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MODELING AND CONTROL OF SOUND RADIATION BY SIMPLY SUPPORTED AND CANTILEVER BEAM COUPLED WITH SMART MATERIAL

In this paper forced vibration of the beam due to bonded piezoelectric patch is considered. When an external excitation is applied to the beam, it starts to vibrate, and the resulting acoustic response is predicted from the analytical model, which is based on Bernoulli-Euler theory of beam vibration. Analytical research of the sound radiation by a finite elastic beam is done for criteria based on minimal total sound power level. Helmholtz equation and inhomogeneous differential equation for beam transverse motion defines the solution to this problem. Such solutions were found for boundary conditions of simply supported and cantilever beams. In order to solve the task two analytical methods were used for simply supported beam. The solutions received by Fourier transform and Green functions approach give the very similar results, thus, proving methods reliability. At the case studies the exerted voltage, phase, location and piezoelectric actuator length are varied in order to establish their influence on noise attenuation.

Розглянуто змушену вібрацію балки завдяки прикріпленому п’єзоелектрику за допомогою аналітичної моделі, що ґрунтується на теорії вібрації балки Бернуллі-Ейлера. Аналітичне дослідження звукового випромінювання пружною балкою кінцевих розмірів виконано для критерію, що оснований на мінімальному сумарному рівні звукової потужності. Задачу для граничних умов вільно обпертої та консольної балок розв’язано за рівнянням Гельмгольца та неоднорідним диференціальним рівнянням поперечного руху балки. Для розв’язання задачі для вільно обпертої балки використовувано два аналітичних методи. Доведено надійність методів за схожими результатами розв’язків задач отриманих перетворенням Фур’є та підходом функцій Гріна. Показано зміну прикладеної напруги, фази, розміщення та довжини п’єзоелектричного актуатора з метою визначення їх впливу на ослаблення шуму при параметричному дослідженні.

Introduction

With the development of aviation industry, aircraft structures are becoming larger and more flexible. Their eigenfrequencies and damping loss factors are relatively low. Due to increasing requirements to aircraft structures, the application of active control for vibration suppression becomes more and more important. The smart materials, such as piezoelectric actuators, have been used extensively as sensors and actuators in order to control the flexible structures.

Many studies have been devoted to the control of sound radiation from vibrating structures with piezoelectric actuators bonded to the surface of the structure [1]. Designing features of the control include some subsystems: piezoelectric patches, sensors and automatic control system. There has been a number publication for this problem.

In this work we use closed-form solutions of a Bernulli-Euler equation of a beam with different boundary conditions. Using them the investigation of actuator parameters influence on total sound power level (SPL) and on each mode separately was done. Classical task of elastic beam vibration in such formulation of the problem gives a possibility to indicate influence of the considered parameters on local optimal solution.

As far as the use of piezoelectric ceramic materials for structural actuation is a fairly well developed practice their parameters are investigated. This study includes the research of the actuator voltage, phase, location and length influence on the sound power level, which is produced with beam oscillations.

The research is done for two types of boundary conditions, namely simply supported (SS) and cantilever. Some similarities of influence of the parameters under research have bean found. Especially this can be seen for force and actuator value influence on sound radiation by SS and cantilever beam. Also the similarities are reported for the different modes of beam oscillations, for example similar acoustic response of SS beam is reported depending on actuator length relation to mode length. The general conformities to the law pointed in conclusions may be important for further optimization of actuator parameters.

The analytical models of beam vibration due to concentrated load have been presented in the papers [2] for both SS and cantilever beam [3]. Also an investigation was provided for different type of distributed loadings on SS [2] and cantilever beam [4]. The solutions received in the above mentioned articles can be combined with those that are obtained in present paper.

Beam vibration due to piezoelectric actuator

Consider the beam with thin layer of piezoelectric material bonded to one side. We consider that the thickness of piezoelectric strip is small compared to that of the beam. The piezoelectric strip is used as actuator by controlling the voltage U applied to electrodes. Analytically the piezoelectric actuator is represented with the model of the couple of torques acting at the actuator ends (fig. 1).

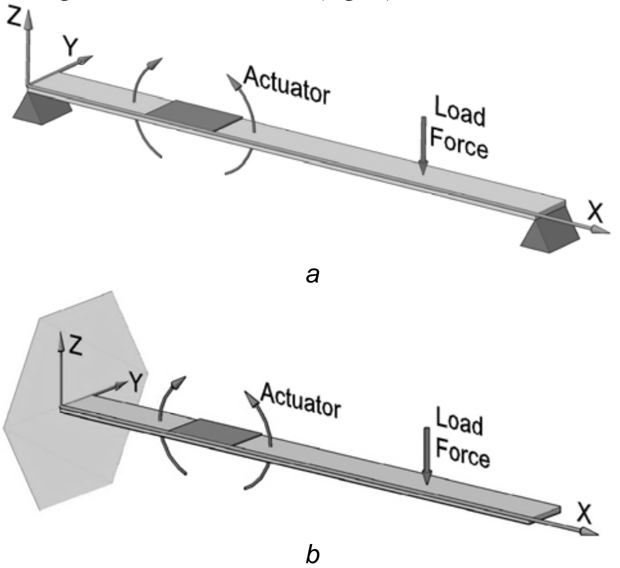


Fig. 1. The model for SS (a) and cantilever (b) beam acoustic radiation investigation

