dpk}\left(\|H_{UZ}\|_2^{dpk}\right) + \sum_{k=1}^{N-1}\lambda_{spk}\left(\|H_{UZ}\|_2^{spk}\right) +$$
$$+ \sum_{k=1}^{N-1}\lambda_{\infty pk}\left(\|T_{wZ}\|_\infty^{pk}\right),$$

where

$\|H_{UZ}\|_2^{dn}$ defines the H_2 – norm of the nominal model in deterministic case;

$\sum_{k=1}^{N-1}\|H_{UZ}\|_2^{dpk}$ stands for summation of the H_2 – norms of $(N-1)$ perturbed models;

$\|T_{wZ}\|_\infty^{n}$ is the H_∞ – norm and gives the estimation of the robustness of the nominal controlled plant;

$\sum_{k=1}^{N-1}\|T_{wZ}\|_\infty^{pk}$ computes the summation of the H_∞ – norm for all $(N-1)$ parametrically disturbed plants.

$\|H_{UZ}\|_2^{sn}$ defines the performances of the nominal stochastic model, the same summation of the H_2 – norm being defined for all perturbed models with the expression $\sum_{k=1}^{N-1}\|H_{UZ}\|_2^{spk}$.

The LaGrange factors λ_{dn}, λ_{sn}, λ_{dpk}, λ_{spk}, $\lambda_{\infty n}$, $\lambda_{\infty pk}$ weight the contribution of each term in the cost function.

So far as the computation of H_2 is based on the controllability Gramian the closed loop system defined in the equations (3), (4) should be stable and fully controllable over the whole optimization procedure, therefore the total cost function should include another term called penalty function, restricting location's area of the closed loop system poles in the predefined region in the complex plan given in fig. 2, *a* [3]. The penalty $PFi(dm)$ as a function of minimal distance could be graphically shown in fig.2, *b* and defined over area D for its 1st border as follows [3]:

$$PFi(d_m)=\begin{cases}0, \text{if } d_m \ge d_{m1}\\ \frac{P}{2}\left[1+\cos\left(\frac{\pi(d_m-d_0)}{d_{m1}-d_0}\right)\right] & if\ d_0<d_m<d_{m1}\\ P, \text{if } d_m \le d_0\end{cases} \quad (5)$$

where

P is a very large value (for example, $P=10^4 \div 10^6$).

This function is smooth and differentiable inside the unit circle. It is necessary to find the minimal value d_m of all distances from all poles of nominal and perturbed models to the 1st and the 2nd borders of area D in complex plane z.

Penalty function of this type is described in more details in [3].

After adding this term to the objective function defined in (5), the total cost function to be optimized becomes:

$$J_\Sigma = J + PF_i. \quad (6)$$

The amount of parameters involved in the optimization procedure increases with the number of estimated states and the number of perturbed models. Therefore the optimization could take very long time and large memory space. In order to reduce the number of variable parameters in the optimization procedure, the determination of sensitivity of the cost function to small variations of the optimization variables is proposed. The method is based on small deviation of each entry of L and K matrices from their nominal values, defined after LQG-synthesis, and compute the increment of total cost function (6) for each change:

$$S_L=\frac{J_i-J_0}{L\pm \Delta L_i} \qquad i=1:length(L); \quad (7)$$

$$S_K=\frac{J_j-J_0}{K_j\pm \Delta K_j} \qquad j=1:length(K).$$

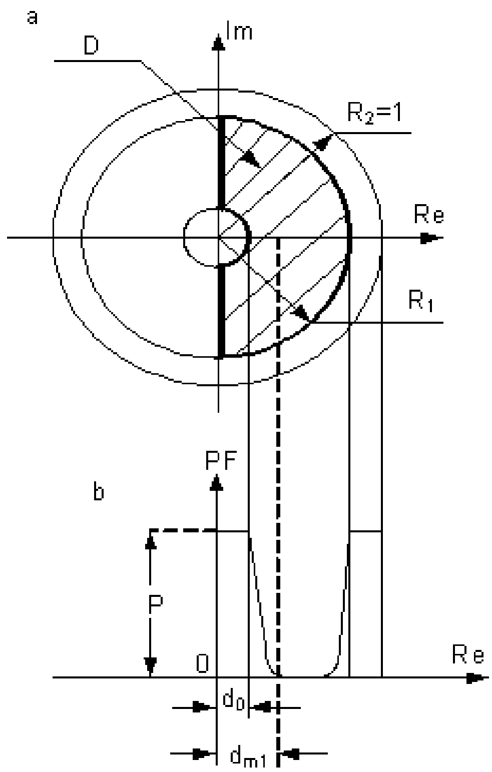

Fig.2. Penalty function in the complex z-plane

If cost function increment is significant, then this parameter is taken as the optimization variable, otherwise it is set to constant. As it was mentioned before, complicated cost function (5) in practical cases is not convex, that is why local minima could take place.

