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NEW FORMULAS FOR THE CRITICAL FORCES OF CYLINDRICAL SHELLS CALCULATION

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Abstract

Purpose: To show that within the framework of the linear theory, it was possible to obtain formulas for axial critical loads, the calculation results for which are in good agreement with experimental data. **Method:** An energetic solution method using the general linear theory of thin-walled shells. **Results:** New formulas for the critical loads of cylindrical shells are obtained. The analysis of the obtained results is carried out. Recommendations for their use are given. **Discussion:** Difficulties in theoretical determination of the critical loads of cylindrical shells under axial compression, which are close to the experimental data, forced researchers to seek empirical solutions. Many empirical relationships have been obtained that give different results and describe known experiments. However, there remains a need to theoretically find formulas that allow calculating the critical loads of cylindrical shells of any geometric parameters. Such formulas have been obtained. A comparison of the critical loads calculated using these formulas with empirical and experimental critical loads is carried out. The differences between them are minor.

Keywords: bending, critical load, displacement, stability, experiment, energy

1. Introduction

Loss of stability of a structure as a whole, or of its individual elements (shells, plates, rods) is one of the reasons for the exhaustion of the bearing capacity of engineering structures. However, there are no exact formulas for the critical loads of plates and especially shells. This article shows that such formulas can be obtained within the framework of linear theory.

2. Analysis of recent research and publications

The first results of studying the stability of structural elements were obtained by L. Euler [1], Brian [2], Lorentz [3] and S.P. Tymoshenko [4]. Leonard Euler (1744) was the author of the theoretical formulation of the problem of the stability of centrally compressed rods. When solving the problem, he used a linear approach and a static Euler criterion. According to this criterion, the critical load is calculated as the smallest load at which, simultaneously with the initial form of equilibrium, an adjacent form of equilibrium is possible, infinitely close to it. Using this approach, he obtained the famous Euler formula:

$$P = EI \frac{\pi^2}{l^2},$$

where P – compressive force; E – Young's modulus; I – moment of inertia of the cross-section of the rod; l – length of the rod.

For a long time (150 years), the correctness of this approach was in doubt.

The carefully carried out experiments of I. Bauschinger (1887), M. Considere (1891), L. Tetmayer (1890, 1896) showed good agreement between theoretical and experimental data and put an end to the era of doubts in the theory and Euler's formula. In his article "Some theoretical problems of elastic stability" (1910), S. P. Timoshenko gives an energy derivation of the Euler formula for the case when the distance between the ends of the rod does not change during the loss of stability and, as a result, the longitudinal compressive force during buckling slightly decreases. The critical state is found from the condition of equality of the decrease in the compression energy upon buckling of the potential bending energy.

In case of loss of stability of the rod, two cases are possible:

- the ends of the bar move along the axis;
- the distance between the ends of the bar does not change.

In both cases, the stability criterion is the same [5].

$$\frac{1}{2} P \int_0^l \left(\frac{dw}{dx} \right)^2 dx = \frac{P^2}{2EI} \int_0^l w^2 dx \quad , \quad (1)$$

where w – rod deflection.

Assuming that when the rod buckles, the compressive force P does not change, and setting the displacement w satisfying the boundary conditions determines the critical force P_* . The P_* value is the same for both cases, since the stability criterion is the same. If both ends of the bar are freely supported, the displacement is given in the form:

$$w = \sum_1^{\infty} w \sin \frac{m\pi x}{l} dx \quad (2)$$

and the Euler formula is obtained.

Let's look at the second case in more detail. When, during bending, the distance between the ends of the bar does not change, as a result of this: the axis of the bar lengthens, the compression force decreases, the compression energy decreases. The critical force is determined from the condition of equality of the decrease in the compression energy during bending of the potential bending energy, i.e. from equality (1). Assuming that the compressive force P decreases slightly, it is assumed to remain constant. Using representation (2), we find P . Using only the first term of the series, we get the smallest value of P_* corresponding to the critical force, which coincides with Euler's formula.

