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NEW APPROACH TO DETERMINING AXIAL CRITICAL LOADS SHELLS, PLATES AND RODS

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Abstract

Purpose: To show that one of the reasons for the large difference between the calculated and experimental critical loads is the incorrect interpretation of the buckling process. Methods: The energy criterion of stability and the relation of the general linear theory of thin-walled structures are used. Results: New formulas for critical loads of shells and plates have been obtained. Discussion: To estimate the bearing capacity of engineering structures, precise formulas are needed to calculate the critical loads under axial compression. Such formulas have not yet been obtained. The reason for the large discrepancies between the theoretical and experimental values of the axial critical loads of cylindrical shells was not found. In this paper, an attempt was made to solve this problem. In contrast to the usual approach, it is assumed here that when the structure is buckled, the distance between the loaded ends does not change. This allowed us to obtain new formulas for axial critical loads. The values of critical loads calculated by these formulas are close to the experimental data. Based on this, it was concluded that the formulas obtained can be used for real calculations of the critical loads of cylindrical shells and plates, and the proposed approach can be used to continue studies of the stability of thin-walled structures.

Keywords: bending; critical load; displacement; stability; experiment; energy

1. Introduction

One of the reasons for the exhaustion of the bearing capacity of engineering structures is the loss of stability of the structure as a whole, or of its individual elements (shells, plates, rods). However, there are no reliable methods for determining the critical loads of the plates and especially the shells. This problem is most acute in the design of aircraft, when it is necessary to fulfill strict requirements: to ensure minimum weight on the one hand and maximum load-carrying capacity of the structure on the other. This article is an attempt to solve this problem.

2. Analysis of recent research and publications

The first results of the study of the stability of structural elements were obtained by L. Euler [6], Brian [2], Lorentz [7], and S.P. Tymoshenko [10]. Leonard Euler owns the theoretical formulation of the problem of the stability of centrally compressed rods. In solving the problem, he used the linear approach and the static Euler criterion. According to this criterion, the critical load is calculated as the smallest load at which, simultaneously with the original form of equilibrium, an adjacent, infinitely close form of equilibrium is possible. Using this approach, he obtained the famous Euler formula Thoroughly carried out experiments by I. Bauschinger (1887), M. Consiger (1891), L. Tetmayer (1890, 1896) showed a good agreement between the theoretical and experimental data and put an end to the era of doubt in Euler's formula.

In the article “Some Theoretical Problems of Elastic Stability” (1910) S. P. Timoshenko gives an energetic derivation of the Euler formula for the case when, with a loss of stability, the distance between the ends of the rod does not change and, as a result, the longitudinal compressive force decreases slightly. The critical state is found from the condition of the equality of the decrease in the compression energy during the buckling of the potential bending energy. The identity of the results, apparently, was the reason that such an approach to solving problems of stability of thin-walled structures did not receive further development J. Brian was the first to use the energy method for solving stability problems and obtained a formula for the critical forces of a hinge plate compressed in one direction

$$N_* = k\frac{\pi^2 D}{b^2}, \quad (1)$$

where $D = \frac{Eh^3}{12(1-\nu^2)}$, $k = \left(\frac{mb}{a} + \frac{a}{mb}\right)^2$,

E - Young's modulus, ν – Poisson's ratio.

Lorenz and S.P. Tymoshenko in a linear formulation based on L. Euler's static criterion, considered the stability of a pivotally supported circular cylindrical shell under axial compression. Rudolf Lorenz (1908) found the critical compressive stress of a thin cylindrical shell, but neglected the transverse compressibility of the material. This was taken into account by S. P. Tymoshenko (1910), who, in the framework of the Kirchhoff-Love conceptions, obtained for critical stress a formula bearing his name:

$$\sigma_* = \frac{Eh}{\sqrt{3(1-\nu^2)}R} \quad (2)$$

or $\sigma_* = 0{,}605\frac{Eh}{R}$ at $\nu = 0{,}3$,

where R and h – the radius and thickness of the shell.

The critical axial force is calculated by the formula $N_*^{в} = \frac{Eh^2}{\sqrt{3(1-\nu^2)}R}$, (3)

it is called the classic or upper critical compressive force. This effort is the most famous and most indicative in terms of the discrepancy between theoretical and experimental values. The critical loads observed in the experiments are much smaller than the upper critical loads.

All further development of the theory of stability of the shells was aimed at identifying the causes of this discrepancy. Different directions of the theory developed, but two directions caused the greatest interest.