As it is shown in [7–9], genetic algorithm optimization procedure is the most adequate and suited procedure to solve such problems.

Overview of genetic algorithms optimization

Genetic algorithms are a search procedure based on Darwinian “survival of the fittest” theory.

GAs were developed to solve difficult problems with objective function that do not possess ‘’nice’’ properties such as continuity, differentiability, etc. based on the idea of natural selection and genetics [8].

GAs are part of broad class of search techniques within the field of evolutionary algorithms or evolutionary computing [8; 9; 13].

These algorithms maintain and manipulate a family, or population of candidate solutions and implement a ‘’survival of the fittest’’ strategy in their search for better solution. GAs work from a population comparing to other methods work from a single point, this provides an implicit as well as explicit parallelism that allows for the exploitation of several promising areas of the solution space at the same time [9].

In our case the initial population is generated for the Kalman gain matrix L and static gain matrix K, the fitness function given in the equation (6).

Several selection methods were developed, in our case the normalized geometric distribution [9] is used.

In this study the arithmetic crossover was adopted as a crossover function.

The multi-non uniform mutation distribution is used to mutate the individual.

Case study

In this paper we consider longitudinal channel of nonlinear Aerosonde UAV model linearized in different trimmed conditions [14]. The state space vector of the linearized model is given by $\overline{X} = [u \quad w \quad q \quad \theta \quad h \quad \Omega]$; u, w are horizontal and vertical velocity components, respectively; q is pitch rate, θ is pitch angle, h is altitude and Ω is engine spin (r.p.m.).The control vector is given by $\vec{U} = [\delta_e \quad \delta_{th}]$, where δ_e is elevator angle deflection, δ_{th} is thrust control (engine throttle deflection).

The range of the uncertainty is made for the true airspeed, which is given by the following expression $V = \sqrt{u^2 + v^2 + w^2}$,

where v defines the lateral velocity component.

We suppose that V changes in the interval $25 \le V \le 35$ m/s, for the sake of simplicity and without loss of generality three $(N = 3)$ models were defined in our study, the nominal model is taken at $V_n = 30$ m/s, the first perturbed model is defined for $V_n = 25$ m/s and for the second perturbed model $V_n = 35$ m/s.

The following matrices give the respective states models:

$$A_n = \begin{bmatrix} -0.293 & 0.38 & -0.55 & -9.78 & 0 & 0.01 \\ -0.55 & -5.36 & 30 & -0.18 & 0 & 0 \\ 0.33 & -5.63 & -6.19 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0.01 & -1 & 0 & 30 & 0 & 0 \\ 41.53 & 0.78 & 0 & 0 & -0.63 & -3.85 \end{bmatrix}$$

$$B_n = \begin{bmatrix} -0.3 & 0 \\ -3.7 & 0 \\ -50 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 2664 \end{bmatrix}$$

$$A_{p1} = \begin{bmatrix} -0.24 & 0.53 & -1.19 & -9.80 & 0 & 0.01 \\ -0.56 & -4.47 & 25 & -0.47 & 0 & 0 \\ 0.43 & -4.48 & -5.15 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0.04 & -1 & 0 & 25 & 0 & 0 \\ 35 & 1.68 & 0 & 0 & -0.03 & -3.23 \end{bmatrix}$$

$$B_{p1} = \begin{bmatrix} 0.35 & 0 \\ -2.54 & 0 \\ -35.21 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 390 \end{bmatrix}$$

$$A_{p2}=\begin{bmatrix}-0.35 & 0.28 & -0.05 & -9.82 & 0 & 0.01\\ -0.55 & -6.25 & 35 & -0.01 & 0 & 0\\ 0.28 & -6.43 & -7.21 & 0 & 0 & -0.01\\ 0 & 0 & 1 & 0 & 0 & 0\\ 0 & -1 & 0 & 35 & 0 & 0\\ 48.5 & 0.08 & 0 & 0 & -0.78 & -4.43\end{bmatrix}$$

$$B_{p2}=\begin{bmatrix}0.5 & 0\\ -5 & 0\\ -68.2 & 0\\ 0 & 0\\ 0 & 0\\ 0 & 3040.3\end{bmatrix}$$

In series the models of actuators are connected to the model of the UAV and are approximated by the first order model given in the following:

$$\left[\begin{array}{c|c}A_{act} & B_{act}\\ \hline C_{act} & D_{act}\end{array}\right]=\left[\begin{array}{c|c}-1/\tau_{\delta act} & 1/\tau_{\delta act}\\ \hline 1 & 0\end{array}\right]$$

where

$\tau_{\delta act}=0.25$ s stands for the time constant of the actuator and the subscript δact can be either for elevator or throttle.