Let us solve this problem taking into account the decrease in the compressive force during the buckling of the rod. The decrease in the compression energy during bending of the rod is equal to the work of the compressive force on the approach of the ends of the rod due to the curvature of the axis. Taking into account the fact that the static problem is considered and that $P = P_* - P_1$, where P_1 is the force that appears during the loss of stability, which changes from 0 to P_1 during buckling, the work of the force P is equal to:

$$\Delta A = \frac{1}{2} \left(P_* - \frac{1}{2} P_1 \right) \int_0^l \left(\frac{dw}{dx} \right)^2 dx$$

The potential bending energy is equal to:

$$\Delta V = \frac{EI}{2} \int_0^l \left(\frac{d^2w}{dx^2} \right)^2 dx.$$

The force P_1 is numerically equal to the internal forces T_1 at the ends of the bar. Taking

$w = f \sin \frac{m\pi x}{l}$, from the equality $\Delta A = \Delta V$ we

find $P_* = E \left(I + \frac{1}{4} f^2 F \right) \frac{\pi^2}{l^2}$. Considering that the displacements are small (f is a small value) and that I is expressed in units of length to the fourth power, and F – in units of length in the square power, then $\frac{1}{4} f^2 F$ is infinitesimal in comparison with I and can be neglected. As a result, we get the Euler formula.

Consequently, Leonard Euler and S.P. Timoshenko were right in the assumption $P = \text{const}$. However, this only applies to the rod.

It is believed that the effectiveness of the static Euler criterion is indisputable. This approach is widely used in solving problems of the stability of rods, thin-walled plates, and shells. However, the experimental critical loads of a closed circular cylindrical shell compressed along the generatrix by forces N uniformly distributed along the arc edges are often several times less than the theoretical ones obtained using the static Euler criterion. At the same time, a circular shell, compressed along the generatrix, is a kind of standard that serves to compare theoretical and experimental data and to test various approaches in the theory of shells stability. This fact casts doubt on the fruitfulness of the static Euler criterion. The correctness of the statement that at a critical load simultaneously with the initial form of equilibrium, an infinitely close adjacent form of equilibrium is statically possible is doubtful. The shells behave very differently. This is shown in works [6,7,8] and will be shown in this article.

J. Brian was the first to use the energy method to solve stability problems and obtained a formula for the critical forces of a hinge plate compressed in one direction:

$$N_* = k \frac{\pi^2}{b^2} D$$

$$k = \left(\frac{mb}{a} + \frac{a}{mb} \right)^2 ; \quad D = \frac{Eh^3}{12(1-\nu^2)} ;$$

where ν – Poisson's ratio; a – plate length; b – plate width.

Lorenz and S.P. Timoshenko in a linear formulation based on the static Euler criterion considered the stability of a hinged circular cylindrical shell under axial compression. Rudolf Lorentz (1908) found the critical compressive stress of a thin cylindrical shell, but neglected the

transverse compressibility of the material. This was taken into account by S.P. Timoshenko (1910), who, within the framework of the Kirchhoff - Love hypotheses, obtained a formula for critical stress bearing his name:

$$\sigma_* = \frac{Eh}{R\sqrt{3(1-\nu^2)}} \quad \text{or} \quad \sigma_* = 0,605 \frac{Eh}{R} \quad \text{at } \nu = 0,3,$$

where R and h are the radius and thickness of the shell. The critical axial force is calculated by the formula:

$$N_*^c = \frac{Eh^2}{R\sqrt{3(1-\nu^2)}} \quad (3)$$

It is called the classical or upper critical compression force. This effort is the most famous and most revealing in terms of the discrepancy between theoretical and experimental values. The critical loads observed in experiments are much less than the upper critical loads.

All further development of the theory of shell stability was aimed at identifying the reasons for this discrepancy. Various directions of research developed, but two directions aroused the greatest interest.

The first direction is associated with the use of the nonlinear theory of shells and recommendations for assessing the stability of shells for the lower critical load. These recommendations turned out to be wrong.

The second direction is connected with the study of the influence of the initial imperfections of the shell on the value of the upper critical load. Unfortunately, this line of research did not bring positive results either. An analysis of the experimental data shows that small deviations of the shell geometry from the ideal shape reduce the critical load, but not by several times, which is often observed in experiments.

The most complete and detailed first, second and other directions of the study of the stability of shells, plates and rods are presented in [5,9,10,11]. No new work has been found to help solve the problem of the stability of a cylindrical shell under axial compression.

The difficulties that have arisen forced the researchers to look for empirical dependences for calculating critical loads on the basis of experimental data. There are many such dependencies. We will dwell on some of them below when analyzing the results of experimental studies.

A large number of research works did not solve

the problem. It is necessary to continue looking for the cause of the large discrepancies between the calculated and experimental data.