The first direction is connected with the use of the nonlinear theory of shells and recommendations to evaluate the stability of the shells by the lower critical load. These recommendations turned out to be erroneous.

The second direction is connected with the study of the influence of the initial imperfections of the shell on the value of the upper critical load. Unfortunately, this line of research has not brought positive results. Analysis of the experimental data shows that small deviations of the shell geometry from the ideal form reduce the value of the critical load, but not by several times, which is often observed in experiments.

The most complete and detailed first, second and other directions of studies of the stability of shells, plates and rods are described in [2, 5, 11, 12]. However, a large number of research papers did not solve the problem. It is necessary to continue to look for the cause of large discrepancies between the calculated and experimental data.

In [3, 4, 13-15], a new approach was proposed to solve the stability problem of a hinged cylinder, which differs from the classical approach.

The classical approach assumes that the transition from the initial to the curved form of equilibrium occurs without a change in the compression energy accumulated in the subcritical state, and the value of the critical compression force N_* does not change. This means that the length L shell remains constant. In this case, the edges of the shell receive some displacement in the axial direction, and the force $N_* = const$ does extra work $\Delta A \neq 0$ on these movements. Due to this work, additional energy appears $\Delta V \neq 0$ shell, but the potential of the system shell - external load does not change, i.e. $\Delta U = \Delta V - \Delta A = 0$.

The proposed approach describes the buckling process in a completely different way. When buckling occurs, the redistribution of the energy of compression accumulated in the subcritical state. Compression energy decreases, and bending and shear energies appear. At the same time, the potential energy of the shell does not change, i.e. $\Delta V = 0$. The edges of the shell remain in place, because the convergence of the edges due to radial movements is compensated by the elongation of the forming shell. The elongation of the generators occurs because the compression energy decreases. At the same time, the compressive forces also decrease and become equal $N_* - N_1$. Since there are no end shifts, the extra work of these forces $\Delta A = 0$. The potential of the system does not change $U = const$, i. e. $\Delta U = \Delta V - \Delta A = 0$.

Thus, the theorems and principles underlying the energy method are fully observed in the proposed approach.

In accordance with the general theorem of mechanics, the total potential energy of any system has a stationary value. $U = const$, when this system is in balance. The beginning of possible displacements. G. Lagrange (1788) says that the work of all forces at any infinitely small possible displacements $\Delta U = \Delta V - \Delta A = 0$. Since the state of the indifferent equilibrium of the system is considered, to determine the critical forces, in accordance with the Lejeune-Dirichlet principle (1846), the condition $\delta\Delta U = 0$. As a result, we have: $U = const$, $\Delta U = 0$, $\delta\Delta U = 0$. Based on this, we conclude: all the prerequisites for using the proposed approach in determining the critical loads of thin-walled structures are available.

In [14], the stability of a cylindrical shell, the edges of which are fixed on fixed hinges, was considered. Formula for axial critical load is obtained.

$$N_* = \frac{Eh}{1-\nu^2}\frac{1}{a_4}\left(a_3 - \frac{a_2^2}{a_1}\right); \quad \overline{N_*} = \frac{N_*}{N_*^в}, \qquad (5)$$

where

$$a_1 = \frac{1-\nu}{2}\lambda^2 + n^2 + \frac{1}{12}\left(\frac{h}{R}\right)^2[2(1-\nu)\lambda^2 + n^2];$$

$$a_2 = n\left[1 + \frac{1}{12}\left(\frac{h}{R}\right)^2[(2-\nu)\lambda^2 + n^2]\right];$$

$$a_3 = 1 + \frac{1}{12}\left(\frac{h}{R}\right)^2(\lambda^2 + n^2)^2;$$

$$a_4 = \lambda^2 + \frac{1-\nu}{6}n^2.$$

Fig. 1 shows the results of minimizing expression (5) with integer parameters m and n at ν = 0.3.

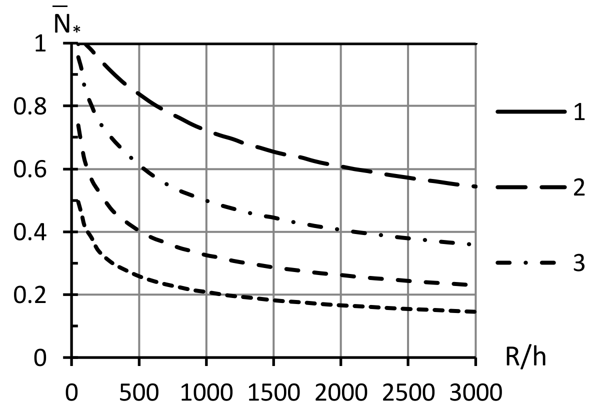

Fig.1. Dependence of a cylindrical shell supported on movable hinges on changes in the ratios L/R and R/h

1 - $\bar{N}_*$ =1; 2 – L/R = 1; 3 – L/R =2; 4 – L/R =4; 5 – L/R = 8

The analysis of the calculated values of critical loads showed that they are close to the experimental data given in [5]. A significant dependence was found on the relative parameters $\frac{L}{R}$ and $\frac{R}{h}$. The classic solution gives $\overline{N_*} = 1$.

