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THE ANALYSIS OF TEMPERATURE AND SHRINKAGE-RHEOLOGICAL STRESS IN TWO-DIMENSIONAL SYSTEMS WITH AN EXTERNAL CERAMIC-METAL LAYER

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In the work the temperature and shrinkage-rheological stresses in friction surfaces with ceramic-metal layer that is obtained at using of tryboretrofitting mixes are researched. The results presented by authors allow to improve mathematical models of stress-deformation condition of ceramic-metal system available on today.

Досліджено температурні та усадочно-реологічні напруження в поверхнях тертя з металокерамічним шаром, одержаним у процесі використання трибовідновлювальних сумішей. Отримані результати дозволяють удосконалювати математичні моделі напружено-деформаційного стану металокерамічної системи, що існують на сьогодні.

Introduction

The friction units repair of the wide circle of technical products without disassembling with the use of tryboretrofitting mixes (TRM).

These mixes have received a wide application recently because they make possible to produce "repair" or restoration of the friction surfaces' properties of tribotechnical systems without the fulfillment of sufficiently labor-consuming operations, which are connected with dismantling of friction units [1].

The analysis of the contemporary state of the world market for similar mixes shows the presence of their broad spectrum, characterized by a significant quantity of trade marks [1; 2].

In the place with the fact, the existing recommendations regarding their application do not consider the specific character of the working temperature of the friction pairs, for example, of the temperature in the friction units of the gas turbine engine transmission can have a spread from 60 to 350^{0}C [3].

This leads to a change in structure and physicomechanical characteristics of surface layers of friction units, and as the consequence a change nature and value of their wear.

Therefore the application of one or the other types TRM requires not only the optimization of their selection for the friction pairs, prepared from the specific materials, but also the optimization of the operating conditions of tribotechnical systems [1; 2].

Thus, the obtaining of the objective information about the stress-deformable condition of the ceramic-metal (CM) layer formed at using of the given sort of mixes, is an actual problem which solving will allow to improve the given technology.

Statement of a problem and theoretical research

Consider the two-layer system (the base layer the CM layer) as structural- orthotropic shell. It works in stationary heating conditions, thus the temperature represents function $t = f(\alpha_1, \alpha_2, Z)$.

We shall designate factors of linear expansion of the CM layer and a base surface accordingly β_1, β_2.

If to accept Dugemal-Neyman's thermoelastic hypotheses it is necessary to consider the fact, that in two-layer systems, the thermal deformations and stresses arise at uniform heating. In this case factor of internal forces in view of thermal deformations:

$$\begin{cases} T_1 = B\left[\varepsilon_1 + \nu_{np}\varepsilon_2 - \left(1+\nu_{np}\right)\beta t_0\right]; \\ T_2 = B\left[\varepsilon_2 + \nu_{np}\varepsilon_1 - \left(1+\nu_{np}\right)\beta t_0\right]; \\ S = B\left(1-\nu_{np}\right)\varepsilon_3; \\ G_1 = -D\left[\chi_1 + \nu_{np}\chi_2 - \left(1+\nu_{np}\right)\beta t_1\right]; \\ G_2 = -D\left[\chi_2 + \nu_{np}\chi_1 - \left(1+\nu_{np}\right)\beta t_1\right]; \\ H = -D\left(1-\nu_{np}\right)\chi_3. \end{cases} \qquad (1)$$

The following generalized temperature characteristics are inducted into system of the equations (1):

$$N = B\left(1+\nu_{np}\right)\beta t_0;$$

$$M = B\left(1+\nu_{np}\right)\beta t_1,$$

where t_0, t_1 – initial and final temperatures.