The mathematical model of elastic beam vibrations is based on Bernoulli-Euler theory of the bending and represented by following differential equation (longitudinal axis is oriented along the x axis and passing through the center of the cross section):

$$\rho_s(x)S(x)\frac{\partial^2 u(x,t)}{\partial t^2}+\frac{\partial^2}{\partial x^2}[E(x)I(x)\frac{\partial^2 u(x,t)}{\partial x^2}]=$$
$$=F(x,t)+U(x,t), \qquad (1)$$

where $\rho_s(x)$ is a beam material density;

$F(x,t)$ is a concentrated load;

$U(x,t)$ is piezoelectric actuator excitation;

$I(x)$ is a moment of inertia for the cross section $S(x)$ of the beam;

$E(x)$ is an Young's modulus, which is defined as $E=E'(1+i\eta)$. In this model of thin beam transverse vibration the beam width b is supposed to be much smaller of its length L: $\frac{b}{L}<<1$.

For harmonic vibration the transverse motion $u(x,t)$ and external excitations $F(x,t)$, $U(x,t)$ have the next form [5]:

$$u(x,t)=u(x)\exp(-i\omega t);$$
$$F(x,t)=F(x)\exp(-i\omega t);$$
$$U(x,t)=U(x)\exp(-i\omega t).$$

In the case if $\rho_s(x)$, $I(x)$, $S(x)$, $E(x)$ are constant the equation (1) can be written in form of biharmonic equation:

$$\frac{d^2u}{dx^2}-k_f^4u=F(x)+U(x), \qquad (2)$$

$$F(x)=\sum_{j=1}^{M}F_j\exp(i\varphi_j)\delta(x-x_j);$$

$$U(x)=\sum_{m=1}^{M}\frac{bh_aE_ad_{31}U_m}{EI}\times$$
$$\times\left[\delta'(x-x_{1m})-\delta'(x-x_{2m})\right]\exp(i\phi_m), \qquad (3)$$

$$k_f^4=\frac{\omega^2\rho_sS}{EI},$$

where ω is an angular frequency;

F_j,φ_j are the amplitude and phase of j-th external point force respectively;

$\delta(x)$, $\delta'(x)$ is a Dirac function and its derivative with respect to x, x_j are location of force;

h_a, E_a, d_{31} are the distance from midplane of actuator to beam axis, Young's modulus of the actuator, the piezoelectric strain constant, respectively;

x_{1m},x_{2m} are the coordinates of the two ends of m-th actuator ($x_{2m}>x_{1m}$);

ϕ_m is phase of actuator.

Equation (3) assumes that the actuators are of the same type and have the same thickness h_a.

Solution of equation (2) for boundary conditions of SS beam is

$$u(x)=-\frac{2}{\rho_sSL}\sum_{n=1}^{\infty}\left\{\sum_{j=1}^{J}F_j\exp(i\varphi_j)\sin\frac{n\pi x_j}{L}-\right.$$
$$-\frac{bh_aE_ad_{31}n\pi}{L}\sum_{m=1}^{M}U_m\exp(i\phi_m)\times$$
$$\left.\times\left[\cos\frac{n\pi x_{1m}}{L}-\cos\frac{n\pi(x_{1m}+L_{am})}{L}\right]\right\}\frac{\sin\frac{n\pi x}{L}}{\omega^2-\omega_n^2}, \qquad (4)$$

where L_{am} is length of m-th actuator;

ω_n are the eigenfrequencies of SS beam vibration.

With the Green functions approach the universal solution to equation (2) can be received.

This solution is applicable to both SS and cantilever beam. Transverse motion of the beam for few forces and actuators may be computed as

$$u(x)=\left\{\frac{1}{EI}\sum_{j=1}^{J}F_j\exp(i\phi_j)G(x,x_j)-\right.$$
$$-bh_aE_ad_{31}\sum_{m=1}^{M}U_m\exp(i\phi_m)\times$$
$$\left.\times\left[\frac{\partial G(x,x_{1m})}{\partial x_{1m}}-\frac{\partial G(x,x_{2m})}{\partial x_{2m}}\right]\right\}; \qquad (5)$$
$$x_{1m}=x_{cm}-\frac{L_{am}}{2};$$
$$x_{2m}=x_{cm}+\frac{L_{am}}{2},$$

where $G(x,x_i)$ is piecewise function;

x_{cm} is the location coordinate of actuator centre.

For cantilever beam it's given in paper [3]. For SS beam $G(x,x_i)$ has the next form:

$$G_-(x,x_i)=\frac{L^3}{4k_b^3\sin(k_b)\sinh(k_b)}\times$$
$$\times\left\{\left[\cos\left(k_b\frac{x+x_i-L}{L}\right)-\cos\left(k_b\frac{x-x_i+L}{L}\right)\right]\sinh(k_b)+\right.$$
$$\left.+\left[-\cosh\left(k_b\frac{x-x_i+L}{L}\right)+\cosh\left(k_b\frac{x+x_i-L}{L}\right)\right]\sin(k_b)\right\};$$
$$G_+(x,x_i)=\frac{L^3}{4k_b^3\sin(k_b)\sinh(k_b)}\times$$
$$\times\left\{\left[\cos\left(k_b\frac{x+x_i-L}{L}\right)-\cos\left(k_b\frac{x-x_i-L}{L}\right)\right]\sinh(k_b)+\right.$$
$$\left.+\left[\cosh\left(k_b\frac{x+x_i-L}{L}\right)-\cosh\left(k_b\frac{x-x_i-L}{L}\right)\right]\sin(k_b)\right\},$$

where $k_b=Lk_f$.

