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GUARANTEED ADAPTIVE TERMINAL CONTROL OF AN AEROSTATIC AIRCRAFT BASED ON DIFFERENTIAL GAME APPROACH

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Abstract

Purpose: *The present paper is aimed to the problem solution of game algorithms construction for the injection control of an unmanned aerostatic aircraft into the desired terminal conditions under the influence of unknown turbulences.* ***Methods:*** *The problem is solved on the basis of multistep differential transform method and the theory of differential games.* ***Results:*** *The proposed approach does not require numerical integration of differential equations of aircraft motion, reduces the problem of control algorithms synthesis to the solution of final system of equations concerning control variables and turbulence parameters, allows for analytical transformations and allows to synthesize control algorithms that own the property of adaptation to the turbulence action and provide a guarantee of the aircraft injection into desired terminal conditions. The solution of the synthesis problem of guaranteed adaptive control of the process of multistep injection of an unmanned aerostatic aircraft into the desired terminal conditions in the form of mathematical model of differential game is considered.* ***Discussion:*** *Application of the differential-game approach to the control algorithms synthesis of dynamic objects under the action of undetermined turbulences allows to carry out continuous calculations of program strategies for players in real time and get the opportunity to control dynamic objects with feedback, taking into account the action of different turbulences.*

Keywords: unmanned aerostatic aircraft; differential game; guaranteed adaptive control; multistep differential transform method

1. Introduction

Recently, there has been a growing interest in creation and application of aeronautical complexes based on unmanned lighter-than-air aircrafts (LTA UAVs), which include autonomous (robotic) airships, aerostatic platforms based on airships etc. Their main purpose is solving tasks of ambient monitoring, operational control over the state of ground infrastructure, forecasting and monitoring of natural phenomena, telecommunications. In front of the modern LTA UAVs making a demand also requirements related to the need of aircraft injection into the given terminal conditions (e.g. emergency flight to an emergency zone, etc.). The injection process of LTA UAV into the given terminal conditions is carried out on a multistep trajectory with taking into account changes in their mass inertia characteristics and operating modes of the aircraft systems.

Algorithm synthesis of optimal multistep control of LTA UAV injection into the given terminal conditions under the action of unknown turbulences (parametric, external) is a complex problem. The main factors of objective complexity are the high order of nonlinear differential equations of LTA UAV spatial motion, the complexity of intra-system relationships, the unknown stochastic characteristics of turbulences. At the same time, the requirements to terminal parameters and a significant duration of the LTA UAV flight, require consideration of the impact of turbulences to achievement of control aims. One way for evaluation of indeterminate form associated with the unpredictable influence of disturbances is to apply a strategy of guaranteed adaptive approach to the control algorithms synthesis. This strategy uses the principle of maximum guaranteed result, as control process is seen in the most adverse conditions, that may occur when exposed turbulences [1-4]. The problem of guaranteed adaptive control synthesis under uncertain impact of turbulences requires a transition from optimization problems to biderectional optimization problems as discussed in the theory of differential games [1].

To solve terminal control problems using the differential games use methods of R. Isaacs [1,2], L.M. Pontryagin [3,4], N.N. Krasovsky [5] and others. The majority of known methods are used for linear systems of differential equations, require solving of differential equations in partial derivatives, use the necessary conditions of optimality similar to Pontryagin's maximal principle or are based on calculation of attainability domains.

Application of the theory of differential games together with the mathematical apparatus of differential transformations (DT) to the synthesis of guaranteed adaptive control algorithms allows to solve complex differential game problems in the field of images with missing time argument and reduce them to simpler problems, which are easily solved by known methods [3].

2. Research task

Questions of game algorithms construction for LTA UAV injection control into the given terminal conditions under the influence of unknown turbulences on the basis of multistep DT and the theory of differential games are considered.

3. Differential transformations

The DT allow replacing in the mathematical model of dynamic process the functions $x(t)$ of continuous argument t by their spectral models in the form of discrete functions $X(k)$ of integer argument $k = 0,1,2,...$.

The differential transformations of function $x(t)$ are defined as [6]:

$$\underline{x(t)} = X(k) = \frac{h^k}{k!}\left[\frac{d^k x(t)}{dt^k}\right]_{t=0}, \qquad (1)$$

where $x(t)$ is the original function; $\underline{x(t)}, X(k)$ are the differential image of original (differential spectrum), representing discrete function of integer argument $k = 0,1,2,...$; h is the scale stationary value having dimensionality of argument t and usually chosen equal to the interval $0 \le t \le h$, on which the function $x(t)$ is considered; the line below is the character of differential transformations. The values of the function *X(k)* at concrete value of the argument k are called discrete.

The inverse transformations allow obtaining the original $x(t)$ by the image $X(k)$ as a Taylor series:

$$x(t) = \sum_{k=0}^{\infty} X(k)\left(\frac{t}{h}\right)^k. \qquad (2)$$

Generally, in actual application of differential transformations, the function *x(t)* is defined as finite series:

$$x(t) \approx y(t) = \sum_{k=0}^{N} X(k)t^k. \qquad (3)$$

4. The multistep DT

Consider the nonlinear differential equation of order *m*:

$$f(t, \mathrm{x}, x', ..., x^{(m)}) = 0, \ t \in [0, T] \qquad (4)$$

subject to the given initial conditions:

$$x^{(p)}(0) = c_{\mathrm{p}}, \ p = 0,1,...,m-1. \qquad (5)$$

Let's divide the initial time interval $[0,T]$ into r given subintervals of length $T_i = t_i - t_{i-1}$, $i = \overline{1, \mathrm{r}}$, $\sum_{i=1}^{\mathrm{r}} T_i = T$. Applying the differential transform (1) to the problem (4) - (5) over the first subinterval $[0, t_1]$ we will obtain the solution in the form:

$$x_1(t) \approx y_1(t) = \sum_{k=0}^{N} X_1(k)\left(\frac{t}{h}\right)^k, \ t \in [0, t_1].$$

