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RESEARCH OF THE MODAL RESPONSE OF SIMPLY SUPPORTED AND CANTILEVER BEAMS SUBJECTED TO POINT AND DISTRIBUTED HARMONIC EXCITATIONS

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In this paper the parameters of vibration control system, including placement and size of actuators was investigated. The results show that location, value, width of distribution, distribution type and phase difference between the load and actuator have a great influence on noise produced by the beams. The investigation also indicates differences in the laws of control for minimizing sound emission by simply supported and cantilever beams. The influence of actuator parameters are presented on noise cancellation and discussed.

Досліджено параметри систем контролю вібрації, зокрема, розташування та розмір актуаторів. Показано, що розміщення, величина, ширина розподілу, тип розподілу, різниця фаз між навантаженням і компесуючою силою мають значний вплив на шум, що створюється балками. Розглянуто результати досліджень, які вказують на різниці в законах контролю, що використовуються для зменшення випромінювання звуку вільно обпертою та консольними балками, та вплив параметрів актуаторів на зменшення шуму.

Introduction

Vibration control of flexible structures has been a major research topic over the past few years [1; 2]. In recent years, a great number of research results have been presented in active structural vibration control using distributed actuators.

The acoustic field characteristics for some simplest models and methods for their investigation have been proposed [3].

In this study the parametric investigation of acoustical characteristics of simply supported (SS) and cantilever beams are done.

Mathematical formalization of noise emission by a beam includes a model and methods of acoustic characteristics evaluation, that was studied [4; 5].

Comparing sound power levels for simply supported and cantilever beams

The main goal of this article is parametrical research of acoustical characteristics of the noise, produced by SS and cantilever beams with the purpose of finding the best set of actuator parameters, that effectively reduces created noise.

The method of noise assessment, created by a beam [4; 5] was realized in computer program.

It was used for the parametrical investigation of forces' location, parameters of distribution and phase influence on sound radiation characteristics.

Therefore acoustical emission is the function of these parameters and frequency.

In order to find the best set of actuator parameters the sound power levels (SPL) was summarized in the frequency range under investigation.

After that total SPL was defined for every set, and then the set with minimal total SPL was chosen.

For the computations the steel beam with the following parameters is used:
– length is 0,61 m;
– width is 0,051 m;
– height is 0,00635 m;
– young modulus is $2{,}1\cdot10^{11}$ Pa;
– density 7800 kg/m^3;
– damping loss factor 0,011.

Next steps were used for calculations:
– frequency step is equal to 5 Hz;
– step for forces' group location is equal to 0,05 of relative beam length (the beam is considered to have a unit length);
– phase step is equal to π/6;
– step between forces in both load and actuator groups is equal 0,1.

The analysis of the obtained results shows, that the same load produce almost the same noise on these two objects the laws of this load compensation is different.

The case of objects compensation loaded with the 1N force at 0,75 of relative beam length is shown on fig. 1.

The length origin for cantilever beam is its clamped end.

This load is compensated by the point actuator of value 0,8N with the variable location. The phase difference between load and compensating force is π. For SS beam the noise is reduced in any actuator position.

But for cantilever beam the noise increases within the intervals from 0,1 to 0,55 and from 0,9 to the free end. SS beam also has some local minimum in 0,25. And the obvious minimum for both beams is 0,75 of relative beam length.

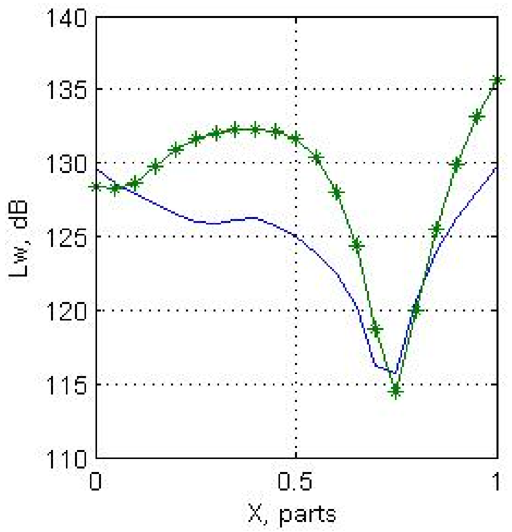


Fig. 1. Total SPL for cantilever (¯*¯) and SS (---) beam at variable actuator position

The influence of actuator distribution

As it was discussed in the previous article the usage of different types of forces value distribution has different effect on noise reduction. For the fig. 2 the load of value 1 N was located at 0,75 of relative beam length.