The most popular piezoelectric materials are Lead-Zirconate-Titanate (PZT) which is ceramic, and Polyvinylidene Fluoride (PVDF) which is a polymer.

For the piezoelectric actuator phase conventions are accepted. Whenever the force is acting downward $\varphi_j=0$, and when actuator enlarges $\varphi_m=0$. The actuator to which the negative voltage is applied acts in the same way as actuator having the same, but positive, voltage and opposite in phase. Thus we automatically subtract π radians from the actuator located on upper surface of the beam.

Sound power level computation procedure is the same as described in articles [2; 3].

Initial data for investigation

The above mentioned solutions (4), (5) were used for the case study of actuators location, their voltage, length and phase influence on sound radiation characteristics. Hence, SPL is the function of these parameters and frequency. To find piezoelectric patch parameters, which lead to the minimum acoustic radiation, the SPL was adaptively integrated in the frequency range under investigation and the total SPL was defined for every set of parameters. The minimum total SPL defines the best set of piezoelectric actuator parameters.

The dimensions and material properties of the beam considered in this study is summarized in tab. 1.

Table 1

Properties of a beam under consideration

Property	Value of parameters
Mass density ρ_s, kg/m^3	7800
Length L, m	0.61
Width b, m	0.051
Thickness h, m	0.00635
Young's modulus E', Pa	2.1×10^{11}
Damping loss factor η	0.0001

The properties of PZT actuator used for computations are shown in tab. 2.

Table 2

Properties of piezoelectric PZT actuator

Property	Value of parameters
Young's modulus E_a, Pa	5×10^{10}
Piezoelectric constant d_{31}, m/V	-1.5×10^{-10}

Actuator parameters influence on vibration and noise of simply supported beam

With the employing adaptive algorithm for obtaining Total SPL the greater values of SPL is obtained due to more accurate integration of mode peaks. Fig. 2 shows the location of load and actuator for the case of variable actuator voltage. A small triangle marks the actuator centre x_c.

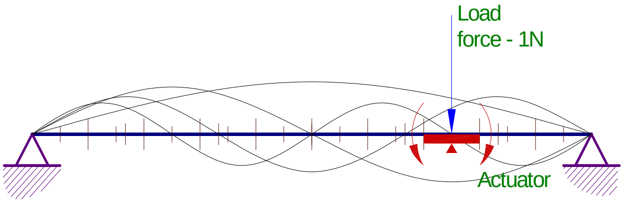
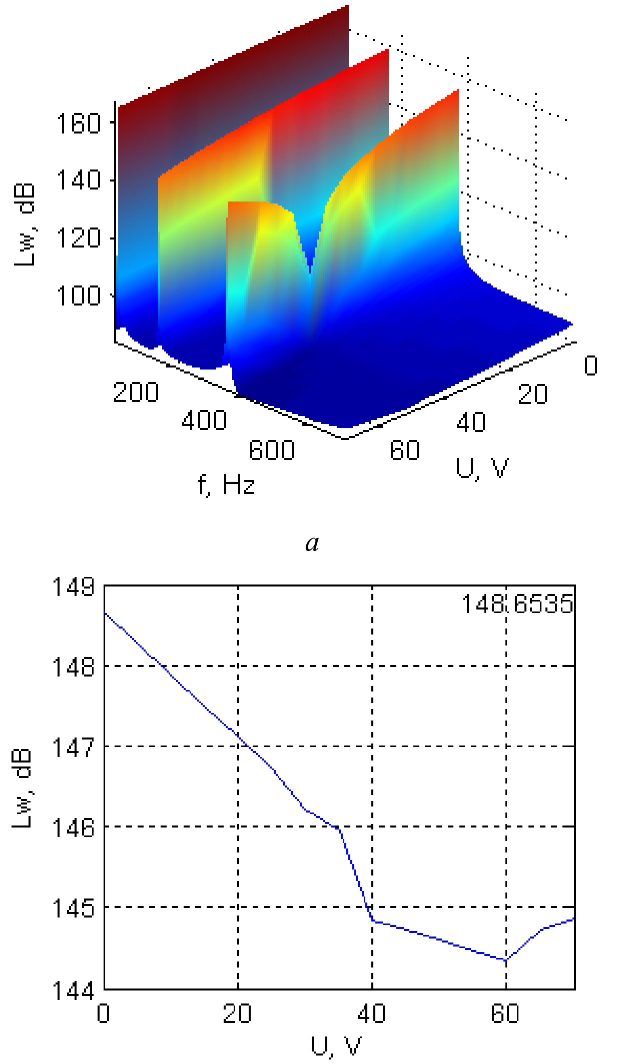


Fig. 2. Scheme of SS beam loading with load and actuator having variable voltage

The results are shown on fig. 3. The digits in the top-right corner of figures indicate total SPL for the beam loaded only with the single load without any actuators.



a

b

Fig. 3. Sound spectrum (*a*) and total SPL (*b*) of SS beam for the piezoelectric actuator with variable location voltage

As far as both load and actuator are located at fourth mode node the SPL on this mode frequency occurs due to influence of other modes of SS beam oscillations. The minimum mainly is produced with 50 V minimum of the third mode and decrease of the second mode shifts total SPL to 60 V. This is a good example of overloading on third mode. Comparing with the case of load compensation with the point force the complete SPL reduction with piezoelectric actuator isn't possible due to the different nature of excitations.