Taking into account the initial condition $x_1^{(p)}(t_0) = c_p$ and the expression (1) we can find for the first subinterval all values of differential spectra $X_1(k)$, $k = 0,1,2,...$. For $i \ge 2$ and at each following subinterval $[t_{i-1}, t_i]$ we will use the initial conditions, which are which the final conditions of the previous subinterval, i.e. $x_i^{(p)}(t_{i-1}) = x_{i-1}^{(p)}(t_{i-1})$. Then the expression (1) for the i-th subinterval will be following:

$$X_i(k) = \frac{h^k}{k!}\left[\frac{d^k x_{i-1}(t)}{dt^k}\right]_{t=t_{i-1}}, k \ge 0.$$

Now applying the DT to the problem (4) - (5) over the interval $[t_{i-1}, t_i]$. The process is repeated and, in result, we obtain the sequence of approximate solutions $x_i(t)$, $i = 1,...,r$ for the solution $x(t)$, where

$$x_i(t) \approx y_i(t) = \sum_{k=0}^{N} X_i(k)(t_i - t_{i-1})^k, \ t \in [t_{i-1}, t_i].$$

Finally, at using of the multistep DT we obtain the following solution [7]:

$$x(t) = \begin{cases} x_1(t) \approx y_1(t), \ t \in [0, t_1] \\ x_2(t) \approx y_2(t), \ t \in [t_1, t_2] \\ ..... \\ x_r(t) \approx y_r(t), t \in [t_{r-1}, t_r] \end{cases}. \qquad (6)$$

If $r=1$ then $h=T$ and the multistep DT reduces to the traditional DT. In the case of dividing the interval into subintervals of the same length, the use of multistep DT reduces the upper bound of the solution estimate in r^s time, where r is the quantity of subintervals into which given solution interval is divided, s is a quantity of accounted discretes of differential spectrum $X(k)$ [8].

5. Differential game model for multistep control process

The whole control process of LTA UAV motion is divided into r given time frames, inside which the mass-inertia parameters of aircraft and operation modes of its propulsion system have no jump changes. All changes in the form of given springs happen at boundaries of given time subintervals

$$T_i = t_i - t_{i-1},\ i=\overline{1,r},\ \sum_{i=1}^{r} T_i = T,$$

where T – a duration of control process.

The mathematical model of differential game describing the LTA UAV trajectory motion at each segment of its injection into the given terminal conditions under the influence of turbulences we shall present as the vector differential equation:

$$\frac{dx_i}{dt} = f_i(t, x_i, u_i, \nu_i),\ x_i(t_{i-1}) = x_i^0,\ i=\overline{1,r}, \quad (7)$$

where $x_i = x_i(t)$ – n-measurement of state vector; $u_i(t)$ – m-measurement of control vector (first player strategy); $\nu_i(t)$ – ℓ-measurement vector of turbulence (second player strategy); f_i – continuous and continuously differentiable on plurality variable t, x_i, u_i, ν_i the vector function of generalized force, $t \in (t_i - t_{i-1})$.

The problem of terminal control consists in determination the vector of optimal program control $u_i^*(t)$ of the phase trajectory $x_i^*(t)$, which for given differential constraints (7) provide optimal multistep translation of LTA UAV from the initial state $x_1(0)$ to final (terminal) state $x_i(T_i)$, which is determined in the point of time $t = T_i$ by q-measurement $(q \le n)$ vector equation:

$$S_i\left[x_{T_i}(T_i), T_i\right] = 0 \quad (8)$$

and ensure minimization of the functional:

$$I_i = G_i\left[x_i(T_i), T_i\right] + \sum_{i=1}^{r} \int_{t_{i-1}}^{t_i} \Phi_i(t, x_i, u_i, \nu_i)\,dt,\ i = 1,2,3,\dots,r, \quad (9)$$

where the given functions G_i and Φ_i have continuous partial derivatives on x_i, u_i, ν_i

Assume, that restriction on state and control vectors are taken into account during the selection of the functional type (9).

The conjugation of final (terminal) and starting conditions of segments of the control process is set in the form of given boundary conditions [9]:

$$\varphi_i\left[x_i(T_i), x_{i+1}^0; u_i(T_i), u_{i+1}^0; T_i\right] = 0,\ i=\overline{1,r}. \quad (10)$$

The mathematical model (7) - (10) describes the terminal control process in conditions of uncertainty regarding the action of turbulences on the object.

We consider the terminal control problem (7)-(10) as a mathematical model of differential game of two players with opposite interests. The LTA UAV motion which is described by differential equation (7), depends from control strategy of first player $u_i(t)$ and on the choice of strategy by the second player (vector of turbulence) $\nu_i(t)$.

The task of first player consists in such translation of control object (7) from the given initial state $x_1(0)$ to final $x_r(T)$, which ensures minimum (maximum) of functional (9) upon condition of its maximization (minimization) during vector of turbulence $\nu_i(t)$ selection by second player.

Functions $u_i(t)$ and $\nu_i(t)$ are termed as program strategies of players.