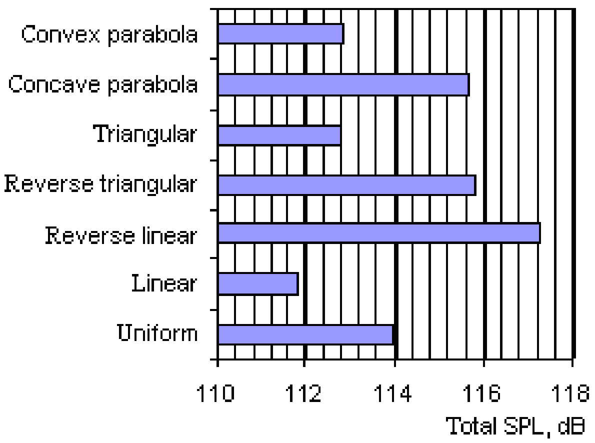


Fig. 2. Total SPL for the point load compensation by actuators distributed in different ways

Investigation was dedicated to the problem of choosing the best type of actuator value distribution for compensating point load. For all types of distribution the phase difference between the actuators and the load was π.

The best SPL reduction is done by convex parabola, triangular and linear loadings. 0,7 is quite close to the minima 0,75, but this distance is not the same for linear and reverse linear loading. That is we have the most good and bad results for this ways of actuator value distribution.

Now let us consider the situation when the load is also distributed fig. 3.

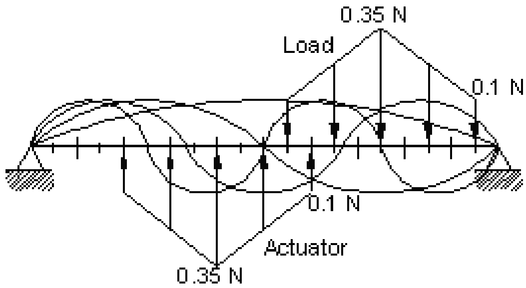


Fig. 3. Scheme for triangular load and actuator beam loading

The curves depicted on the schemes of loadings represent the shapes of oscillations corresponding to different modes. Both load and actuator are of triangular type distribution and the position of actuator group is variable. The load group has the fixed position and distributed between 0,55 and 0,95 of relative beam length with central force applied to 0,75. Phase difference between load and actuator equals π. The results of investigation are depicted on the fig. 4.