For the case depicted on fig. 4 the actuator phase is investigated.

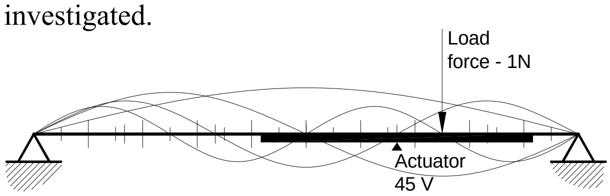
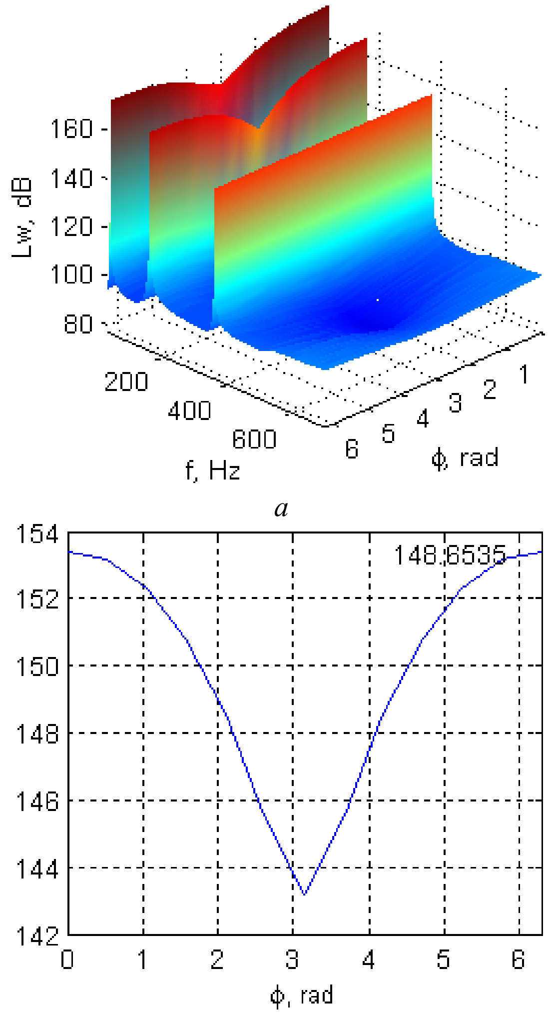


Fig. 4. Scheme of SS beam loading with load and actuator located at third mode node

All modes of oscillations as it can be seen from fig. 5 have a minimum corresponding to π radian. This is the case of actuator shortening when load is acting downward. No influence of actuator on third mode is explained with actuator location at its node. The absence of acoustic radiation on the fourth mode is explained by the equality of actuator length to the wavelength of the fourth mode shape and load location at the node of this mode.



a

b

Fig. 5. Sound spectrum (*a*) and total SPL (*b*) of SS beam for the piezoelectric actuator with variable phase

The research dedicated to the actuator location is presented on fig. 6 and 7.

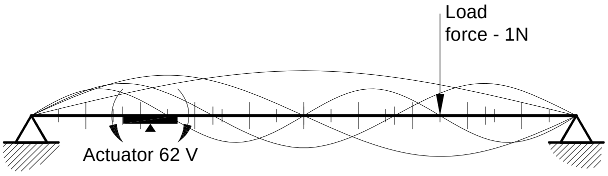
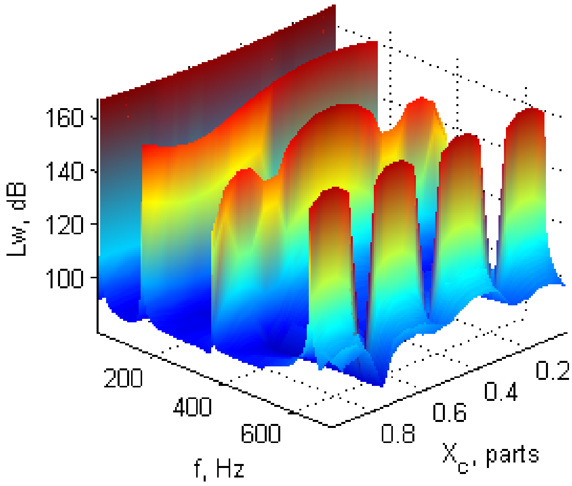


Fig. 6. Scheme of SS beam loading with load and actuator having a variable location