Pair of player strategies u_i^* and ν_i^* is termed as optimal, if there is the ratio (saddle point) [10]:

$$I(u_i^*, \nu_i) \le I(u_i^*, \nu_i^*) \le I(u_i, \nu_i^*). \quad (11)$$

A differential game in which has a saddle point (11) has the property that any deviation from the optimal control of one player leads to a decrease in his gain, provided that the optimal control of the other player is chosen.

The necessary conditions of strategies optimality u_i^* and ν_i^* are [12]:

$$\frac{\partial I_i}{\partial u_i} = 0,\ \frac{\partial I_i}{\partial \nu_i} = 0, \quad (12)$$

$$\frac{\partial^2 I_i}{\partial u_i^2} \ge 0,\ \frac{\partial^2 I_i}{\partial \nu_i^2} \le 0, \quad (13)$$

and sufficient conditions are the ratio (12) and condition (13), which has the strict inequality. Player strategies u_i^* and v_i^*, that satisfy the sufficient conditions, ensuring the existence of saddle point (11) of differential game (7)-(10). The control process of trajectory motion we shall consider within the frame of such mathematical models of differential games, which satisfy conditions (12) and (13).

From ratio (11) follows, that random law of vector of turbulence variation, other than optimal v_i^* doesn't impair the quality of object process control, which is achieved under the optimal control u_i^*. Therefore, the control u_i^* is guaranteed the quality of control process no worse of definition (u_i^*, v_i^*) at conditions of restricted random turbulences. Taking into account, that control u_i^* ensures obtaining of guaranteed assessment of control quality and adaptability to the specific type of turbulences action, we will call the control u_i^* as guaranteed adaptive control [3].

Simulation of LTA UAV control process in the form of differential game removes the uncertainty caused by the influence of turbulences. However, the disclosure of uncertainty is achieved at the cost of complicating of the mathematical model and the simulation process, as a result, in addition to optimal control u_i^*, it is necessary to determine the law of vector of turbulence v_i^* variation that describes the maximum opposition of purposes of terminal control.

6. Method of guaranteed adaptive control algorithms synthesis

For the motion control algorithms optimization of multistep aircraft, we use the differential-game approach based on the mathematical apparatus of DT [9]. This allows us to reduce the problem of terminal control synthesis to solving a system of nonlinear algebraic equations without numerically integrating or differentiating the equations of aircraft trajectory motion, which significantly reduces the amount of necessary calculations.

Mathematical models obtained on the basis of DT (1) of the original mathematical model are called spectral models. Further, we will assume that the time functions that describe the control processes in problem (7) - (10) in the middle of each motion section are analytical.

Synthesis of guaranteed adaptive control algorithms we will realize in two stages. At the first stage will perform a synthesis of optimal gaming algorithms of program control $u_i^0(t)$ and opposing turbulence $v_i^0(t)$, which satisfy the conditions (12) and (13), in the middle of each control segment in the class of analytic functions $u_i(\tau, A_i)$ and $v_i(\tau, B_i)$ where $A_i = (a_{i1}, a_{i2}, ..a_{iN})$ and $B_i = (b_{i1}, b_{i2}, ..b_{iM})$ are vectors of free parameters, τ is a local time argument.

Let's choose a scale stationary value $h = T_i$ and assume $\tau = 0$. Applying the DT (1) to functions $u_i(\tau, A_i)$ and $v_i(\tau, B_i)$, we obtain their differential spectra in the form:

$$\underline{u_i}(\tau, A_i) = U_i(k, A_i) = \frac{T_i^k}{k!}\left[\frac{d^k u_i(t_{i-1} + \tau, A_i)}{dt^k}\right]_{\tau=0} . \quad (14)$$

$$\underline{v_i}(\tau, A_i) = V_i(k, B_i) = \frac{T_i^k}{k!}\left[\frac{d^k v_i(t_{i-1} + \tau, B_i)}{dt^k}\right]_{\tau=0} .$$

Differential equation (7) in the image field on the basis of transformations (1) is written as the following spectral model:

$$X_i(k+1, A_i, B_i, X_i^0) = \\ = \frac{T_i}{k+1} \underline{f_i}\begin{bmatrix} T_i, X_i(k, A_i, B_i, X_i^0), \\ U_i(k, A_i), V_i(k, B_i) \end{bmatrix} \quad (16)$$

$$X_i(0) = X_i^0(A_{i-1}, A_{i-2}, ...A_1, B_{i-1}, B_{i-2}, ..., B_1); \\ X_1(0) = X_1^0 = x_0; \, i = \overline{1, r} .$$

The spectral model (16) is universal in nature and can be used to solve the problems of trajectory motion of different types of multistep aircraft, which differ both in their layout and in the degree of multistep. Note that since DT (1) are an exact operational method, the spectral model (16) has no methodological errors and potentially allows us to obtain an exact solution of differential equation (7).

Recursion expression (16) allows finding the differential spectra $X_i(k, A_i, B_i, X_i^0)$ of state vector $x_i(t)$ in the differential spectra (14) and (15).