In [6,7,8], a new approach to solving the problem of stability of a hinged supported cylinder is proposed, which differs from the classical approach.

The classical approach assumes that the transition from the initial to the curved form of equilibrium occurs without changing the critical compression force N_* . This means that the length of the generatrices of the shell L remains constant. In this case, the edges of the shell receive some displacement in the axial direction and the force $N_* = const$ performs additional work $\Delta A \neq 0$. Due to this work, additional energy of the shell appears $\Delta V \neq 0$, but the potential of the shell - external load system does not change, i.e. $\Delta U = \Delta V - \Delta A = 0$.

The proposed approach describes the process of loss of stability in a completely different way. When buckling, the edges of the shell remain in place because the convergence of the edges due to radial movements is compensated by the elongation of the generatrices of the shell. The lengthening of the generatrices occurs because the compression energy decreases. In this case, the compressive forces also decrease and become equal to $N_* - N_l$. There is a redistribution of the compression energy accumulated in the subcritical state. Compression energy decreases and bending and shear energies appear. In this case, the potential energy of the shell does not change, i.e. $\Delta V = 0$. Since there are no end displacements, additional work of forces $N_* - N_l$, $\Delta A = 0$. System potential $U = const$, because $\Delta U = \Delta V - \Delta A = 0$. To determine the critical forces, the condition is used $\delta \Delta U = 0$. Thus, the general theorem of mechanics, the beginning of possible displacements of G. Lagrange (1788) and the Lejeune-Dirichlet principle (1846), which underlie the energy method, are fulfilled. Based on this, we conclude: all the prerequisites for using the proposed approach in determining the critical loads of thin-walled structures are available.

In [11] the conclusion was made: "It is now quite clear that good agreement between experiment and calculation can be achieved within the framework of linear theory. In this case, the classical linear theory should be revised taking into account a number of factors."

In works [6,7,8] it is shown that the most important factors are: taking into account changes in the external load during buckling of the shell and boundary conditions. The obtained theoretical values of the axial

critical load of a cylindrical shells are in good agreement with the experimental data given in [11].

In [6] the main results of solving the problem of stability of a hinged cylindrical shell under axial compression are briefly presented, taking into account changes in the external load during buckling of the shell. This paper provides a complete solution to this problem. In addition, the formula for the critical load of a cylindrical shell obtained on the basis of the theory of shallow shells is given. The analysis of the obtained results is carried out on the basis of their comparison with the classical solution and experimental data. A general conclusion is made on the stability of cylindrical shells under axial compression and the prospects for using the proposed approach in studying the stability of plates and shells.

3. Purpose of work

Demonstrate that within the framework of the linear theory it was possible to obtain formulas for the axial critical loads, the calculation results for which are in good agreement with the experimental data.

4. Solution method

The energy solution method using the general linear theory of thin-walled shells.

The accuracy of calculating critical loads by this method depends on the accuracy of the description of the behavior system the shell – external force during buckling and on the successful choice of the buckling shape. The selected shape of the bulging surface must strictly observe the boundary conditions and, if possible, correspond to the real form of buckling, since the accepted expression for the deflection imposes additional constraints on the elastic surface of the shell, which leads to an overestimation of the values of the found critical values of the external force.

5. Solution of the problem

Cylindrical shell of length L , radius R , with wall thickness h , loaded along the edges by uniformly distributed compressive forces N (Fig. 1).

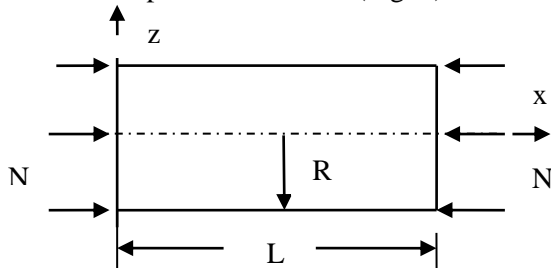


Fig. 1. Cylindrical shell

Initial prerequisites: the shell is geometrically perfect, ideally elastic, the subcritical state has no moment, the edges freely rest on the diaphragms, rigid in their plane and flexible out of their plane.

In this case, the boundary conditions are as follows:

$$u = v = w = \frac{\partial^2 w}{\partial x^2} = 0$$

Here u , v , w are the displacements of the points of the middle surface of the shell in the direction of the coordinates x , y , z .