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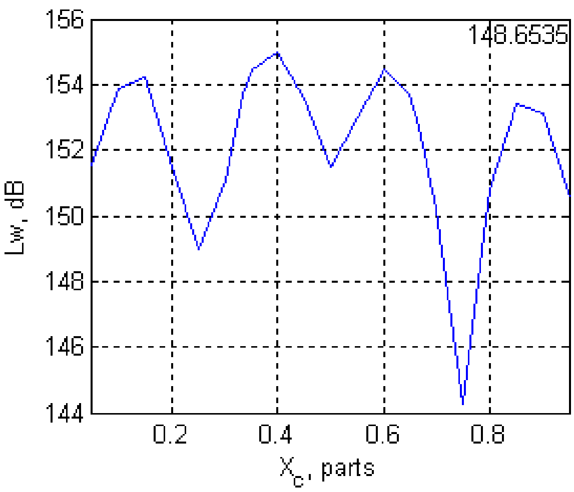


b

Fig. 7. Sound spectrum (*a*) and total SPL (*b*) of SS beam for the piezoelectric actuator with variable location

Load phase φ the actuator voltage and length is too small to influence the first mode. The second mode possesses the increase of SPL at the half wave of the second mode where there is no force, and reduction of SPL for actuator located in the same half wave with force. The minima of the third mode are close to the nodes. This effect is usual for the case of the mode loading with excessive compensating excitation.

Because in this way actuator influence is reduced. The fourth mode shape on fig. 7, *a* is produced with the actuator only, as far as the load is located at the mode node. A lot of local minima thus appear on total SPL graph. The global minimum is 0,75 mainly due to the second mode. This graph clearly illustrates that actuator influence on each mode should be analysed separately. Another important parameter is actuator length. The case corresponding to this research is shown on fig. 8 SPL graph (fig. 9) indicates that with the increase of actuator length the influence on the first mode of oscillations increases.

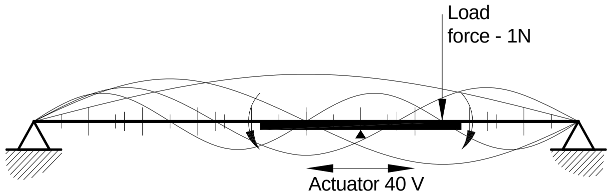
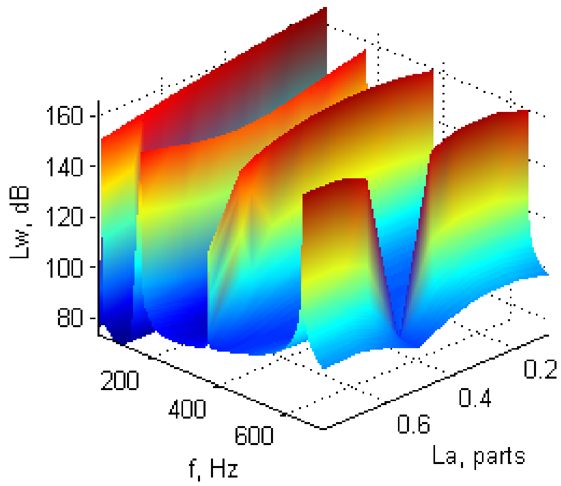


Fig. 8. Scheme of SS beam loading with load and actuator of different lengths



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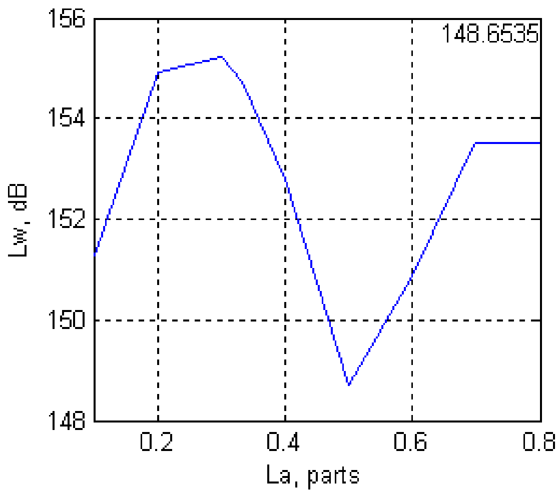


b

Fig. 9. Sound spectrum (*a*) and total SPL (*b*) of SS beam for the piezoelectric actuator with variable length

The actuator length is relative (with respect to unit beam length). The minimum on the second mode correspond to the actuator length equal to the half of the beam, i.e. to the half of wavelength of the second mode. The similar situation is observed at fourth mode for actuator length 0,25.

The influence of piezoelectric actuator parameters on vibration and noise of cantilever beam

The next set of cases shown on fig. 10, illustrate the significant property of piezoelectric actuators.

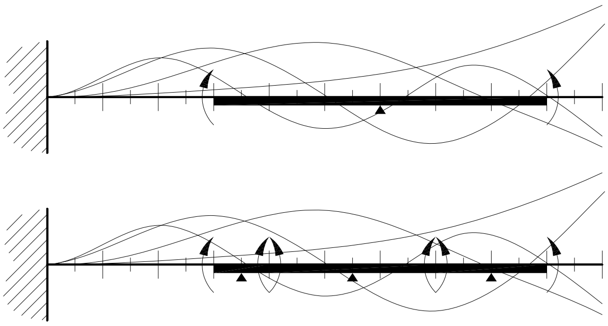


Fig. 10. Scheme of actuator substitutions

The large actuator can be substituted with a set of smaller ones, all of which have the same voltage and phase as the original actuator. That is why for all cases depicted on fig. 10 SPL is the same (fig. 11).