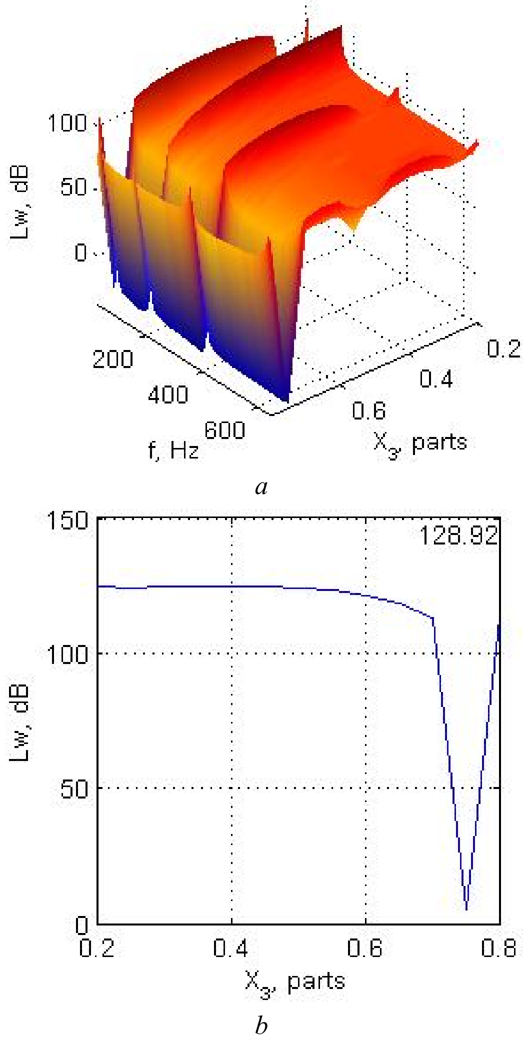


Fig. 4. Sound spectrum (*a*) and total SPL (*b*) for varied location of triangular actuator group

The X_3 coordinate on the figure represents the location of the central force in the actuator group. The digits in the upper-right corner of the total SPL graph correspond to the total SPL produced by the beam subjected to the distributed load of triangular type without the actuator group. As it can be seen the minimum corresponds to 0,75 of relative beam length.

In this position of actuator group 4,88 dB refers more to the calculation error than to the real SPL radiated by the beam.

Calculations show that the shift of actuator position on 0,05 of relative beam length leads to the change of SPL on 109 dB. The values in distributed loadings are chosen in such way that the sum of all forces in the group equals to unit.

For the case shown on fig. 5 the load is also distributed by the triangular law, its position is fixed. The actuator value distribution is uniform. The phase in the actuator group changes linearly from 120 to 240° (2/3π — 4/3π). In this way the phase difference between the central force in the actuator and forces of load group equals π. The location of actuator is variable.

The results of investigation are depicted on the fig. 6.

The first mode minimum corresponds to the location of actuator group at the half of the beam.

This situation is usual for the case when actuator is of less value compared to the load value. But in the considered case they are equal.

The minimum of the second node corresponds to 0,75, and the third mode has the minima at the beam ends.

After summation of the received spectrum we get the minimum corresponding to 0,65 of relative beam length.

Thus we can see that force value distribution cannot be efficiently substituted with the phase distribution.

In the case of uniform actuator value distribution, shown on fig. 7, we also vary the position of actuator group.

Investigations (fig. 8) shows that there are two minima of the first mode corresponding to 0,25 and 0,75. Second mode minimum is 0,75 and third is 0,25. For the considered case noise level reduction is 22 dB. This is worth compared with the situation depicted on fig. 3, 4. The fig. 9 represents the case of beam loading with the fixed triangular load and variable position convex parabolic actuator.

The phase difference between forces in excitation group and forces in compensating group is equal to π. There are no phase difference between forces within one group.

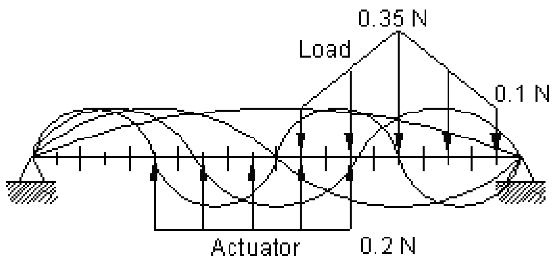
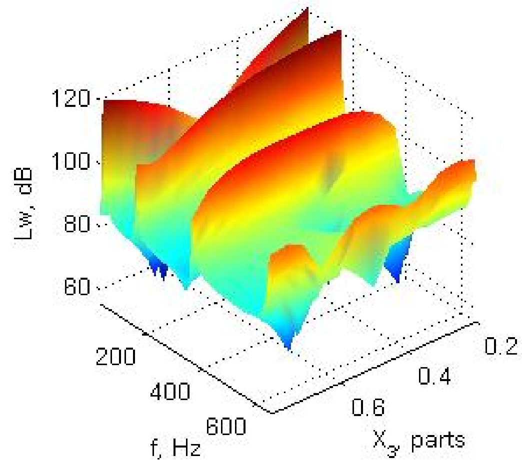
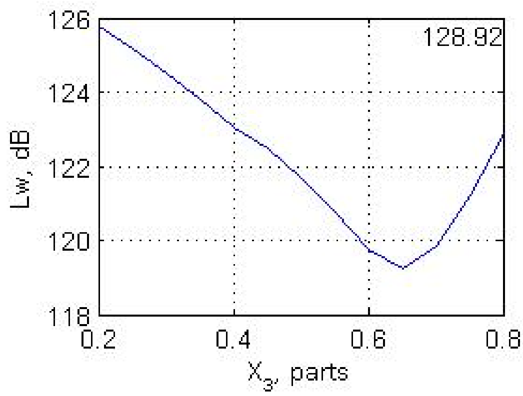