In [15], the stability of a thin-walled cylinder, the edges of which are fixed to movable hinges, was considered. Formula for axial critical load is obtained:

$$N_* = \frac{Eh}{1-\nu^2}\frac{1}{a_5}\left(a_3 - \frac{a_2^2}{a_1}\right); \quad \overline{N_*} = \frac{N_*}{N_*^в}, \qquad (6)$$

where $a_5 = \frac{3}{4}\lambda^2 + \frac{1-\nu}{6}n^2\left(\frac{2m\pi}{3} - \frac{3}{4}\right)$.

Fig. 2 shows the results of calculations using formula (6) .

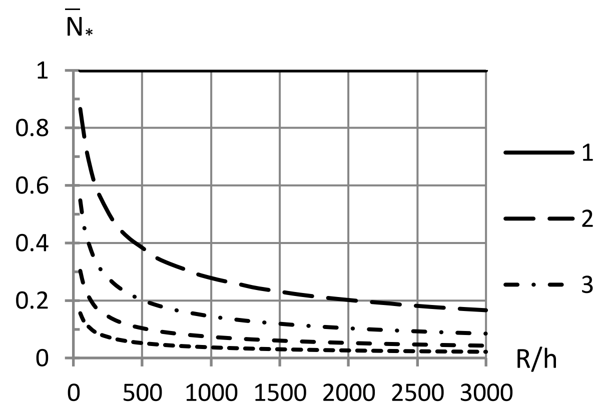

Fig. 2. Dependence of a cylindrical shell supported on movable hinges on changes in the ratios L/R and R/h

1 – $\bar{N}_*$ =1; 2 – L/R = 1; 3 – L/R =2; 4 – L/R =4; 5 – L/R = 8.

Comparison of the values of critical loads calculated by (5) and (6) showed a large influence of the boundary conditions on $\overline{N_*}$.

In this paper, these studies are continued. The essence of this method is more fully expounded.

Stability under axial compression is considered: cylindrical shells with rigidly embedded edges; rigidly supported, rigidly fixed and with a combined fixing of platinum; hinged and rigidly fixed rods. The accuracy of calculations by this method was estimated on the basis of a comparison of the results obtained with the results obtained with the classical approach and with experimental data.

3. The purpose and objectives of the study

Show that one of the reasons for the large difference between the calculated and experimental critical loads is the incorrect interpretation of the buckling process.

4. Materials and research methods

To solve the problem, the energy stability criterion and the relation of the general linear theory of thin-walled structures are used.

The accuracy of determining the critical loads by the energy method depends on how close the shape of the buckling is close to the actual form of buckling. As in this paper, set the movement u, v, w, in the direction of the axes x, y, z respectively, described below.

5. Problem solving

5.1. Cylindrical shell with hard-edged edges

Cylindrical shell length L, radius R, with wall thickness *h*, loaded along edges evenly distributed compressive forces *N* (Fig.3).

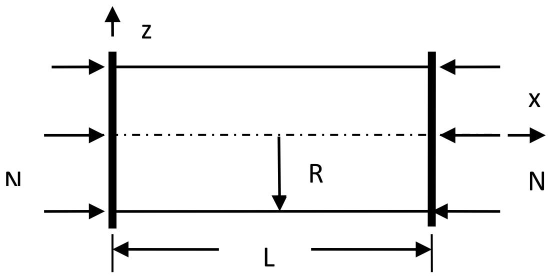

Fig. 3. Cylindrical shell

Initial assumptions: the shell is geometrically perfect and ideally elastic, the subcritical state is momentless, the edges of the shell are rigidly sealed.