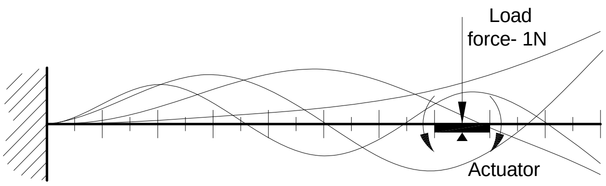


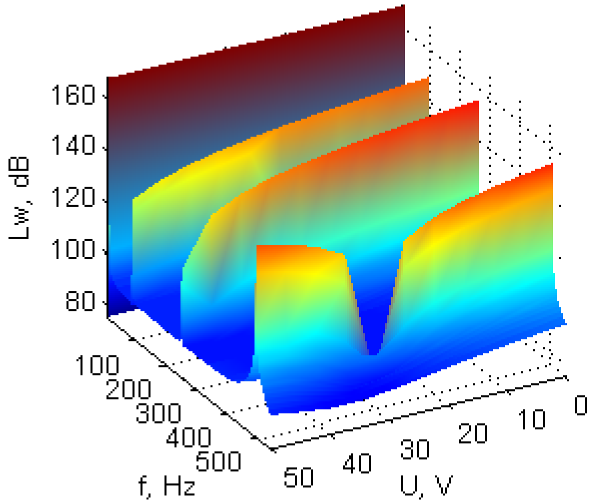
Fig. 11. Scheme of cantilever loading with load and actuator having the same location

This property also can be used for SS beam. It enables to analyze large actuator by parts. For instance, by dividing the actuator on parts in different half waves of the mode we can separate actuators with opposite influences on the mode.

In the study described with fig. 11 the influence of actuator voltage on oscillations of cantilever beam is investigated. For the considered situation load phase $\varphi_j = 0$ and actuator phase $\varphi_m = \pi$, therefore $\Delta\varphi=\pi$.

The results shown on fig. 12 indicate that no influence can be observed on the first mode of oscillations. The other modes decrease up to some voltage, which is different for each mode. This can clearly be seen on the fourth mode of oscillations, where the minimum corresponds to 30 V.

After the integration of the received spectrum we get the minimum corresponding to 35 V. thus noise level reduction is 7 dB.



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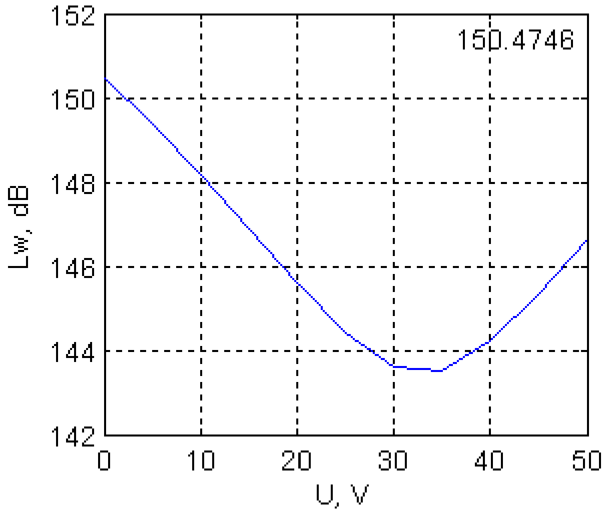


b

Fig. 12. Sound spectrum (*a*) and total SPL (*b*) for the piezoelectric actuator with variable voltage

The next investigated case on fig. 13 describes the influence of actuator location on cantilever beam oscillations.

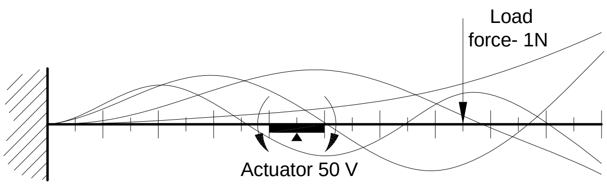
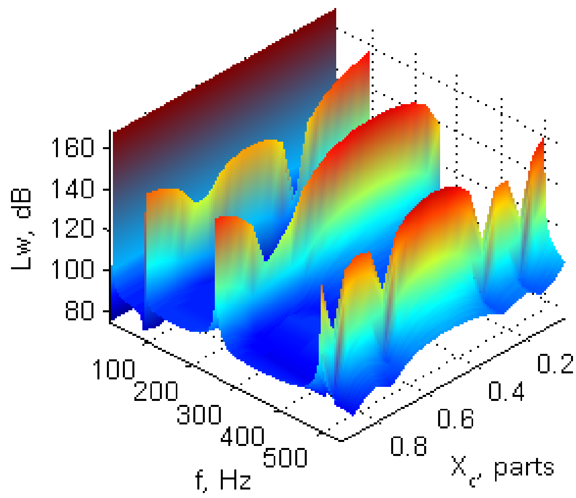


Fig. 13. Scheme of cantilever beam loading with load and actuator having different position

The results of research, which are shown on fig. 14, indicate that there is no significant influence on the first mode. The deep minimum is observed on the second mode corresponding to 0,35 of relative beam length. The third mode minimum is at 0,75. And the fourth mode minima are 0,15; 0,3, 0,7 and 0,9. This leads to the few local minima and one global corresponding to 0,75.