Fig. 5. Scheme for triangular load and linear phase actuator beam loading



a



b

Fig. 6. Sound spectrum (*a*) and total SPL (*b*) for varied location of actuator group with linear phase distribution

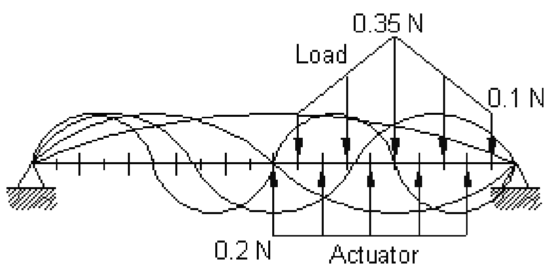
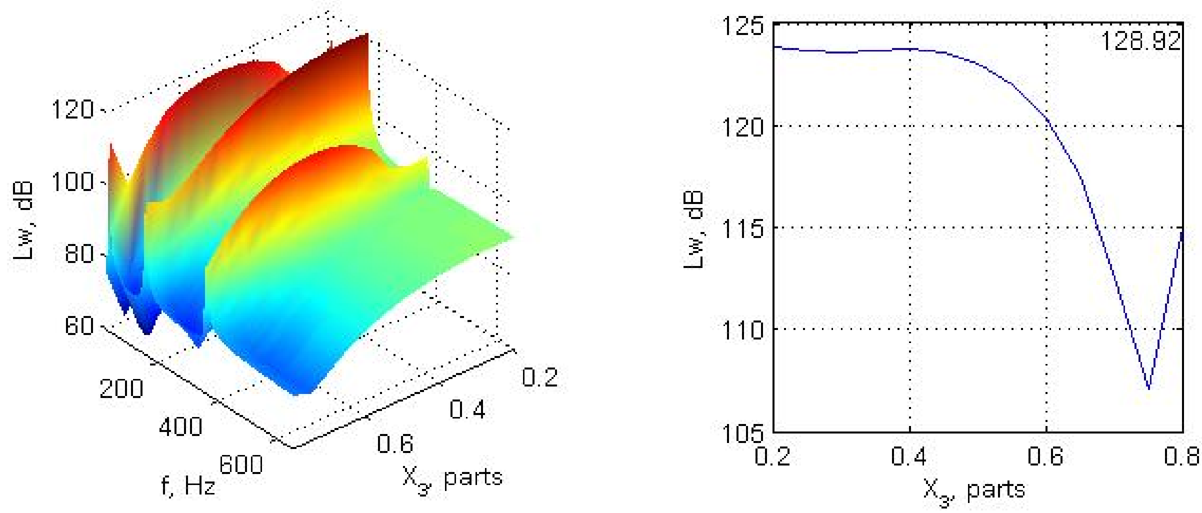


Fig. 7. Scheme for triangular load and uniform actuator beam loading



a *b*

Fig. 8. Sound spectrum (*a*) and total SPL (*b*) for varied location of uniform actuator group, compensating triangular distributed loading

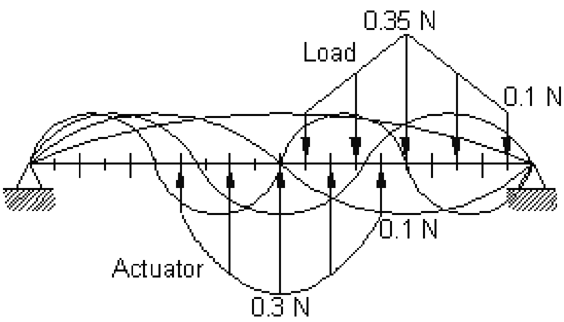


Fig. 9. Scheme for triangular load and convex parabola actuator beam loading

The step between forces in actuator group is fixed to be 0,1 of relative beam length.