According to [8], if the edges of the shell are rigidly embedded, the boundary conditions have the form

$$u = v = w = \frac{\partial w}{\partial x} = 0 .$$

Satisfying the boundary conditions of displacement, in the direction of the axes y and z set

$$v = f_2 \sin^2 \frac{m\pi x}{L} \cos \frac{ny}{R} ;$$

$$w = f_3 \sin^2 \frac{v\pi x}{L} \sin \frac{ny}{R} , \qquad (7)$$

where f_2, f_3 – amplitudes in the direction of the axes y and z. Move in the direction of the axis x we find from the condition

$$\frac{\partial u}{\partial x} + \frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2 = f(x, y) . \qquad (8)$$

Integrating expression (8), we get

$$u = \int f(x, y) -$$

$$\frac{1}{4} f_3^2 \left(\frac{m\pi}{L}\right)^2 \left(x - \frac{L}{4m\pi} \sin 4\frac{m\pi}{L}\right) \sin^2 \frac{ny}{R} . \quad (9)$$

In accordance with the boundary conditions, $u(0) = u(L) = 0$, in this case, $u(x, y) \neq 0$. These conditions are met if:

$$\int f(x, y) =$$

$$\frac{1}{4} f_3^2 \left(\frac{m\pi}{L}\right)^2 \left(x + \frac{L}{4m\pi} \sin 4\frac{m\pi}{L}\right) \sin^2 \frac{ny}{R} , \qquad \text{or}$$

$$\int f(x, y) dx = \frac{1}{4} f_3^2 \left(\frac{m\pi}{L}\right)^2 x \sin^2 \frac{ny}{R} . \qquad (10)$$

Substituting (10) in (9) we find:

a) $u = \frac{1}{8} f_3^2 \frac{m\pi}{L} \sin 4\frac{m\pi x}{L} \sin^2 \frac{ny}{R}$;

b) $u = \frac{1}{16} f_3^2 \frac{m\pi}{L} \sin 4\frac{m\pi x}{L} \sin^2 \frac{ny}{R}$.

However, the displacement should always be negative since it compensates for the positive displacements due to bending. Therefore, we ask

$$u = -\frac{1}{8} f_3^2 \frac{m\pi}{L} \sin 4\frac{m\pi x}{L} \sin^2 \frac{ny}{R} . \quad (11)$$

Case b) is not considered, since it gives a greater value of the critical effort N_*.

Change in shell deformation energy due to loss of stability

$$\Delta V = \frac{Eh}{2(1-\nu^2)} \int_0^{2\pi R} \int_0^L \left\{ \begin{array}{l} \varepsilon_1^2 + 2\nu\varepsilon_1\varepsilon_2 + \varepsilon_2^2 + \\ \frac{1-\nu}{2}\varepsilon_{12}^2 + \frac{h^2}{12}[\chi_1^2 + \\ 2\nu\chi_1\chi_2 + \chi_2^2 + \\ 2(1-\nu)\chi_{12}^2 \end{array} \right\} dxdy , \quad (12)$$

where $\varepsilon_1 = \frac{\partial u}{\partial x}; \varepsilon_2 = \frac{\partial v}{\partial y} + \frac{w}{R}; \varepsilon_{12} = \frac{\partial v}{\partial x} + \frac{\partial u}{\partial y};$

$$\chi_1 = -\frac{\partial^2 w}{\partial x^2}; \chi_2 = -\frac{\partial^2 w}{\partial y^2} + \frac{1}{R}\frac{\partial v}{\partial y};$$

$$\chi_{12} = -\frac{\partial^2 w}{\partial x \partial y} + \frac{1}{R}\frac{\partial v}{\partial x}.$$

Substituting (7), (11) into (12) and performing the operations of differentiation and integration, we obtain

$$\Delta V = \frac{Eh}{1-\nu^2}\begin{pmatrix} \frac{3}{16} f_3^4 \frac{\lambda^2}{R^2} b_4 + f_2^2 b_1 + \\ 2 f_2 f_3 b_2 + f_3^2 b_3 \end{pmatrix}, \quad (13)$$

where

$$b_1 = \frac{1-\nu}{2}\lambda^2 + \frac{3}{4}n^2 + \frac{1}{12}\left(\frac{h}{R}\right)^2 [2(1-\nu)\lambda^2 + \frac{3}{4}n^2];$$

$$b_2 = n\left[\frac{3}{4} + \frac{1}{12}\left(\frac{h}{R}\right)^2 [(2-\nu)\lambda^2 + \frac{3}{4}n^2]\right];$$

$$b_3 = \frac{3}{4} + \frac{1}{12}\left(\frac{h}{R}\right)^2 \left[4\lambda^4 + 2\lambda^2 n^2 + \frac{3}{4}n^4\right];$$

$$b_4 = \lambda^2 + \frac{1-\nu}{24}n^2.$$

The work of the external load is equal to

$$\Delta A = \frac{1}{2}\int_0^{2\pi R}\int_0^L \left(N_* - \frac{1}{2}N_1\right)\left(\frac{\partial w}{\partial x}\right)^2 dxdy. \quad (14)$$

Multiplier $\frac{1}{2}$ at N_1 appeared because the transition of the shell from the initial non-deformed to the deformed state is accompanied by a change in N_1 from zero to its maximum value by module [9].