The displacements v and w corresponding to the boundary conditions are given in the form:

$$v = f_2 \sin \frac{m\pi x}{L} \sin \frac{ny}{R};$$

$$w = f_3 \sin \frac{m\pi x}{L} \cos \frac{ny}{R}, \quad (4)$$

where m is the number of half-waves along the length of the shell; n is the number of waves in the circumferential direction; f_2 , f_3 – amplitudes of displacements in the direction of the x and y axes.

We find the displacement u as follows. Let the sum of the elongation of the median surface of a single shell element in the axial direction due to stretching and convergence of its opposite faces during bending is equal to some function $f_1(x, y)$, i.e.

$$\frac{\partial u}{\partial x} + \frac{1}{2} \left(\frac{\partial w}{\partial x} \right)^2 = f_1(x, y) \quad (5)$$

By integrating expression (5) by parts and fulfilling the condition $u(0)=u(L)=0$ for $u(x, y) \neq 0$ we find:

$$u = -\frac{1}{4} f_3^2 \frac{m\pi}{L} \sin \frac{2m\pi x}{L} \cos^2 \frac{ny}{R} \quad (6)$$

Change in the potential energy of deformation of the shell:

$$\Delta V = \frac{Eh}{2(1-\nu^2)} \int_0^{2\pi R} \int_0^L \left[\begin{aligned} &\varepsilon_1^2 + 2\nu\varepsilon_1\varepsilon_2 + \varepsilon_2^2 + \\ &\frac{1-\nu}{2} \varepsilon_{12}^2 + \frac{h^2}{R^2} [\chi_1^2 + \\ &2\nu\chi_1\chi_2 + \chi_2^2 + \\ &2(1-\nu)\chi_{12}^2 \end{aligned} \right] dx dy, \quad (7)$$

where

$$\varepsilon_1 = \frac{\partial u}{\partial x}; \quad \varepsilon_2 = \frac{\partial v}{\partial y} + \frac{w}{R}; \quad \varepsilon_{12} = \frac{\partial v}{\partial x} + \frac{\partial u}{\partial y};$$

$$\chi_1 = -\frac{\partial^2 w}{\partial x^2}; \quad \chi_2 = -\frac{\partial^2 w}{\partial y^2} + \frac{1}{R} \frac{\partial v}{\partial y};$$

$$\chi_{12} = -\frac{\partial^2 w}{\partial x \partial y} + \frac{1}{R} \frac{\partial v}{\partial x}.$$

From (7) we obtain:

$$\Delta V = \frac{Eh}{1-\nu^2} \left[\frac{1}{16} f_3^4 \frac{\lambda^2}{R^2} \left(3\lambda^2 + \frac{1-\nu}{2} n^2 \right) + \right] \frac{\pi L}{4R}, \quad (8)$$

where

$$a_1 = \frac{1-\nu}{2} \lambda^2 + n^2 + \frac{1}{12} \left(\frac{h}{R} \right)^2 \left[2(1-\nu)\lambda^2 + n^2 \right];$$

$$a_2 = n \left[1 + \frac{1}{12} \left(\frac{h}{R} \right)^2 \left[(2-\nu)\lambda^2 + n^2 \right] \right];$$

$$a_3 = 1 + \left(\frac{h}{R} \right)^2 (\lambda^2 + n^2); \quad \lambda = \frac{m\pi R}{L}.$$

Additional work ΔA of external compressive load:

$$\Delta A = \frac{1}{2} \int_0^{2\pi R} \int_0^L \left(N_* - \frac{1}{2} N_1 \right) \left(\frac{\partial w}{\partial x} \right)^2 dx dy. \quad (9)$$

External load N_l is equal to internal forces T_l at the edges of the shell:

$$N_1 = -\frac{Eh}{2(1-\nu^2)} f_3^2 \left(\frac{m\pi}{L} \right)^2 \cos^2 \frac{ny}{R}. \quad (10)$$