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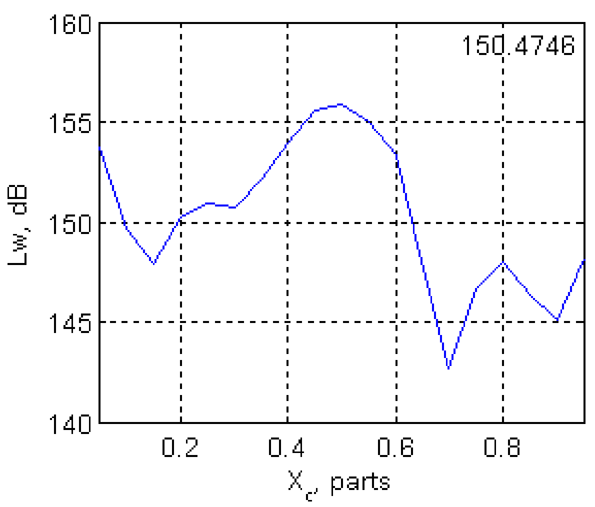


b

Fig. 14. Sound spectrum (*a*) and total SPL (*b*) of cantilever beam for the piezoelectric actuator with variable location

The phase difference between the disturbance and compensating actuator is one of the parameters we should choose to perform the compensation correctly. For the case shown on fig. 15 the actuator phase is varied within the range from 0 to 2π radians.

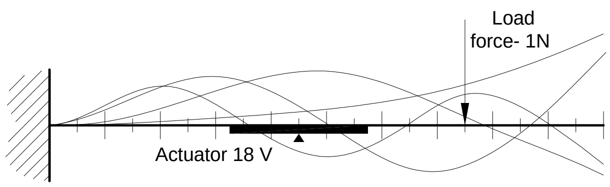
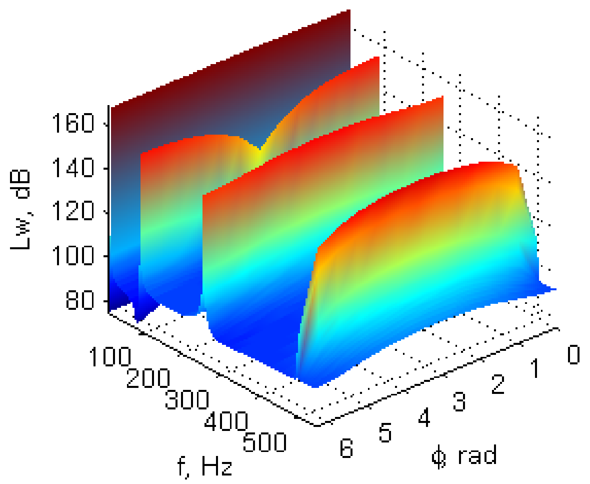


Fig. 15. Scheme of load and actuator location on cantilever beam (the case of various phases)

Minimum on the total SPL graph (fig. 16) corresponds to 0 radians. This is defined by the third and fourth mode of cantilever beam vibration.



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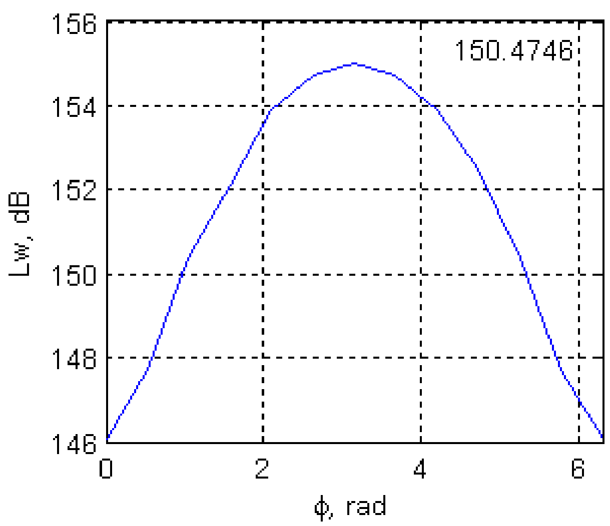


b

Fig. 16. Sound spectrum (*a*) and total SPL (*b*) for the piezoelectric actuator with variable phase

Fig. 17 shows the actuator of variable length applied to cantilever beam. Phase of actuator is π rad.