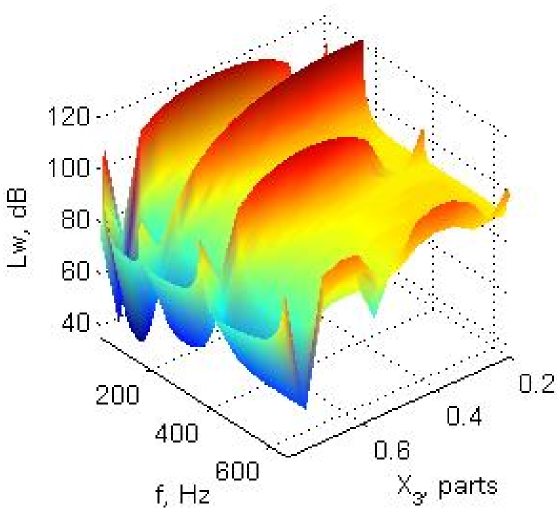
The loading results in the very similar spectrum fig. 10, *a* to the case with triangular actuator group, but now minimum SPL corresponding to all modes of simply supported beam oscillations (0,75) is not so deep.

After summation of the received sound spectrum, that is on fig. 10, *b* the minimum corresponding to 0,75 can be observed.

The total SPL reduction is 45 dB. This is better than reduction at uniform loading and worth compared to triangular one.

In the case depicted on fig. 11 we investigate cantilever beam.

The load is distributed by concave parabolic law and fixed in the position between 0,55 and 0,95 with the central force located in 0,75 of relative beam length. The beam origin is in the clamped end.



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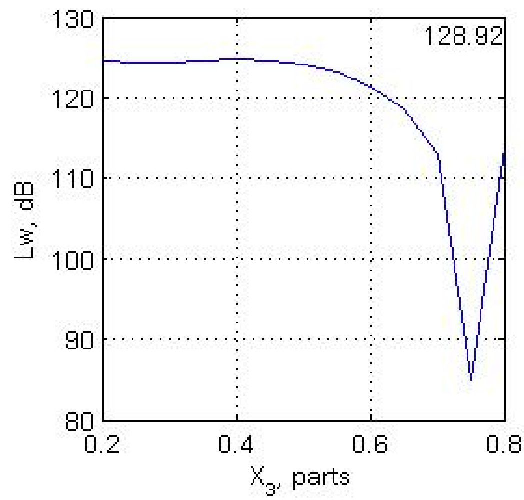


b

Fig. 10. Sound spectrum (*a*) and total SPL (*b*) for varied location of convex parabola actuator group, compensating triangular load

Actuator distribution is also convex parabolic. The location of actuator group is variable. After some calculations the SPL spectrum depicted on fig. 12 can be obtained. The only one minimum can be seen for all modes of oscillations, it corresponds to 0,75. The digits in the upper-right corner of the total SPL graph correspond to the total SPL produced by the cantilever beam excited by the distributed load of concave parabolic type without the actuator group. It enables us to estimate total SPL reduction.