External load N_1 numerically equal to internal efforts T_1:

$$N_1 = \frac{Eh}{1-\nu^2}[\varepsilon_1(0) + \nu\varepsilon_2(0)] = \frac{Eh}{1-\nu^2}[\varepsilon_1(L) + \nu\varepsilon_2(L)] = -\frac{Eh}{2(1-\nu^2)} f_3^2 \left(\frac{m\pi}{L}\right)^2 \sin^2\frac{ny}{R}.$$

From the condition $\Delta A = 0$ we find

$$\frac{Eh}{1-\nu^2}\frac{3}{16} f_3^4 \frac{\lambda^4}{R^2} + N_* f_3^2 \lambda^2;$$

$$\frac{3}{16} f_3^2 \frac{\lambda^2}{R^2} = -\frac{1-\nu^2}{Eh} N_*. \quad (15)$$

In view of (13) and (15) we find

$$\Delta U = \Delta V - \Delta A;$$

$$\Delta U = \frac{Eh}{1-\nu^2}\left[\begin{matrix} f_2^2 b_1 + 2 f_2 f_3 b_2 + \\ f_3^2\left(b_3 - \frac{1-\nu^2}{Eh} N_* b_4\right) \end{matrix}\right]\frac{\pi L}{4R} \quad (16)$$

N_* we obtain from the condition of the Minimum potential energy displacements

$$\frac{\partial \Delta U}{\partial f_2} = 0; \frac{\partial \Delta U}{\partial f_3} = 0:$$

$$N_* = \frac{Eh}{1-\nu^2}\frac{1}{b_4}\left(b_3 - \frac{b_2^2}{b_1}\right); \overline{N_*} = \frac{N}{N_*^в}. \quad (17)$$

In Fig. 4 presents the results of minimizing $\overline{N_*}$ for the integer parameters m and n, when $\nu = 0,3$

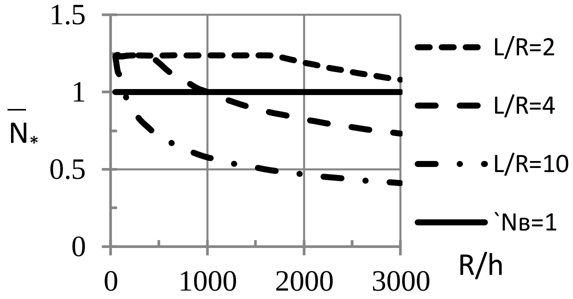

Fig. 4. Dependence $\overline{N_*}$ on relationships L/R and R/h

5.2. The stability of rectangular plates uniformly compressed in the direction of the axis

The plate with sides a, b and thickness h is compressed in the median plane by forces N, evenly distributed on sides $x = 0$ and $x = a$.

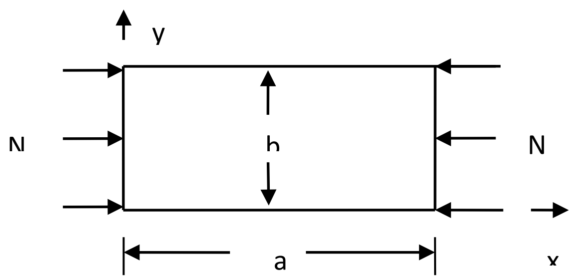

Fig. 5. Rectangular plate

5.2.1. Hinged plates on all edges

Offsets w set; u we find also as in shells, ν we find similarly, however, the amplitude value f_2 is set arbitrarily, since edges $y = 0$ and $y = b$ are not loaded

$$u = -\frac{1}{4} f_3^2 \frac{m\pi}{a}\sin 2\frac{m\pi x}{a}\sin^2\frac{n\pi y}{b};$$