Thus, the system of the equations (1) can be written down in the form of:

$$\begin{cases} T_1 = B\left(\varepsilon_1 + \nu_{np}\varepsilon_2\right) - N; \\ T_2 = B\left(\varepsilon_2 + \nu_{np}\varepsilon_2\right) - N; \\ G_1 = -D\left(\chi_1 + \nu_{np}\chi_2\right) + M; \\ G_2 = -D\left(\chi_2 + \nu_{np}\chi_1\right) + M; \\ S = B\left(1-\nu_{np}\right)\varepsilon_3; \\ H = -D\left(1-\nu_{np}\right)\chi_3. \end{cases}$$

Passing from internal force factors (consider that components of deformations ε_1, ε_2, ε_3, χ_1, χ_2, χ_3 are known), we shall define stress components in view of temperature deformation – in the top MC layer:

$$\sigma_1^{(1)} = \overline{E}_1\left[\varepsilon_1 + \nu_1\varepsilon_2 + Z(\chi_1 + \nu_1\chi_2) - (1+\nu_1)\beta_1 t\right];$$
$$\sigma_2^{(1)} = \overline{E}_1\left[\varepsilon_2 + \nu_1\varepsilon_1 + Z(\chi_2 + \nu_1\chi_1) - (1+\nu_1)\beta_1 t\right]; \quad (2)$$
$$\tau^{(1)} = \overline{E}_1(1-\nu_1)(\varepsilon_3 + Z\chi_3).$$

In the bottom basis of system:

$$\sigma_1^{(2)} = \overline{E}_2\left[\varepsilon_1 + \nu_2\varepsilon_2 + Z(\chi_1 + \nu_2\chi_2) - (1+\nu_2)\beta_2 t\right];$$
$$\sigma_2^{(2)} = \overline{E}_2\left[\varepsilon_2 + \nu_2\varepsilon_1 + Z(\chi_2 + \nu_2\chi_1) - (1+\nu_2)\beta_2 t\right]; \quad (3)$$
$$\tau^{(2)} = \overline{E}_2(1-\nu_2)(\varepsilon_3 + Z\chi_3).$$

The general scheme of stress calculation from temperature influence on two-layer system «the MC–layer–base» is those.

Let's consider a concrete case of asymmetric heating of two-layer system which takes place at friction of two-layer system on all length of contact to a counterbody.

Components of deformations results:

$$\varepsilon_1 = \frac{\partial U}{\partial X}; \quad \varepsilon_2 = \frac{\omega}{R}; \quad \chi_1 = -\frac{\partial^2\omega}{\partial\chi^2}, \quad (4)$$

where R is radius of the reduction surface's curvature. Thus internal force factors:

$$\begin{cases} T_1 = B\left(\dfrac{\partial U}{\partial X} + \nu_{np}\dfrac{\omega}{R}\right) - N; \\ T_2 = B\left(\dfrac{\omega}{R} + \nu_{np}\dfrac{\partial U}{\partial X}\right) - N; \\ G_1 = D\dfrac{\partial^2\omega}{\partial X^2} + M; \\ G_2 = D\nu_{np}\dfrac{\partial^2\omega}{\partial X^2} + M. \end{cases} \quad (5)$$

The equations of balance of the reduction surface at asymmetric heating:

$$T_1 = 0; \quad \frac{\partial G_1}{\partial X} = Q_1; \quad \frac{\partial^2 G_1}{\partial X^2} + \frac{T_2}{R} = 0. \quad (6)$$

From the first equation (6) it is defined:

$$\frac{\partial U}{\partial X} = \frac{N}{B} - \nu_{np}\frac{\omega}{R}. \quad (7)$$

Substituting (7) in the second equation of system (5) we receive:

$$T_2 = B\left(1-\nu_{np}^2\right)\frac{\omega}{R} - (1-\nu_{np})N. \quad (8)$$

Substituting (8) and two last equations of system (5) in last equation of balance (4), we receive the differential equation describing asymmetric heating of two-layer system:

$$\frac{\partial^4\omega}{\partial X^4} + 4K^4\omega = \frac{(1-\nu_{np})N}{DR} - \frac{1}{D}\frac{\partial^2 M}{\partial X^2}, \quad (9)$$

where K is parameter of attenuation of regional effects:

$$K^4 = \frac{B\left(1-\nu_{np}^2\right)}{4DR^2}.$$

If two-layer system with free edges is hearted regularly up to t = const, the generalized temperature characteristics become:

$$\begin{cases} N = \left(\dfrac{E_1\delta_1\beta_1}{1-\nu_1} + \dfrac{E_2\delta_2\beta_2}{1-\nu_2}\right)t; \\ M = \dfrac{E_1E_2\delta_1\delta_2\delta}{E_1\delta_1 + E_2\delta_2}\left(\dfrac{\beta_1}{1-\nu_1}\right)\dfrac{t}{2}. \end{cases} \quad (10)$$

The solution of the differential equation (9) disappearing on infinity, looks like:

$$\omega = c_1\Phi_1(\kappa x) + c_2\Phi_2(\kappa x) + \frac{NR}{B(1+\nu)},$$

where Φ_1, Φ_2 are decreasing exponential-trigonometrical functions:

$\mathbf{\Phi}_1 = e^{-kx}\cos kx$; $\mathbf{\Phi}_2 = e^{-kx}\sin kx$.

Constants of integration are defined from boundary conditions on the shell's free end t: at $x = 0$, $G_1 = Q_1 = 0$, the intense-deformed condition of the system will be symmetric concerning the middle of system.

From boundary conditions we receive:

$$c_1 = -\frac{M}{2k^2D}; \quad c_2 = -\frac{M}{2k^2D}.$$

The shell's deflection from edge up to the middle will be defined by expression:

$$\omega = \frac{M}{2\kappa^2D}e^{-\kappa x}(\sin\kappa x - \cos\kappa x) + \frac{NR}{B(1+\nu_{np})}. \quad (11)$$

Deformation of shell's free edge ω_0 is defined so:

$$\omega_0 = \frac{R}{\sqrt{B(1+\nu_{np})}}\left[\frac{N}{\sqrt{B(1+\nu_{np})}} - \frac{M}{\sqrt{D(1-\nu_{np})}}\right].$$

Components of other deformations of free edge are defined, proceeding from (11) and (10):

$$\begin{cases} \chi_1 = \dfrac{M}{D}e^{-\kappa x}(\sin\kappa x + \cos\kappa x); \\ \varepsilon_1 = \dfrac{N}{B(1+\nu_{np})} - \dfrac{\nu_{np}M}{\sqrt{BD\left(1-\nu_{np}^2\right)}}e^{-\kappa x}(\sin\kappa x - \cos\kappa x); \\ \varepsilon_2 = \dfrac{M}{\sqrt{BD\left(1-\nu_{np}^2\right)}}e^{-\kappa x}(\sin\kappa x - \cos\kappa x) + \dfrac{N}{B(1+\nu_{np})}. \end{cases}$$

The stresses arising in normal cross-sections of two-layer system, will be defined under formulas (2) and (3).

Thus the interlayer shifts which are proportional to the intersected force Q should appear in the CM layer.

As to internal shrinking-rheological stresses in two-layer systems with an external CM layer, the shrinkage of CM layer (more than 20 %) takes place as a result of rheological phenomena.

Because the strong connection of layers interferes with free shrinkage and to temperature expansion in two-layer system there are the additional internal stresses causing the warpage of the CM layer.

Name these stresses as shrinking-rheological stresses and assume, that the phenomenon of shrinkage represents quasi-elastic is volumetric-deformation similarly to temperature expansion. The value of relative change of linear dimensions because of the elastic shrinkage Δ is the analogue of linear expansion coefficient. This new mechanical characteristic of the CM layer depends on material nature of CM layer, propensity to rheological properties, etc. We shall apply the hypotheses similar to accept Dugemal-Neyman's thermoelastic hypotheses, etc. we shall assume, that shrinking-rheological stresses are connected with elastic components of deformations according to Guk's law.

The system of coordinates and position of a reduction surface are defined as well as in case of definition of temperature stresses.