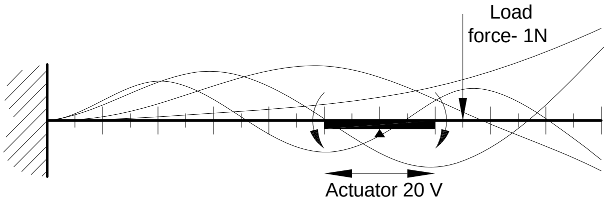
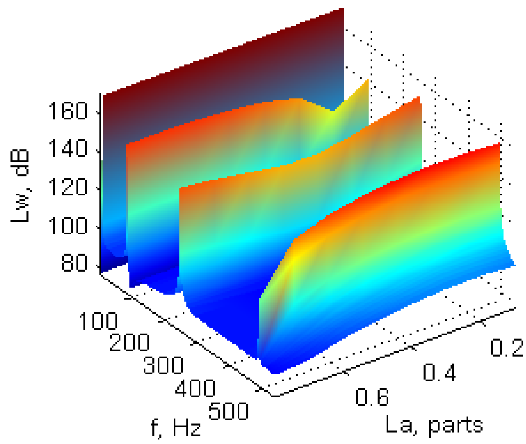


Fig. 17. Scheme of cantilever beam loading with load and actuator having different length

From the fig. 18 we can see that there is no influence on the first mode. Minimum for the second mode is 0,2 of relative beam length. This can be explained by overloading of the second mode with the actuator at lengths greater than 0,2. The minimum of the third mode correspond to 0,4 of relative beam length. The fourth mode decreases at lengths 0,7-0,8.



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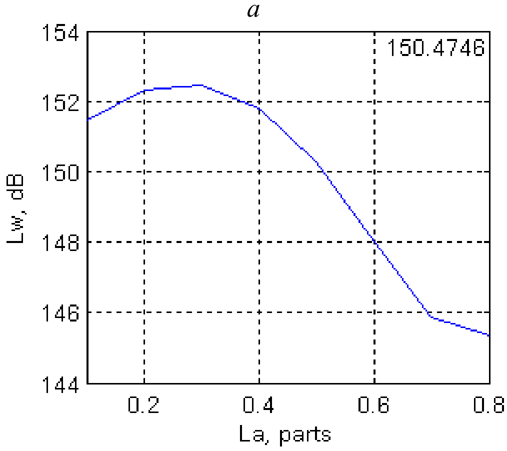

b

Fig. 18. Sound spectrum (*a*) and total SPL (*b*) for the piezoelectric actuator with variable length

Conclusions

As far as the control of acoustic radiation is defined by actuator parameters, their influence was investigated in the present paper. They include actuator voltage, location, length and phase.

The basic results of investigation of sound radiation by beams with piezoelectric actuator are the following:

1. The analytical method of sound radiation evaluation was elaborated for the beams with different boundary conditions excited by concentrated load and actuator.
2. For solution of tasks two methods where used: Fourier transform and Green function approach:

a) advantages of Fourier transform are:

– the solution is received in the form of mode superposition, which allows analyzing analytically each mode separately;

– the solution obtained with Fourier transform requires less computation time compared to that which is obtained by the method of Green functions;

b) advantages of Green functions method are:

– the received solution is more precise compared to that which is obtained by Fourier transform (the difference in total SPL is 10^{-4} dB for the force);

– the Green functions method is universal: it allows the usage of the same formulas for the beams with different boundary conditions.

3. The case studies show that, when actuator centre is located at the inflection points the influence of actuator on corresponding mode equals to zero.
4. The phase difference between load and actuator voltage required for load compensation is different for different half-waves of the mode shape. With $\Delta\varphi=\pi$ for the actuator location in the same half-wave as the load location.
5. If the actuator phase was chosen correct then SPL decreases with the actuator value increase up to some limiting value. This value is different for different modes of plate oscillations.
6. If the actuator phase was chosen wrong then SPL increases with the actuator value increase without any limitations.
7. The SPL compensation on the separate mode caused by the change of actuator voltage is limited due to the actuator influence on the other modes and due to SPL decay from these modes.
8. For the simplest case of one load and one actuator the optimal actuator location is always found in the position of load.
9. The results of investigation exhibits that with the use of actuator pairs (or set of actuators that have the same voltage) the significant reduction of sound power level on certain mode can be get, with almost no influence on other modes.

References

1. *Preumont A.* Vibration control of active structures. – Kluwer Academic Publishers, Dordrecht, 2003.
2. *Parametric* investigation of acoustic radiation by a beam under load and actuator forces / Alexander Zaporozhets, Vadim Tokarev, Werner Hufenbach et al. // Вісн. НАУ. − 2005. − № 4. − С. 122–133.
3. *Investigation* of sound radiation by cantilever beam / Olexandr Zaporozhets, Vadim Tokarev, Werner Hufenbach et al. // Вісн. НАУ. − 2006. − № 1. − С. 169–172.
4. *Makarenko V.* Research of the modal response of simply supported and cantilever beams subjected to point and distributed harmonic excitations // Proc. of the NAU. − 2006. − №3. − P. 101–107.
5. *Junger M.G., Feit A.* Sound, Structures and their Interaction. – The MIT Press, Cambridge, 1986.
6. *Modern* methods in analytical acoustics / A.G. Crighton et al. – Springer-Verlag, London, 1992.
7. *Modeling* of sound radiation by a beam / O. I. Zaporozhets, V. I. Tokarev, Werner Hufenbach et al. // Вісн. НАУ. − 2005. − №4. − С. 160–163.
8. *Graff K.F.* Wave motion in elastic solids, Clarendon Press, Oxford, 1975.

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