$$v = f_2 \sin 2\frac{m\pi x}{a}\sin^2\frac{n\pi y}{b};$$

$$w = f_3 \sin \frac{m\pi x}{a} \sin \frac{n\pi y}{b}.$$

Plate deformation energy

$$\Delta V = \frac{Eh}{2(1-\nu^2)} \int_0^b \int_0^a \left\{ \begin{matrix} \varepsilon_1^2 + 2\nu\varepsilon_1\varepsilon_2 + \varepsilon_2^2 + \\ \frac{1-\nu}{2}\varepsilon_{12}^2 + \frac{h^2}{12}[\chi_1^2 + \chi_2^2 + \\ 2\nu\chi_1\chi_2 + 2(1-\nu)\chi_{12}^2] \end{matrix} \right\} dxdy, \quad (18)$$

where $\varepsilon_1 = \frac{\partial u}{\partial x}; \varepsilon_2 = \frac{\partial v}{\partial y}; \varepsilon_{12} = \frac{\partial v}{\partial x} + \frac{\partial u}{\partial y};$

$$\chi_1 = -\frac{\partial^2 w}{\partial x^2}; \chi_2 = -\frac{\partial^2 w}{\partial y^2}; \chi_{12} = -\frac{\partial^2 w}{\partial x \partial y}.$$

The work of the external load is equal to

$$\Delta A = \frac{1}{2} \int_0^b \int_0^a \left(N_* - \frac{1}{2} N_1 \right) \left(\frac{\partial w}{\partial x} \right)^2 dxdy. \quad (19)$$

Further solution was carried out as in the case of shells. As a result, we have

$$N_* = k_1 \frac{\pi D}{b^2}, \quad (20)$$

where

$$k_1 = \frac{\left(\frac{mb}{a} + \frac{a}{mb} \right)^2}{1 + \frac{1-\nu}{6}\left(\frac{a}{mb} \right)^2 - \frac{(1+\nu)^2}{12\left[3 + \frac{1-\nu}{2}\left(\frac{mb}{a} \right)^2 \right]}}.$$

In Fig. 6 presents the results of calculations by (20) and below obtained formulas of coefficients k_1, k (k is the coefficient obtained by Brian) and k_i, if $\nu = 0{,}3$.

5.2.2 Plate with loaded clamped and unloaded hinged edges

In this case, the bulging surface can be represented as follows.

$$u = -\frac{1}{8} f_3^2 \frac{m\pi}{a} \sin 4\frac{m\pi x}{a} \sin^2 \frac{n\pi y}{b};$$

$$v = f_2 \sin^4 \frac{m\pi x}{a} \sin 2\frac{n\pi y}{b}$$

$$w = f_3 \sin^2 \frac{m\pi x}{a} \sin \frac{n\pi y}{b}.$$

Similar to the above, we get $N_* = k_2 \frac{\pi D}{b^2}$; (21)

$$k_2 = \frac{4\left(\frac{mb}{a} \right)^2 + \frac{3}{4}\left(\frac{a}{mb} \right)^2 + 2}{1 + \frac{1-\nu}{24}\left(\frac{a}{mb} \right)^2 - \frac{(1+\nu)^2}{60\left[7 + 2(1-\nu)\left(\frac{mb}{a} \right)^2 \right]}}.$$

Solving the problem by the energy method, but in the traditional way, when the change in the external load during the buckling of the plate is not taken into account, we obtain

$$N_* = \frac{\pi^2 D}{b^2}\left[4\left(\frac{mb}{a} \right)^2 + \frac{3}{4}\left(\frac{a}{mb} \right)^2 + 2 \right] = k_{21} \frac{\pi^2 D}{b^2}$$

5.2.3. Plate with pivotally loaded and unloaded pinched edges

In this case

$$u = -\frac{1}{4} f_3^2 \frac{m\pi}{a} \sin 2\frac{m\pi x}{a} \sin^4 \frac{n\pi y}{b},$$

$$v = f_2 \sin^2 \frac{m\pi x}{a} \sin 4\frac{n\pi y}{b},$$

$$w = f_3 \sin \frac{m\pi x}{a} \sin^2 \frac{n\pi y}{b}.$$

Critical load $N_* = k_3 \frac{\pi D}{b^2}$, (22)

where

$$k_3 = \frac{\left(\frac{mb}{a} \right)^2 + \frac{16}{3}\left(\frac{a}{mb} \right)^2 + \frac{8}{3}}{1 + \frac{2(1-\nu)}{7}\left(\frac{a}{mb} \right)^2 - \frac{(1+\nu)^2}{35\left[12 + \frac{1-\nu}{2}\left(\frac{mb}{a} \right)^2 \right]}}.$$

When not taking into account changes in external loads

$$k_{31} = \left(\frac{mb}{a} \right)^2 + \frac{16}{3}\left(\frac{a}{mb} \right)^2 + \frac{8}{3}.$$