Let's take advantage of the property of the DT, according to which the algebraic total of all discretes of differential spectra of any analytical function in point $t = t_\nu$, is equal to zero discrete of a differential spectrum of function in point $t_{\nu+1} = t_\nu + h$ or value of the original of function in the same point [6]:

$$\sum_{k=0}^{\infty} X_\nu(k) = X_{\nu+1}(0) = x(t_\nu + h) . \quad (17)$$

From the obtained relation (17) at $t_\nu = t_{i-1}$ and $h = T_i$ we determine a state vector at the end of each control segment:

$$x_i(T_i, A_i, B_i, x_i^0) = \sum_{k=0}^{\infty} X_i(k, A_i, B_i, X_i^0), \quad i = \overline{1, r}. \quad (18)$$

Then the equation of the final state of whole control process (9) in view of the expression for conjugation of final and initial segments (8), and also the expressions for a state vector at the end of each segment (11) is conversed as followed:

$$S_i[A_i, B_i] = 0. \quad (19)$$

The given terminal condition in the implicit shape define q-components of vectors of the free parameters A_i and B_i, $i = \overline{1, r}$ on each subinterval as functions T_i and x_i^0. Remaining M+N-q components of vector of free parameters are determined from the stationary conditions (12) of the functional (10). The DT (1) of functional (10) allow presenting functional (4) as the function of vectors of undetermined parameters A_i and B_i:

$$I_i(A_i, B_i) = G[A_i, B_i] + \sum_{i=1}^{r} T_i \cdot \sum_{k=0}^{\infty} \frac{\Phi_i[T_i, X_i(k, A_i, B_i, X_i^0), U_i(k, A_i), V_i(k, B_i)]}{k+1}. \quad (20)$$

The stationary conditions (12) of function (20) enable to receive the system of equations for determining remaining M+N-q of unknown components of vectors of free parameters A_i and B_i for i-th subinterval:

$$\frac{\partial I_i(A_i, B_i)}{\partial a_{ij}} = 0, \; q+1 \le i \le N, \quad (21)$$

$$\frac{\partial I_i(A_i, B_i)}{\partial b_{ij}} = 0, \; 1 \le j \le M. \quad (22)$$

Solving the system of nonlinear algebraic equations (19), (21) and (22), in the case of their consistency, allows to find the components of vectors of free parameters A_i and B_i of program strategies of both players as functions from a vector of an arbitrary initial state $x_0 = x_1(t_0)$. Then can be verified sufficient conditions (12), (13) of player strategies optimality at strict inequality in the expression (13).

When system of equations (19), (21) and (22) is inconsistency, the differential game (7)-(10) has no solution in selected function types $u_i(t, A_i)$ and $v_i(t, B_i)$ then, the type of function with free parameters should be changed or expand the dimension of vectors of free parameters [3,10].

As a result of execution of the first stage in the implicit form, the nonlinear link of program strategies of both players $u_i(t, A_i)$ and $v_i(t, B_i)$ with a vector of the initial state $x_0 = x_1(t_0)$ is established. These strategies can be utilized only in initial instant t_0 and do not account changes of state during motion. To take into account the current state of the control process is necessary to synthesize control algorithms and maximum counteract turbulences in the form of positional strategies of the players $u_i = u_i(t, x_i), v_i = v_i(t, x_i)$.

At the second stage of synthesis we shall make the following assumption. We shall consider only such models of control process in which there are player strategies and allow to associate an arbitrary initial condition within a given region of state space with given terminal conditions (9). The strategies synthesis beyond a given region of state space is not considered.

The solution of combined equation (19), (21) and (22) for current instant t for each current state of game $x_i(t)$ sets pair of player strategies $u_i[t, A_i(T_i, x_i)]$ and $v_i[t, B_i(T_i, x_i)]$, linking current state of game with given terminal conditions (9). If organize a time continuous process of calculation parameters A_i and B_i of players strategies, then on the set of solution can be formulated player strategies on each motion segment as $u_i^*[t, A_i(T_i, x_i)]$ and $v_i^*[t, B_i(T_i, x_i)]$. The first player who realizes a potential control strategy $u_i^*[t, A_i(T_i, x_i)]$, which continuously determined from combined equations (19), (21) and (22), ensures achievement of the given terminal conditions (9) at the maximum counteract of turbulence which action is modeled by a strategy of second player $v_i^*[t, B_i(T_i, x_i)]$.

The general problem solution is continuous and piecewise continuous functions and is defined as the sum of the solutions on the intervals (6).

If necessary, to find the optimal trajectory $x(t, A, B)$, its components can be identified as a truncated Taylor series (2) or using inverse DT in the form of polynomials of Legendre, Chebyshev, Fourier series [6].

The basic benefit of the considered approach is replacement of operation of integration of the differential equations for dynamic object motion by calculations on the recurrent expression (9) and potential possibility for obtaining exact solution of differential game (7)-(10) under condition of exact display of time functions by a final differential spectrum. This possibility appears due to the fact that differential transformations (1) are an exact operation method.

7. Game algorithm of multistep terminal control by LTA UAV

Consider the task of terminal multistep control of the ALA UAV injection process into the given terminal conditions in the form of mathematical model of differential game. As a mathematical model of trajectory motion, we adopt a simplified mathematical model. We assume that ALA UAV is under the influence of aerostatic lifting force, overall constant thrust force and gravity. The aerodynamic lift force, considering that it is much less than aerostatic force, is not taken into account. To the first approximation, for demonstration the game approach for solving this problem, we do not take into account the drag force.