In our design only four state variables are measured:

$\bar{X}=[u \quad q \quad \theta \quad h]$,

so the observation matrix is given as follows:

$C=\left[\begin{bmatrix}1 & 0_{3\times1}\end{bmatrix}^T \quad 0_{4\times1} \quad \begin{bmatrix}0_{1\times4} & I_{3\times3} & 0_{3\times1}\end{bmatrix}^T\right]$,

where

I represents the unity matrix with appropriate dimension. According to [2–4], the Dryden filter has two inputs: horizontal and vertical wind guests, the outputs are the longitudinal turbulent speed u_g, vertical turbulent speed w_g and turbulent pitch rate q_g.

State space of the Dryden filter is defined by the following matrices:

$$A_{dr}=\begin{bmatrix}-1/\lambda_u & 0 & 0\\ 0 & -1/\lambda_w & 0\\ 0 & -K_q/\lambda_q^2 & -1/\lambda_q\end{bmatrix};$$

$$B_{dr}=\begin{bmatrix}K_u/\lambda_u & 0\\ 0 & K_w/\lambda_w\\ 0 & 0\end{bmatrix};$$

$$C_{dr}=\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & K_q/\lambda_q & 1\end{bmatrix}$$

where the subscript w corresponds to vertical components and u for the longitudinal.

In our case the Aerosonde flies at an altitude of 200m, and in moderate turbulence. The parameters appearing in the state space of Dryden filter are given in the following [2–4]:

$K_u=\sigma_u\sqrt{(2L_u/\pi V)}$;

$\lambda_u=L_u/V$;

$K_w=2.2$;

$\lambda_w=0.6$;

$K_q=1/V$;

$\lambda_q=4b/\pi V$,

where

b is the wing span for the Aerosonde: $b=2.9\text{m}$.

The same parameters are defined for different models with different true airspeed V. The covariance matrices of the process noises and measurement noises are equal to

$R_n=diag([5 \quad 5])$,

$Q_n=diag([2 \quad 0.1 \quad 0.2 \quad 2])$,

and are defined by the corresponding accuracy of the sensors.

The weighting matrices Q_r, R_r for the optimal deterministic performance are given as:

$Q_r=diag([850 \ 100 \ 1 \ 1 \ 800 \ 0.08 \ 0.1 \ 0.1 \ 0.1 \ 0.1])$

$R_r=diag([1 \quad 1])$,

using the above models an extended model is defined containing 11 states, so the Kalman filter is using 4 measured states to restore 11 ones. On the basis of separation theorem the restored states are controlled by the deterministic optimal controller and the gain matrix in the (2) is found as follows:

$$K=\begin{bmatrix}1.45 & 2.44 & -7.47 & -192.5 & -14.48; & 0.002 & -0.61 & -4.33 & -5.43 & 12.13 & 0.07\\ 11.75 & 0.4 & -0.14 & -7.49 & 2.197; & 0.12 & -2.51 & 1.35 & -0.05 & 0.04 & 12.43\end{bmatrix}$$

For the sake of brevity in this paper the Kalman gain L is not given. As it is shown the dimension of the matrix K is of 2×11, and for the matrix L is 4×11, $\dim(L)+\dim(K)=66$, the cost function is depending on 66 parameters, that makes the optimization procedure slow and deteriorates its convergence.

To reduce dimension of the optimization problem the sensitivity of the cost function to each parameter is computed using (6) and (7).
Selection of parameters with significant sensitivity could decrease the number of parameters to 25.
These parameters constitute the initial values to the optimization procedure.

After execution this procedure the optimal values of these parameters have been defined and they were used for simulation of controlled longitudinal dynamics. The simulation results are given in the fig. 3, and table defines the H_2/H_∞-norms for nominal and perturbed models.