According to the accepted elastic-shrinkable hypothesis and Kirgoff-Lyav hypotheses for definition it is elastic-rheological stresses we receive expressions.

In the top layer (CM layer):

$$\sigma_1^{(1)} = \overline{E}_1\left[\varepsilon_1 + \nu_1\varepsilon_2 + Z\left(\chi_1 + \nu_1\chi_2\right) + \left(1+\nu_1\right)\Delta\right];$$

$$\sigma_2^{(1)} = \overline{E}_1\left[\varepsilon_2 + \nu_1\varepsilon_1 + Z\left(\chi_2 + \nu_1\chi_1\right) + \left(1+\nu_1\right)\Delta\right];$$

$$\tau^{(1)} = \overline{E}_1\left(1-\nu_1\right)\left(\varepsilon_3 + Z\chi_3\right).$$

In the bottom basis of system (the base surface):

$$\sigma_1^{(2)} = \overline{E}_2\left[\varepsilon_1 + \nu_2\varepsilon_2 + Z\left(\chi_1 + \nu_2\chi_2\right) + \left(1+\nu_2\right)\beta_2 t_T\right];$$

$$\sigma_2^{(2)} = \overline{E}_2\left[\varepsilon_2 + \nu_2\varepsilon_1 + Z\left(\chi_2 + \nu_2\chi_1\right) + \left(1+\nu_2\right)\beta_2 t_T\right];$$

$$\tau^{(2)} = \overline{E}_2\left(1-\nu_2\right)\left(\varepsilon_3 + Z\chi_3\right),$$

where t_T is temperature of the unit with the CM layer.

Internal forces and the moments are defined so (under condition of Δ=const , that most often meets in an engineering practice):

$$\begin{cases} T_1 = B\left(\varepsilon_1 + \nu_{np}\varepsilon_2 + \varepsilon_\Delta\right); \\ T_2 = B\left(\varepsilon_2 + \nu_{np}\varepsilon_1 + \varepsilon_\Delta\right); \\ S = B\left(1-\nu_{np}\right)\varepsilon_3; \\ G_1 = -D\left(\chi_1 + \nu_{np}\chi_2 + \chi_\Delta\right); \\ G_2 = -D\left(\chi_2 + \nu_{np}\chi_1 + \chi_\Delta\right); \\ H = -D\left(1-\nu_{np}\right)\chi_3. \end{cases} \qquad (12)$$

In system (12) key parameters describing value of shrinkable stresses and warping of two-layer systems at shrinkage (ε_Δ, χ_Δ) are used:

$$\varepsilon_\Delta = \frac{1}{B}\left[\frac{E_1\delta_1\Delta}{1-\nu_1} + \frac{E_2\delta_2\beta_2 t}{1-\nu_2}\right];$$

$$\chi_\Delta = \frac{E_1E_2\delta_1\delta_2\delta}{2BD\left(1-\nu_{np}^2\right)}\left(\frac{\Delta}{1-\nu_1} - \frac{\beta_2 t}{1-\nu_2}\right);$$

$$\delta = \delta_1 + \delta_2.$$

Definition relative elastic shrinkage Δ it is better to make experimentally according to methodology described [4; 5].

Conclusions

The methodology of the temperature shrinking-rheological stresses definition in friction surfaces with CM layer which is developed by authors gives the opportunity to improve current mathematical models of the is intense-deformed condition of ceramic-metal system available today. Precomputations with the use of this model showed that the strength of base layer, its reliability and longevity do not deteriorate. They increase by 3 by 5%. This it proves, that the tryboretrofitting mixes application leads not only to the improvement in the tribological characteristics of friction surfaces, but also they do not worsen the mechanical criteria of the fitness for work of the tribotechnical systems.

Results of the carried out research have shown that in case of a long idle time of friction pairs covered with MCL their operational properties are changed i.e. their resistance to the stretch appearing becomes less.

Such changes of a surface layer can be as a result of relaxation of remaining internal stress of "basic material – MCL" system which can appears due to high ambient temperatures.

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