The overall thrust force $P_{\Sigma} = n \cdot m_i \cdot g \cdot \chi$ is formed by the propulsion system, consisting of n engines of the same thrust; $n \cdot \chi$ is air vehicle thrust/weight, m_i is vehicle mass on the i-th trajectory interval, g is gravity acceleration. Let's us consider the planar motion of the mass point in inertial axes of coordinates OXY. Let's us define the velocity components on the i-th interval of trajectory as V_{X_i} and V_{Y_i}. As a control function $\varphi_i(t)$ we select the direction angle of the overall thrust with the OX axis. We will investigate the effect of variation in the thrust level of one of the n engines to its complete failure. The variation in the thrust level of one of the engines will be dependent on the turbulence function $1-\beta_i^2(t)$, where $\beta_i(t) = \dfrac{P_{1_i}(t)}{P_H}$, $P_{1_i}(t)$ is the thrust of one engine; P_H is nominal thrust of one engine. Under the assumptions made, the trajectory motion of the LTA UAV on the i-th segment is described by the following mathematical model of differential game:

$$\dot{L}_i = V_{X_i}, \tag{23}$$

$$\dot{V}_{X_i} = \varsigma_{i_1}\left(n-1+\beta_i^2\right)\cos\varphi_i, \tag{24}$$

$$\dot{H}_i = V_{Y_i}, \tag{25}$$

$$\dot{V}_{Y_i} = \varsigma_{i_2}\left(n-1+\beta_i^2\right)\sin\varphi_i + \widetilde{\phi}_i - \varsigma_{i_3} g, \tag{26}$$

$$L_1(0)=0,\ V_{X_1}(0)=0,\ H_1(0)=0,\ V_{Y_1}(0)=0, \tag{27}$$

where $\varsigma_{i_1} = \dfrac{1}{m_i+m_x} g\cdot\chi$, $\varsigma_{i_2} = \dfrac{1}{m_i+m_y} g\cdot\chi$, $\varsigma_{i_3} = \dfrac{m_i}{m_i+m_y}$, m_x, m_y is the added masses of aircraft along the axes OX, OY respectively; $\widetilde{\phi}_i = \dfrac{1}{m_i+m_y}\gamma U$ is the relative aerostatic lifting force; U is the hull air vehicle volume; $\gamma = \gamma_в - \gamma_г$; $\gamma_в, \gamma_г$ is air gravity and buoyant gas respectively; $\varphi_i = \varphi_i(t)$ is the program strategy of first player, $\beta_i = \beta_i(t)$ is the program strategy of second player.

The terminal control problem consists in the LTA UAV transfer from the origin of coordinates in the OXY plane (start of takeoff) to a trajectory parallel to the OX axis and located at a given height H from it with maximizing of horizontal speed at the end of the injection process.

The first player selects the LTA UAV control $\varphi_i(t)$ from the condition that the maximum value of the criterion is reached at a time moment T_i:

$$I_i = V_{X_i}(T_i) - \frac{\lambda}{2} T_i^2 \beta_i,\ \lambda \succ 0 = const \tag{28}$$

and the given boundary conditions:

$$H_i(T_i) = H_{T_i}, \tag{29}$$

$$V_{Y_i}(T_i) = V_{Y_{T_i}}. \tag{30}$$

At the end of injection stage $V_{Y_r}(T) = V_Y(T) = 0$.

The aim of the second player is opposite to the aim of LTA UAV control and consists in such choice of a turbulence function $1-\beta_i^2(t)$, simulating a drop in the thrust level by one of the n engines, which provides the minimum value of criterion (28).

In criterion (28), the constant λ takes into account the desire of the first player to minimize the time T of LTA UAV rise, depending on the operating mode of the propulsion system. In the nominal operating mode of the propulsion system, the first player maximizes the criterion (28), and also seeks to fulfill the boundary conditions (29) and

(30). In case of failure of one of the engines on the i-th segment ($\beta_i = 0$), the first player selects control only from the condition of achieving the maximum horizontal speed and fulfilling the boundary conditions (29) and (30), without taking into account the time of rise.

Consider the effect of thrust loss by one of the engines at the instant of move of the air vehicle. Suppose that the turbulence function $\beta_i(t)$ is constant and starts acting at the initial moment of motion time. Based on the mathematical model of the differential game (21) - (30), we perform a synthesis of guaranteed adaptive control algorithms. We assume that the turbulence function $\beta_i(t)$ is constant and starts acting at the initial time $t_0 = 0$ of air vehicle takeoff.

We will look for the control of the first player on the *i*-th segment in the form:

$$\varphi_i(t, A_i) = a_{i_0} + a_{i_1} t \,, \tag{31}$$

and the second player - in the form of a constant function:

$$\beta_i(t, B_i) = b_{i_0} \,. \tag{32}$$

To form a spectral model of the differential game (23)-(30) differentiate the expressions (24), (26) and taking into account (31), (32) we will write them down as second order differential equations:

$$\ddot{V}_{X_i} = -\frac{a_{i_1} \cdot \varsigma_{i_1}}{\varsigma_{i_2}}\left(\dot{V}_{Y_i} + \varsigma_{i_3} g - \dot{\tilde{\phi}}_i\right), \tag{33}$$

$$\ddot{V}_{Y_i} = \frac{\varsigma_{i_2} \cdot a_{i_1}}{\varsigma_{i_1}} \dot{V}_{X_i} \,. \tag{34}$$