From (9), (10) and the conditions $\Delta A=0$ we obtain

$$\Delta A = \left(\frac{Eh}{1-\nu^2} \frac{3}{16} f_3^4 \frac{\lambda^4}{R^2} + N_* f_3^2 \lambda^2 \right) \frac{\pi L}{4R};$$

$$\frac{1}{16} f_3^2 \frac{\lambda^2}{R^2} = -\frac{1-\nu^2}{3Eh} N_*.$$

Change in the potential energy $\Delta U = \Delta V - \Delta A$:

$$\Delta U = \frac{Eh}{1-\nu^2} \left[\frac{f_2^2 a_1 + 2f_2 f_3 a_2 + f_3^2 \left(a_3 - \frac{1-\nu^2}{Eh} N_* a_4 \right)}{1-\nu^2} \right] \quad (11)$$

where

$$a_4 = \lambda^2 + \frac{1-\nu}{6} n^2.$$

From the conditions for the minimum potential energy for displacements

$$\frac{\partial \Delta U}{\partial f_2} = 0; \quad \frac{\partial \Delta U}{\partial f_3} = 0$$

we obtain:

$$N_* = \frac{Eh}{(1-\nu^2)a_4} \left(a_3 - \frac{a_2^2}{a_1} \right) \quad (12)$$

The relative value of the critical compression force:

$$\bar{N}_* = \frac{N_*}{N_*^e} \quad (13)$$

The minimization of expressions (12) and (13) with respect to integer parameters m and n allows one to find the critical loads of cylindrical shells of any geometric dimensions. The results are shown in Fig. 2.

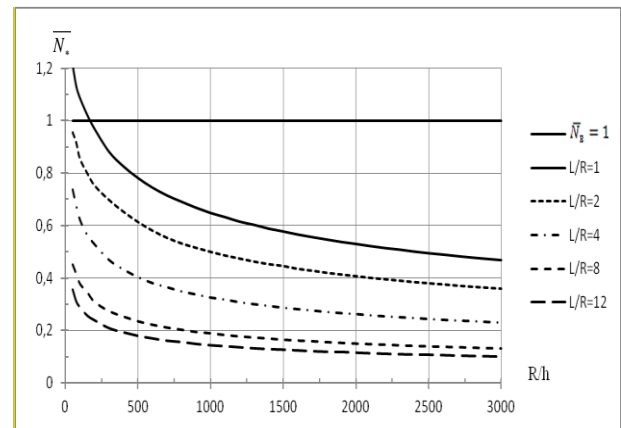


Fig. 2. Theoretical relative axial critical forces of cylindrical shells

There are many empirical dependencies for calculating critical loads. These formulas are obtained by processing numerous experiments. They are different and are recommended for different shells. Let's carry out calculations for some of them. The dependence that gives the maximum values of k_c [11].

$$k_{c\max} = 0,5 - 3,5 * 10^{-3} (R/h)^{1/2} + 4,6(1/R)^{1/2}. \quad (14)$$

The formula that gives the minimum k_c values [11]:

$$k_{c\min} = 3,87(h/R)^{1/2} + 10^{-3} (R/h)^{1/2}. \quad (15)$$

Formula, which is recommended for practical calculations [12]:

$$k_c = 1 - 0,9 \left(1 - e^{-\frac{1}{16} \sqrt{\frac{R}{h}}} \right). \quad (16)$$

In Fig. 3. the dotted line shows the results of calculations using these formulas and the theoretical curves obtained from (13) for $L/R=1$ and $L/R=12$. Most of the experimental data fall within this range.

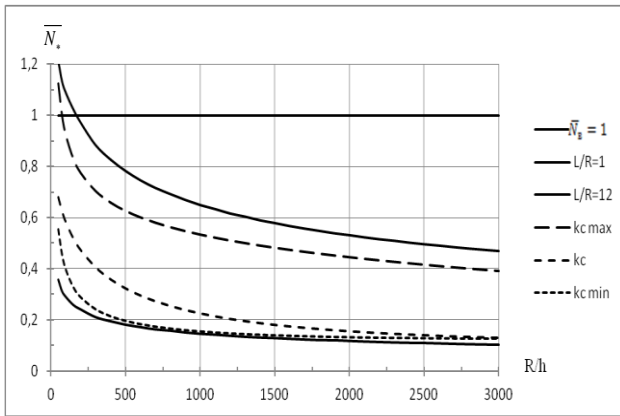


Fig. 3. Theoretical and empirical relative axial critical forces of cylindrical shells