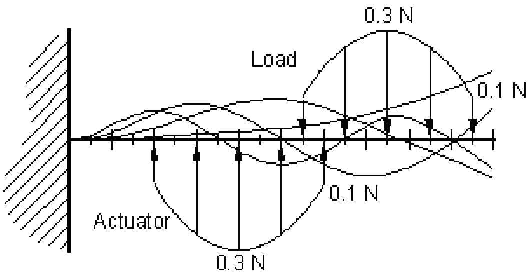
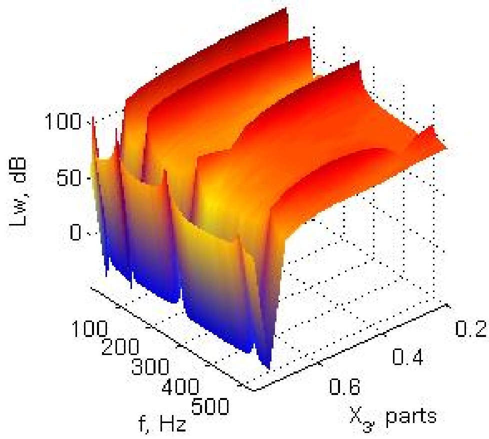
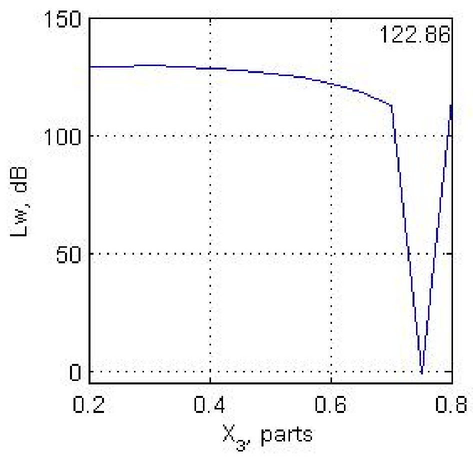


Fig. 11. Scheme for concave parabolic load and convex parabolic actuator cantilever beam loading



a



b

Fig. 12. Sound spectrum (*a*) and total SPL (*b*) for varied location of convex parabola actuator group, compensating concave distributed loading

For this case it's excellent. Fig. 13 shows the case of cantilever beam compensation by the uniformly distributed actuator.

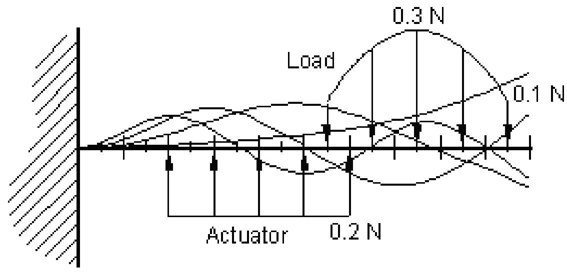
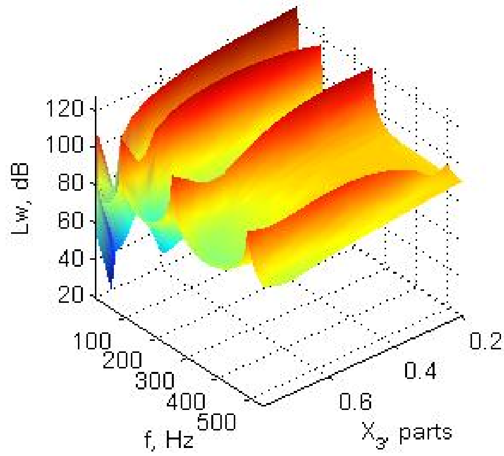
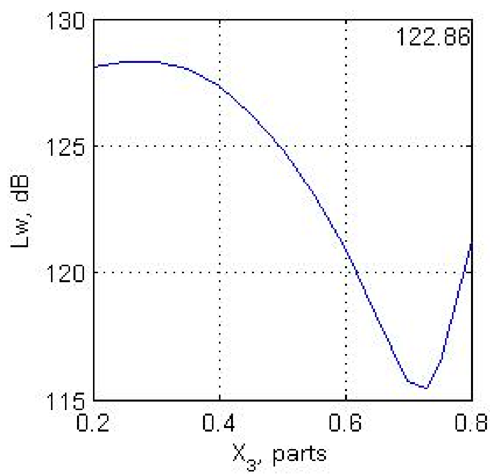


Fig. 13. Scheme for concave parabola load and uniform actuator beam loading

We vary the actuator position. The results depicted on the fig. 14 indicate minima corresponding to 0,75 for the first and second modes of cantilever oscillations.



a



b

Fig. 14. Sound spectrum (*a*) and total SPL (*b*) for varied location of uniform actuator group, compensating concave load

The third mode has the minimum in 0,7.
And 0,25 correspond to the minimum of the fourth mode.
After summation we receive the minimum total SPL 105,27 dB corresponding to 0,73.
For the next investigated case, which is shown on fig. 15, actuator value distribution is linear and actuator location is variable.