Table 1

H_2 and H_∞ of the closed loop system

Plant		H_2 Deterministic	H_2 Stochastic	H_∞
Vn=30 [m/s]	Nominal	2.1619	0.5377	0.7474
Vp_1=25 [m/s]	Perturbed 1	1.2058	0.5648	2.1990
Vp_2=35 [m/s]	Perturbed 2	3.8100	0.5836	0.7415

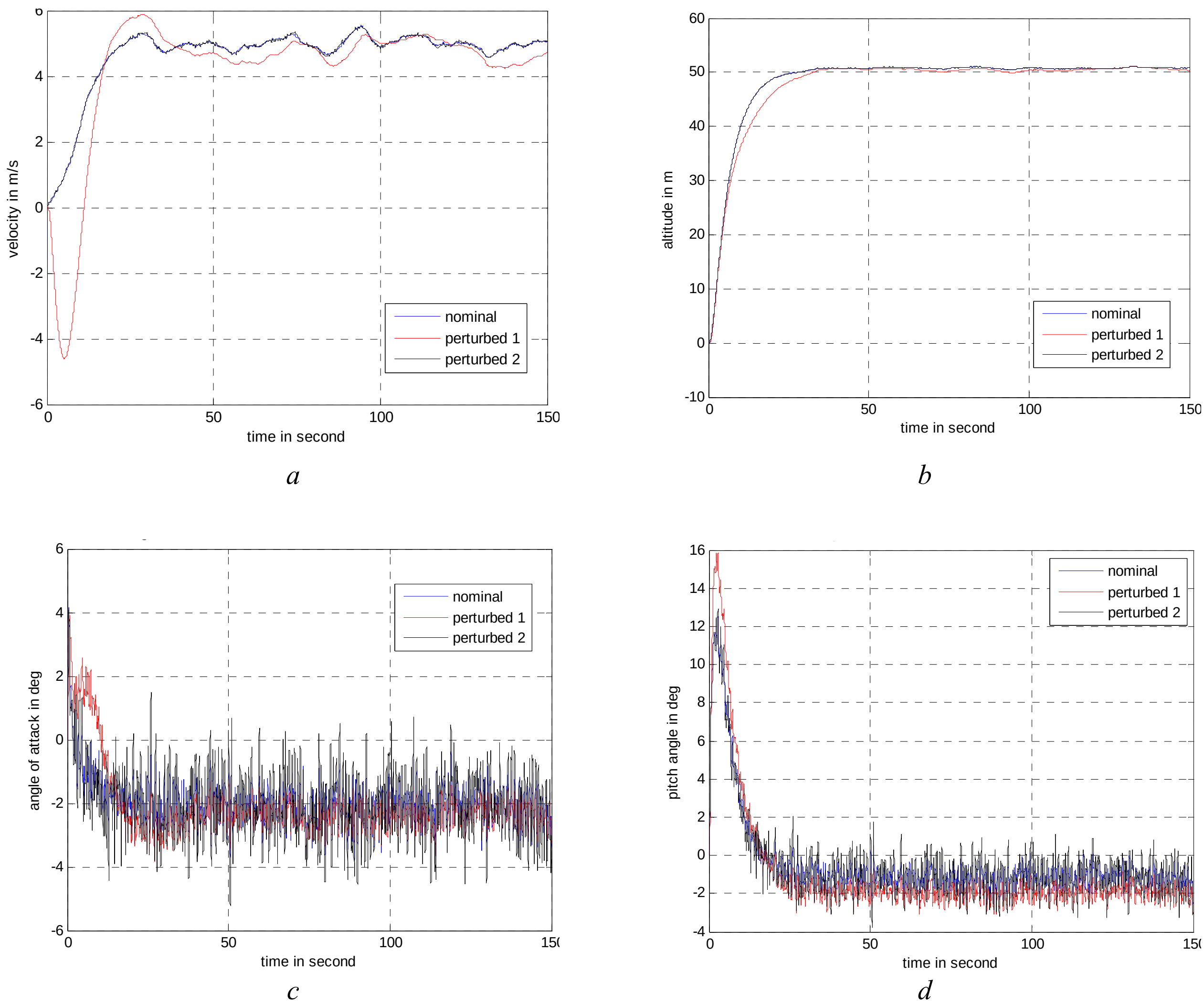

Fig. 3. longitudinal channel simulation results:
a – velocity of the UAV nominal and perturbed models;
b – altitude of the UAV nominal and perturbed models;
c– pitch angle of the UAV nominal and perturbed models;
d – angle of attack of the UAV nominal and perturbed models

Conclusion

The simulation results prove the efficiency of the proposed approach. The flight requirement was respected for the nominal as well as for the perturbed models. The maximum angles deflections are all respected $-5 < \alpha < 5; -4 < \theta < 16$, and the altitude *h* is held at the reference signal (50 m) as shown in the last figure. The velocity reference signal (5 m/s) is also tracked and is given in the first figure. The rade-off between performance and robustness is guaranteed as it is shown in the table.

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