5.2.4. The plate is rigidly clamped along all edges

$$u = -\frac{1}{8} f_3^2 \frac{m\pi}{a} \sin 4\frac{m\pi x}{a} \sin^4 \frac{n\pi y}{b}$$

$$v = f_2 \sin^4 \frac{m\pi x}{a} \sin 4\frac{n\pi y}{b},$$

$$w = f_3 \sin^2 \frac{m\pi x}{a} \sin^2 \frac{n\pi y}{b}.$$

$$k_4 = \frac{4\left[\left(\frac{mb}{a}\right)^2 + \left(\frac{a}{mb}\right)^2 + \frac{2}{3}\right]}{1 + \frac{1-\nu}{14}\left(\frac{a}{mb}\right)^2 - \frac{(1+\nu)^2}{4900\left[1 + \frac{1-\nu}{2}\left(\frac{mb}{a}\right)^2\right]}} \quad (23)$$

When not taking into account changes in external loads $k_{41} = 4\left[\left(\frac{mb}{a}\right)^2 + \left(\frac{a}{mb}\right)^2 + \frac{2}{3}\right]$.

Conclusion: the proposed approach gives good results when solving problems of stability plates.

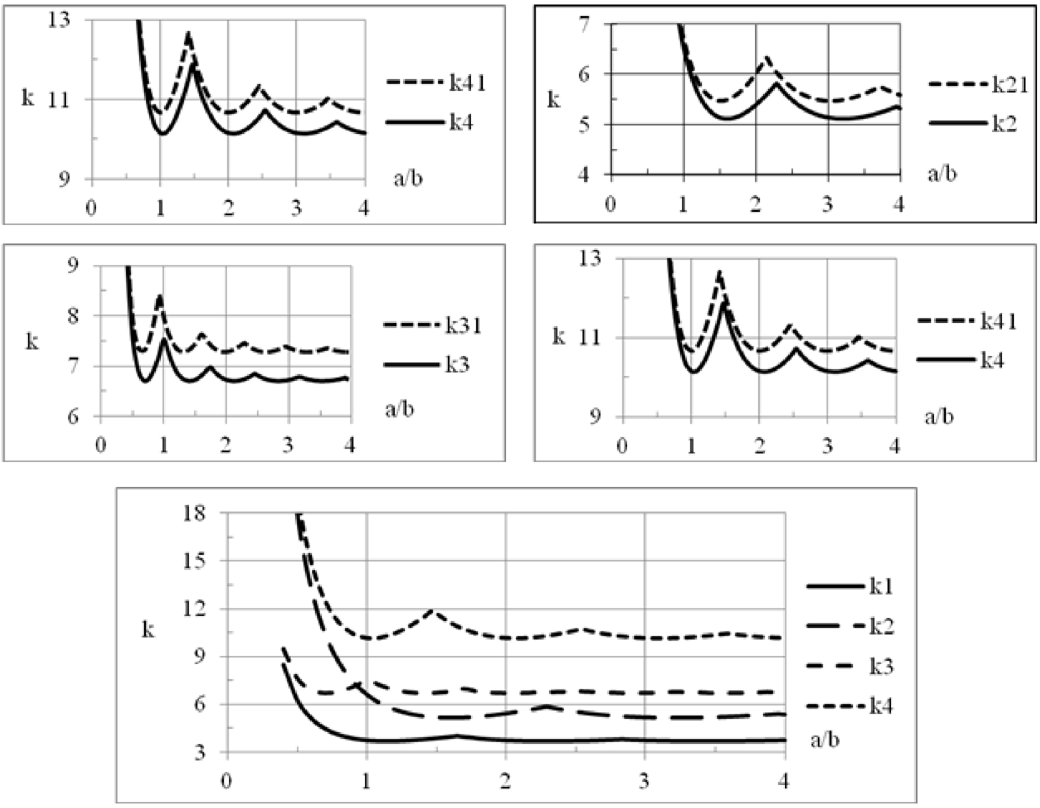

Fig. 6. The coefficient *k* under various conditions of fixing the plate, compressed in one direction

5.3. Pivotally supported rod loaded with force

$$w = f\sin\frac{m\pi x}{L};\ u = -\frac{1}{4}f^2\frac{m\pi}{L}\sin 2\frac{m\pi x}{L};$$

$$P_* = \left(\frac{\pi}{L}\right)^2 EJ. \quad (24)$$

Formula (24) completely coincides with the Euler formula.