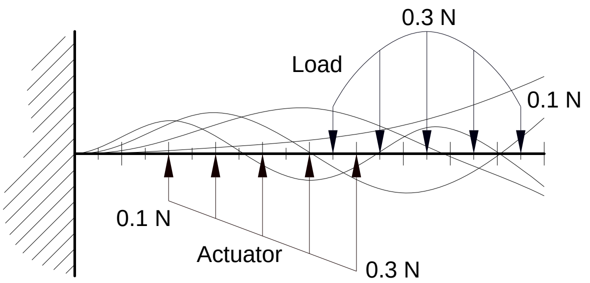
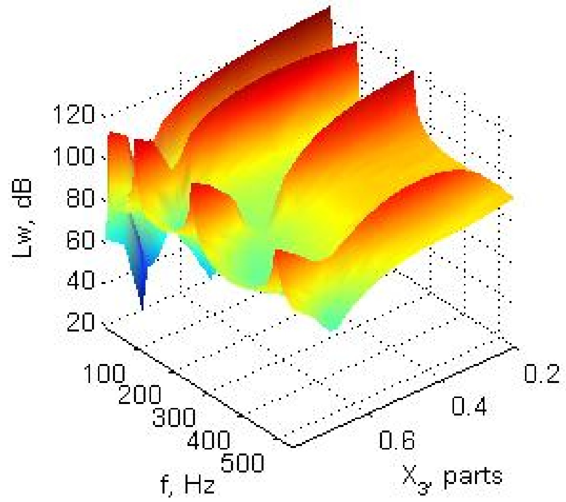
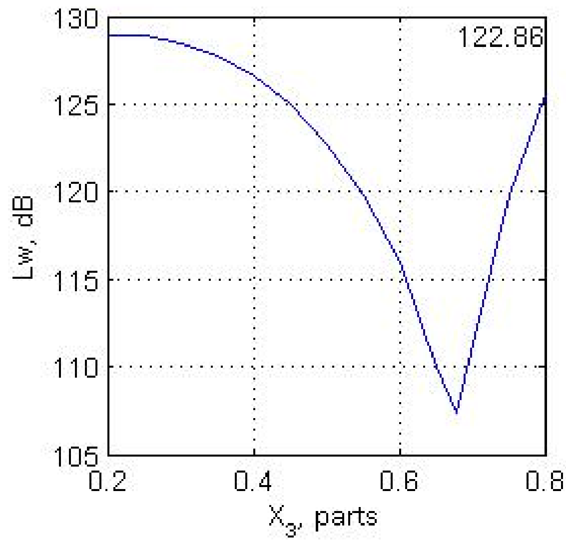


Fig. 15. Scheme for concave parabola load distribution and linear actuator beam loading

The result (fig. 16) is very similar to the previous but now all minima are shifted at 0,05:0,7 of relative beam length is the minimum of first, second and fourth modes, 0,6 is the minimum of the third one. And as a result the total SPL shown on fig. 16, *b* also shifts to position 0,68.
But the total SPL reduction is better than for the uniform actuator distribution.
The next investigation is dedicated to the problem of choosing actuator width corresponding to the best cantilever beam compensation.
For that we place the load group between 0,4 and 0,8 with the central force located at 0,6 of relative beam length (fig. 17).
The step between the forces in the load group is 0,1.
The centre of actuator distribution is also 0,6, but the step between the actuators is variable.
For the loads and actuators the same type of forces distribution is used.
As we can see from the results shown on figure 18 all modes behaves themselves very similar.
All of them have the minimum in 0,1.
In fact zero steps between the actuators on both plots refer to the point loading.
Unfortunately the small change in actuator width on 0,08 of relative beam length (change in step between forces is 0,02) leads to the sharp change in the total SPL (107,97dB).
Also from the fig. 18, *b* we can see that it's better to use the force distribution with the width smaller than the width of the load group.