The DT (1) of equations (23), (25), (33), (34) at $h = T_i$ and $t = 0$ are defined their spectral models as:

$$\underline{L_i}(k+1) = \frac{T_i}{k+1} \underline{V_{X_i}}(k) \,, \tag{35}$$

$$\frac{(k+2)!}{k! T_i^2} \underline{V_{X_i}}(k+2) = -a_{i_1} \frac{\varsigma_{i_1}}{\varsigma_{i_2}} \cdot \left[\frac{k+1}{T_i} \underline{V_{Y_i}}(k+1) + (\varsigma_{i_3} g - \tilde{\phi}_i)\text{ъ}(k)\right], \tag{36}$$

$$\underline{H_i}(k+1) = \frac{T_i}{k+1} \underline{V_{Y_i}}(k) \,, \tag{37}$$

$$\frac{(k+2)!}{k! T_i^2} \underline{V_{Y_i}}(k+2) = a_{i_1} \frac{\varsigma_{i_2}}{\varsigma_{i_1}} \frac{k+1}{T_i} \underline{V_{X_i}}(k+1) \,, \tag{38}$$

where $\text{ъ}(k) = \begin{cases} 1, & k = 0 \\ 0, & k > 0 \end{cases}$, $k = 0,1,2,\ldots$

Using recursion expressions (35) - (38), taking into account controls (31), (32) and zero initial conditions, we obtain differential spectra of variables L, H, V_X, V_Y for the first subinterval:

$$L_1(0) = 0, \; L_1(1) = 0,$$

$$L_1(2) = \frac{\varsigma_{1_1} T_1^2}{2}\left(n - 1 + b_{10}^2\right)\cos a_{1_0}, \tag{39}$$

$$L_1(3) = -\frac{a_{1_1} T_1^3}{6} \varsigma_{1_1}\left(n - 1 + b_{10}^2\right)\sin a_{1_0}, \ldots$$

$$H_1(0) = 0, \; H_1(1) = 0,$$

$$H_1(2) = \frac{T_1^2}{2}\left[\begin{array}{l} \varsigma_{1_2}\left(n - 1 + b_{10}^2\right)\sin a_{1_0} + \\ + \tilde{\phi}_1 - \varsigma_{1_3} g \end{array}\right] \tag{40}$$

$$H_1(3) = \frac{a_{1_1} T_1^3}{6} \varsigma_{1_2}\left(n - 1 + b_{10}^2\right)\cos a_{1_0}, \ldots$$

$$V_{X_1}(0) = 0,$$

$$V_{X_1}(1) = \varsigma_{1_1} T_1\left(n - 1 + b_{10}^2\right)\cos a_{1_0}, \tag{41}$$

$$V_{X_1}(2) = -\frac{a_{1_1} T_1^2}{2} \varsigma_{1_1}\left(n - 1 + b_{10}^2\right)\sin a_{1_0}, \ldots$$

$$V_{Y_1}(0) = 0,$$

$$V_{Y_1}(1) = T_1\left[\begin{array}{l} \varsigma_{1_2}\left(n - 1 + b_{10}^2\right)\sin a_{1_0} + \\ + \tilde{\phi}_1 - \varsigma_{1_3} g, \end{array}\right] \tag{42}$$

$$V_{Y_1}(2) = \frac{a_{1_1} T^2}{2} \varsigma_{1_2}\left(n - 1 + b_{10}^2\right)\cos a_{1_0}, \ldots$$

The boundary conditions (29) and (30) on the basis on the expression (1) we present through thedifferential spectra (40) and (42):

$$\begin{gathered} H_1(T_1) = \frac{T_1^2}{2}\left[\begin{array}{l} \varsigma_{1_2}\left(n - 1 + b_{10}^2\right)\sin a_{1_0} + \\ + \tilde{\phi}_1 - \varsigma_{1_3} g \end{array}\right] - \\ - \frac{a_{1_1} T_1^3}{6} \varsigma_{1_2}\left(n - 1 + b_{10}^2\right)\cos a_{1_0} + \ldots = H_{T_1} \end{gathered}, \tag{43}$$

$$\begin{gathered} V_{Y_1}(T) = T_1\left[\begin{array}{l} \varsigma_{1_2}\left(n - 1 + b_{10}^2\right)\sin a_{1_0} + \\ + \tilde{\phi}_1 - \varsigma_{1_3} g \end{array}\right] + \\ + \frac{a_{1_1} T_1^2}{2} \varsigma_{1_2}\left(n - 1 + b_{10}^2\right)\cos a_{1_0} + \ldots = V_{Y_{T_1}} \end{gathered}. \tag{44}$$

In the last expressions, we restrict ourselves to two nonzero discretes of the differential spectra of variables H, V_Y. As a result, we obtain two equations for determining the free parameters of the control function (41) for the first player. From equation (44) we find the parameter a_{1_1} of the control function (41):

$$a_{1_1} = -\frac{2}{T_1} tga_{1_0} - \frac{2}{T_1} \frac{\left[\widetilde{\phi}_1 - \varsigma_{1_3} g\right]}{\varsigma_{1_2}\left(n-1+b_{10}^2\right)\cos a_{1_0}} + + \frac{2}{T_1^2} \frac{V_{Y_{T_1}}}{\varsigma_{1_2}(n-1+b_{1_0}^2)\cos a_{1_0}}. \quad (45)$$

For special case, at $g = 0$, in the absence of aerostatic force $(\widetilde{\phi}_i = 0)$, added masses $(m_x = m_y = 0, \varsigma_{i_3} = 1)$, turbulence function $(b_{i_0} = 0)$, and under boundary conditions $V_{Y_1}(T_1) = V_{Y_{T_i}} = 0$, the terminal problem (23) - (30) is similar to the terminal problem of injection a mass point into orbit near an atmosphereless planet, considered in [12], in which the following analytical solution was obtained:

$$a_1 = -\frac{2}{T} tga_0,$$

which is part of a more general expression (45).