Using the relations of the theory of shallow shells

$$\varepsilon_1 = \frac{\partial u}{\partial x}; \varepsilon_2 = \frac{\partial v}{\partial y} + \frac{w}{R}; \varepsilon_{12} = \frac{\partial v}{\partial x} + \frac{\partial u}{\partial y};$$

$$\chi_1 = -\frac{\partial^2 w}{\partial x^2}; \chi_2 = -\frac{\partial^2 w}{\partial y^2}; \chi_{12} = -\frac{\partial^2 w}{\partial x \partial y}$$

we get a simpler formula for critical loads:

$$N_*^f = \frac{Eh}{(1-\nu^2)\left(\lambda^2 + \frac{1-\nu}{6}n^2\right)} \left[\frac{\frac{1-\nu}{2}\lambda^2}{\frac{1-\nu}{2}\lambda^2 + n^2} + \frac{1}{12}\left(\frac{h}{R}\right)(\lambda^2 + n^2)^2 \right]; \quad (17)$$

$$\frac{\overline{N}_*^f}{N_*^e} = \frac{N_*^f}{N_*^e} \quad (18)$$

Critical loads calculated by formulas (17) and (18) differ little from the results of calculations by formulas (12) and (13).

Error for relatively thick shells ($R/h \leq 50$) is equal to:

$$\Delta = \frac{N_*^f - N_*^e}{N_*^e} 100 \leq 5\%$$

The rest of the shells have a smaller error. Formulas (17) and (18) are simpler and can be recommended for practical calculations.

6. Results

Analyzing the results obtained, we see:

1. Found theoretically, on the basis of the general linear theory of thin shells, formulas (12) and (13) allow calculating the axial critical loads of cylindrical shells with specific values;
2. Theoretical critical loads are in good agreement with empirical and, therefore, with experimental critical loads;
3. A large dependence of the relative values of the

critical load on the parameter was revealed according to the classical theory;

4. A large dependence of the relative values of the critical load on the parameter was revealed. For a long time, it was believed that length has no or little effect on the magnitude of the critical load.

5. Simplified formulas for critical loads (17) and (18) are obtained on the basis of the theory of shallow shells. The results of calculations using exact and simplified formulas differ little.

7. Conclusions

1. The formulas obtained for the axial critical loads of cylindrical shells can be recommended for practical calculations.

2. The proposed approach can be used to study the stability of thin-walled structures.

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Нові формули розрахунку критичних сил циліндричних оболонок

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Мета: Показати, що в рамках лінійної теорії вдалося отримати формули осьових критичних навантажень, результати розрахунків за якими добре узгоджуються з експериментальними даними.

Метод: Метод вирішення енергетичний з використанням загальної лінійної теорії тонкостінних оболонок. **Результати:** Отримано нові формули критичних навантажень циліндричних оболонок. Проведено аналіз отриманих результатів. Дано рекомендації по їх використанню. **Обговорення:** Труднощі визначення теоретичним шляхом критичних навантажень циліндричних оболонок при осьовому стисненні близьких до експериментальних даних змусили дослідників шукати рішення емпіричним шляхом. Отримано багато емпіричних залежностей, які дають різні результати і описують відомі експерименти. Необхідність теоретично знайти формули, які дозволяли б обчислювати критичні навантаження циліндричних оболонок будь-яких геометричних параметрів, залишається. Такі формули отримані. Проведено порівняння, обчислених за цими формулами, критичних навантажень з емпіричними і експериментальними критичними навантаженнями. Відмінності між ними незначні.

Ключові слова: вигин, критичне навантаження, зміщення, стійкість, експеримент, енергія

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Новые формулы расчета критических сил цилиндрических оболочек

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Цель: Показать, что в рамках линейной теории удалось получить формулы осевых критических нагрузок, результаты расчетов по которым хорошо согласуются с экспериментальными данными.

Метод: Метод решения энергетический с использованием общей линейной теории тонкостенных оболочек. **Результаты:** Получены новые формулы критических нагрузок цилиндрических оболочек. Проведен анализ полученных результатов. Даны рекомендации по их использованию. **Обсуждение:** Трудности определения теоретическим путем критических нагрузок цилиндрических оболочек при осевом сжатии близких к экспериментальным данным вынудили исследователей искать решение эмпирическим путем. Получено много эмпирических зависимостей, которые дают разные результаты и описывают известные эксперименты. Необходимость теоретически найти формулы, которые позволяли бы вычислять критические нагрузки цилиндрических оболочек любых геометрических параметров, остается. Такие формулы получены. Проведено сравнение, вычисленных по этим формулам, критических нагрузок с эмпирическими и экспериментальными критическими нагрузками. Различия между ними незначительны.

Ключевые слова: изгиб, критическая нагрузка, смещение, устойчивость, эксперимент, энергия

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