5.4. Rigidly fixed rod

$$w = f\sin^2\frac{m\pi x}{L};\ u = -\frac{1}{8}f^2\frac{m\pi}{L}\sin 4\frac{m\pi x}{L};$$

$$P_* = 4\left(\frac{\pi}{L}\right)^2 EJ. \quad (25)$$

Formula (25) fully coincides with the solutions obtained by other authors, which confirms the validity of the proposed approach in this case.

6. Results and discussion

Analysis of the results of theoretical calculations using the formulas obtained in [12, 13] and in this work, which are presented in Fig. 2, Fig. 3 and Fig. 4, shows:

1. The absolute and relative values of the axial critical loads of cylindrical shells strongly depend on the ratio of the radius to the thickness and on the ratio of the length to the radius of the shell.

2. Critical loads are highly dependent on boundary conditions.

3. Theoretical critical loads calculated by the obtained formulas are close to the experimental data systematized in [6]. However, for a more accurate assessment, it is necessary to take into account, in each particular case, the geometrical and mechanical characteristics of the shell and the conditions for fastening the edges.

4. The formulas obtained, on the basis of the accepted assumption, more accurately describe the process of buckling of the shells and plates, since

they give lower values of critical loads compared to the classical solution.

5. The coincidence of the obtained formulas for the critical rod forces with the Euler formula and other authors confirms the validity of the proposed approach.

7. Conclusions

1. One of the reasons for the discrepancy between the calculated and experimental critical loads of the shells and plates is the incorrect interpretation of the process of their buckling.
2. The obtained formulas can be used to calculate the axial critical loads of real shells and plates.
3. The proposed approach may bring researchers closer to solving the problem of stability and bearing capacity of engineering structures as a whole**.**

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Новий підхід до визначення осьових критичних навантажень оболонок, пластин і стрижнів

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Мета: Показати, що однією з причин великої різниці між розрахунковими і експериментальними критичними навантаженнями є неправильне трактування процесу випучування. **Метод**: Використовується енергетичний критерій стійкості і співвідношення загальної лінійної теорії тонкостінних конструкцій. **Результати**: Отримано нові формули критичних навантажень оболонок і пластин. **Обговорення**: Для оцінки несучої здатності інженерних споруд, необхідні точні формули для обчислення критичних навантажень при осьовому стисненні. Такі формули поки що не отримані. Причину великих розбіжностей між теоретичними і експериментальними значеннями осьових критичних навантажень циліндричних оболонок не знайдено. У даній роботі зроблена спроба вирішити цю проблему. На відміну від традиційного підходу, тут передбачається, що при випучуванні конструкції, відстань між навантаженими кінцями не змінюється. Це дозволило отримати нові формули осьових критичних навантажень. Критичні навантаження, розраховані за цими формулами, близькі до експериментальних даних. На основі цього зроблено висновок, що отримані формули можна використовувати для реальних розрахунків критичних навантажень циліндричних оболонок і пластин, а запропонований підхід може бути використаний для продовження досліджень стійкості тонкостінних конструкцій.

Ключові слова: вигин, критичне навантаження, зміщення, стійкість, експеримент, енергія

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Новый подход к определению осевых критических нагрузок оболочек , пластин и стержней

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Цель: Показать, что одной из причин большого различия между расчетными и экспериментальными критическими нагрузками является неправильная трактовка процесса выпучивания. **Метод:** Используется энергетический критерий устойчивости и соотношения общей линейной теории тонкостенных конструкций. **Результаты:** Получены новые формулы критических нагрузок оболочек и пластин. **Обсуждение:** Для оценки несущей способности инженерных сооружений, необходимы точные формулы для вычисления критических нагрузок при осевом сжатии. Такие формулы пока не получены. Причина больших расхождений между теоретическими и экспериментальными значениями осевых критических нагрузок цилиндрических оболочек не обнаружена. В данной работе предпринята попытка решить эту проблему. В отличие от обычного подхода, здесь предполагается, что при выпучивании конструкции, расстояние между нагруженными концами не изменяется. Это позволило получить новые формулы осевих критических нагрузок. Значения критических нагрузок, рассчитанные по этим формулам, близки к экспериментальным данным. На основе этого сделан вывод, что полученные формулы можно использовать для реальных расчетов критических нагрузок цилиндрических оболочек и пластин, а предложенный подход может быть использован для продолжения исследований устойчивости тонкостенных конструкций.

Ключевые слова: изгиб, критическая нагрузка, смещение, устойчивость, эксперимент, энергия

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