If in the latter case gravity acceleration g is taken into account, we obtain a solution of the terminal problem of mass point injection into orbit, considered in [10] in the form of a mathematical model of differential game:

$$a_1 = -\frac{2[\alpha(n-1+b_0^2)\sin a_0 - g]}{\alpha T[n-1+b_0^2)\cos a_0}.$$

Note that the solution (45) of the differential game (21) - (30) is approximate, since only the first two nonzero discretes of the differential spectra of functions of variable parameters were used. In addition to the number of discretes taken into account, the solution accuracy is also affected by the vectors dimension selection of free parameters A, B for the analytical description of the strategies of players $\varphi(t, A)$ and $\nu(t, B)$ [10].

Substituting the differential spectrum (41) into expression (1), we find for the first subinterval the boundary values of the forward velocity:

$$V_{X_1}(T_1) = \varsigma_{i1} T_1\left(n-1+b_{10}^2\right)\cos a_{1_0} - -\frac{a_{1_1} T_1^2}{2}\varsigma_{1_1}\left(n-1+b_{10}^2\right)\sin a_{1_0} + \ldots \quad (46)$$

We restrict ourselves in expression (46) to the first two terms and substitute them into criterion (28):

$$I_1 = \varsigma_{1_1} T_1\left(n-1+b_{10}^2\right)\cdot \cdot\left(\cos a_{10} - \frac{a_{1_1}T_1}{2}\sin a_{10}\right) - \frac{\lambda}{2} T_1^2 b_{10}. \quad (47)$$

Taking into account the relation (45), expression (47) can be represented as:

$$I_1 = \frac{\varsigma_{1_1} T_1\left(n-1+b_{10}^2\right)}{\cos a_{10}} + T_1 \frac{\varsigma_{1_1}}{\varsigma_{1_2}} \widetilde{\phi}_1 tga_{10} - - T_1 \frac{\varsigma_{1_1}}{\varsigma_{1_2}} \varsigma_{1_3} g \cdot tga_{10} - \frac{\varsigma_{1_1}}{\varsigma_{1_2}} V_{Y_{T_1}} tga_{1_0} - \frac{\lambda}{2} T_1^2 b_{10}.$$

Substituting function (48) into the stationary conditions $(\frac{\partial I_1}{\partial T_1} = 0)$, we obtain the link between the free parameter a_{10} and the time of rise T_1 on the first takeoff interval:

$$\frac{\varsigma_{1_1}\left(n-1+b_{10}^2\right)}{\cos a_{10}} + \frac{\varsigma_{1_1}}{\varsigma_{1_2}} \widetilde{\phi}_1 tga_{10} - - \frac{\varsigma_{1_1}}{\varsigma_{1_2}} \varsigma_{1_3} g \cdot tga_{10} - \lambda T_1 b_{10} = 0. \quad (49)$$

In view of restriction by two terms in equation (43) and the expulsion of a free parameter a_{11} from it, according to (45), we obtain an expression for determining the duration of the first takeoff interval:

$$T_1 = \frac{-V_{Y_{T_1}} + \sqrt{V_{Y_{T_1}}^2 + 24 H_{T_1} \cdot D_1}}{2D_1}, \quad (50)$$

where $D_1 = \varsigma_{1_2}(n-1+b_{1_0}^2) + (\widetilde{\varphi}_1 - g)$.

When $V_{Y_{T_1}} = 0$ we have:

$$T_1 = \sqrt{\frac{6 H_{T_1}}{\varsigma_{1_2}\left(n-1+b_{10}^2\right)\sin a_{10} + \widetilde{\phi}_1 - \varsigma_{1_3} g}}.$$

From the necessary conditions of optimality (21) of function (48), we find the free parameter of the turbulence function on the first interval:

$$b_{10} = \frac{\lambda T_1}{4\varsigma_{1_1}} \cos a_{10}. \quad (51)$$

Substituting (51) into (49), we obtain the parameter a_{1_0}. Thus, for the first interval of the takeoff process, expressions (45), (49) - (51) are obtained for determining the free parameters a_{10} and a_{11} of control function (31) of the first player, duration T_1 and parameter b_{10} of the strategy of second player (32). A verification of sufficient conditions (21), (22) for function (48) showed that for the differential game (23) - (30) there is a saddle point (20) with respect to the free parameters of the players' strategies (31) and (32).

The components of the vector of optimal trajectory can be restored in the form of segments of Taylor power series from the differential spectra (40) - (42) and expression (2):

$$
\begin{aligned}
&L_1(t)=\frac{\varsigma_{1_1}t^2}{2}\left(n-1+b_{10}^2\right)\cos a_{10}-\\
&-\frac{a_{1_1}t^3}{6}\varsigma_{1_1}\left(n-1+b_{10}^2\right)\sin a_{10},...\\
&V_{X_1}(t)=\varsigma_{1_1}t\left(n-1+b_{10}^2\right)\cos a_{10}-\\
&-\frac{a_{1_1}t^2}{2}\varsigma_{1_1}\left(n-1+b_{10}^2\right)\sin a_{10}+...\\
&H_1(t)=\frac{t^2}{2}\begin{bmatrix}\varsigma_{1_2}\left(n-1+b_{10}^2\right)\sin a_{10}+\\+\tilde{\phi}_1-\varsigma_{1_3}g\end{bmatrix}+\\
&+\frac{a_{1i_1}t^3}{6}\varsigma_{1_2}\left(n-1+b_{10}^2\right)\cos a_{10}+...\\
&V_{Y_1}(t)=t\begin{bmatrix}\varsigma_{1_2}\left(n-1+b_{10}^2\right)\sin a_{10}+\\+\tilde{\phi}_1-\varsigma_{1_3}g\end{bmatrix}+\\
&+\frac{a_{1_1}t^2}{2}\varsigma_{1_2}\left(n-1+b_{10}^2\right)\cos a_{10}+...
\end{aligned}
\qquad (53)
$$

Expressions (53) at $t=T_1$, taking into account the obtained expressions (45), (49) - (51), allow us to determine the final values of the trajectory parameters at the end of the first takeoff interval.