a



b

Fig. 16. Sound spectrum (*a*) and total SPL (*b*) for varied location of linear actuator group

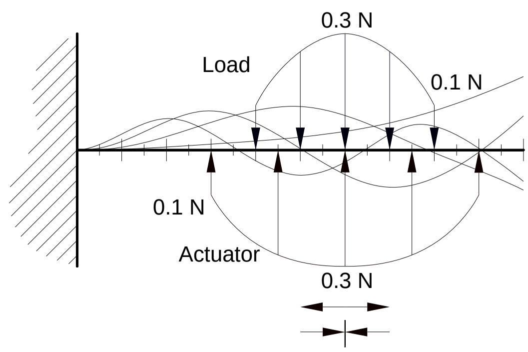
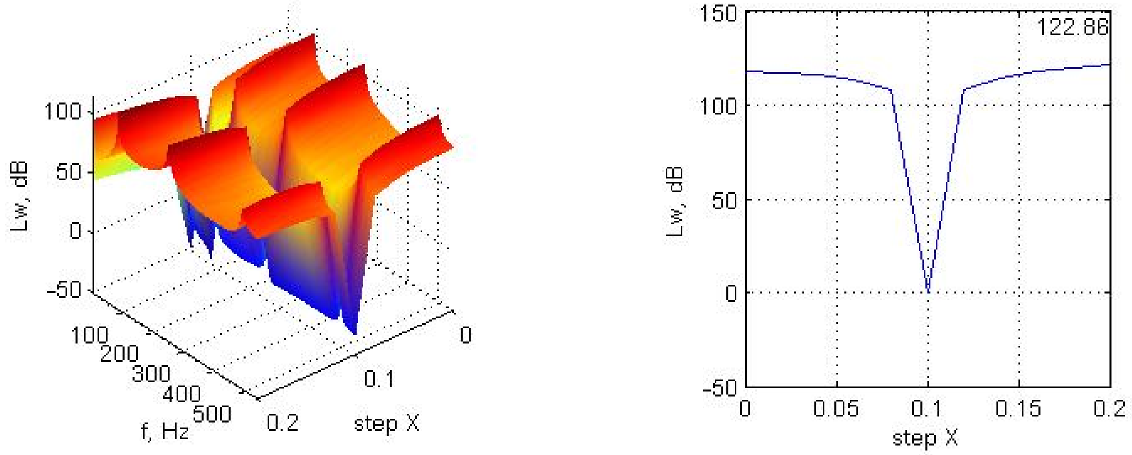


Fig. 17. Scheme for concave parabola load and actuator cantilever beam loading



a *b*

Fig. 18. Sound spectrum (*a*) and total SPL (*b*) for the actuator group with varied width

Conclusions

Five forces are quite enough to substitute forces value distribution. The best compensation can be observed in the case when the load and actuator are distributed in the same way. Namely, both load and actuator must have the same width and types of distribution, but different in phase on π radians.

A small shift of actuator location from its optimal position usually leads to the big changes in SPL.

The same concerns to the actuator width.

Linear and reverse linear forces distributions results in the shifting of the optimal actuator location relative to the minima in the cases of other types of distribution (parabolic, triangular and uniform).

References

1. *Preumont A.* Vibration control of active structures. – Kluwer academic publishers. – Dordrecht, 2003. – 350 p.

2. *Modeling* of sound radiation by a beam / O.I. Zaporozhets, V.I. Tokarev, Werner Huftnbach, a.o. // Вісн. НАУ. – 2005. – № 3. – С. 160–163.

3. *Junger M. G., Feit D.* Sound, Structures and their Interaction. – MIT Press. – Cambridge, 1986. – 448 p.

4. *Parametric* investigation of acoustic radiation by a bean under load and actuator forces / Alexander Zaporozhets, Vadim Tokarev, Werner Hufenbach at al. // Вісн. НАУ. – 2005. – № 4. – С. 122–133.

5. *Investigation* of sound radiation by cantilever beam / O.I. Zaporozhets, V.I. Tokarev, Werner Huftnbach, a.o. // Вісн. НАУ. – 2006. – № 1. – P. 169–172.

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