Applying matching conditions (10), assigning obtained values to the initial values of the second stage and applying the procedure described above to find free parameters of control and turbulence vectors, we determine the optimal program control on the second interval of the trajectory. The general solution of the terminal program control problem of LTA UAV injection from the initial conditions into the given final conditions is a continuous and piecewise continuous function and is defined as a sum of solutions in the intervals:

$$
\begin{aligned}
&L(t)=\sum_{i=1}^{r}L_i(t),\ H(t)=\sum_{i=1}^{r}H_i(t),\\
&V_X(t)=\sum_{i=1}^{r}V_{X_i}(t),\ V_Y(t)=\sum_{i=1}^{r}V_{Y_i}(t),\\
&u=\sum_{i=1}^{r}u_i(t),\ \beta=\sum_{i=1}^{r}\beta_i(t).
\end{aligned}
$$

The solution of differential game (23)-(30) has shown that the main advantage of the proposed approach to the algorithm synthesis for multistep dynamic processes control is the ability to perform analytical transformations, which makes it possible to significantly reduce the amount of calculations for obtaining solution in the numerical form without using numerical methods of integration of differential equations.

Application of differential-game approach to the algorithm synthesis of dynamic processes control under the action of undefined turbulences allows for continuous calculations of the players' program strategies in real time and to obtain the possibility of dynamic objects control with feedback, taking into account the action of different turbulences. Errors arising due to a limited number of considered discretes of differential spectra and vectors dimensionality of arbitrary parameters of program controls can be reduced to the necessary level at the refinement of solution by a gradient method in the field of differential spectra [10].

8. Conclusions

The approach to the synthesis solution problems of guaranteed-adaptive algorithms of multistep control of LTA UAV injection into the given terminal conditions is offered. The approach is based on the application of the theory of differential games and the mathematical apparatus of differential transformations, allows to carry out analytical transformations that gives the possibility considerably reduce the volume of calculations for solution obtaining in the numerical form without application of numerical methods of integration of the differential equations.

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Гарантовано-адаптивне термінальне керування аеростатичним літальним апаратом на основі диференціально-ігрового підходу

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Мета: Метою даної статті є розв’язання задачі побудови ігрових алгоритмів керування виведенням безпілотного аеростатичного літального апарата в задані термінальні умови за дії невідомих збурень. **Методи:** Задача розв’язується на основі застосування багатоетапних диференціальних перетворень і теорії диференціальних ігор. **Результати:** Запропонований підхід не потребує численного інтегрування диференціальних рівнянь руху апарата, зводить проблему синтезу алгоритмів керування до розв’язання скінченої системи рівнянь відносно змінних керування і параметрів збурень, припускає аналітичні перетворення і дає змогу синтезувати алгоритми керування, що володіють властивістю адаптації до дії збурень і забезпечують гарантію виведення апарата в заданні термінальні умови. Розглянуто розв’язання задачі синтезу гарантовано-адаптивного керування процесом багатоетапного виведення безпілотного аеростатичного літального апарата в задані термінальні умови в формі математичної моделі диференціальної гри. **Обговорення:** Застосування диференціально-ігрового підходу до синтезу алгоритмів керування динамічними об’єктами за дії невизначених збурень дає змогу здійснювати неперервні обчислення програмних стратегій гравців у реальному часі та отримати можливість керування динамічними об’єктами із зворотним зв’язком, що враховує дію різних збурень.

Ключові слова: безпілотні аеростатичні літальні апарати; гарантовано-адаптивне керування; диференціальна гра; багатоетапний метод диференціальних перетворень.

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Гарантировано-адаптивное терминальное управление аэростатическим летательным аппаратом на основе дифференциально-игрового подхода

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Цель: целью данной статьи является решение задачи построения игровых алгоритмов управления выведением беспилотного аэростатического летательного аппарата в заданные терминальные условия при воздействии неизвестных возмущений. **Методы:** задача решается на основе применения многоэтапных дифференциальных преобразований и теории дифференциальных игр. **Результаты:** предложенный подход не требует численного интегрирования дифференциальных уравнений движения аппарата, сводит проблему синтеза алгоритмов управления к решению конечной системы уравнений относительно переменных управления и параметров возмущений, допускает аналитические преобразования и позволяет синтезировать алгоритмы управления, владеющие свойством адаптации к действию возмущений и обеспечивающие гарантию выведения аппарата в заданные терминальные условия. Рассмотрено решение задачи синтеза гарантированно-адаптивного управления процессом многоэтапного выведения беспилотного аэростатического летательного аппарата в заданные терминальные условия в форме математической модели дифференциальной игры. **Обсуждение:** Применение дифференциально-игрового подхода к синтезу алгоритмов управления динамическими объектами при действии неопределенных возмущений позволяет осуществлять непрерывные вычисления программных стратегий игроков в реальном времени и получить возможность управления динамическими объектами с обратной связью, учитывающей действие различных возмущений.

Ключевые слова: беспилотные аэростатические летательные аппараты; гарантированно-адаптивное управление; дифференциальная игра; многоэтапный метод дифференциальных преобразований.

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