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PROBLEM OF THE MEASUREMENTS ANGULAR AND LINEAR SIZES OF THE COMPLEX GEOMETRIC FIGURES ON BASE INFORMATION AND MEASURING SYSTEM SIGMA

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Considered problem of the measurement angular and linear sizes large-scale details in aircraft production by means of information and measuring system type Sigma 2030. Motivated use kinematics transformations for decision direct and inverse problem of the kinematics when moving the moving parts IMS in Euclidean space.

Розглянуто задачі вимірювання кутових і лінійних розмірів великогабаритних деталей в авіаційному виробництві з допомогою ІВС типу Sigma 2030. Обґрунтовано використання кінематичних перетворень для вирішення прямої і оберненої задач кінематики при переміщенні рухомих частин ІВС в евклідовому просторі.

Introduction

Given work is dedicated to realization to methodologies of the use information and measuring systems (IMS) of the type Sigma [1] at decision of the problems of the measurement angular and linear sizes of the complex geometric figures.

The essence of these problems of the measurement consists in following.

When use IMS Sigma 2030 for measurement of the details occurs spatial displacement component moving parts main remote measuring module that is to say change the space location of its parts comparatively measured detail and base coordinate system IMS Sigma.

With standpoint of the kinematics main measuring module IMS Sigma 2030 can be considered as open-loop chain of the component parts united consecutively by means of link of the join.

One end chain comparatively free and united with special device, which and is a subsystem sensor. The second end kinematics chain bolted on immovable base – a fixing plate.

Obviously that locations and orientation of the subsystem sensor are a result of the action of the address and carrying each link chain of the joining the main measuring module IMS.

The known that, number of the degree of freedom of the mechanical system names the number of the possible moving the system.

The hard bodies, which fall into mechanical system IMS, shall name the section.

Rolling join two sections, which touch is identified kinematics pair [2].

The positions and orientations of the subsystem sensor main measuring module IMS Sigma 2030 can be described through position and orientation coordinate system, connected with subsystem sensor, to inertial base coordinate system, which takes seats and is fixed by software in computing device of the main measuring module IMS.

The collection of the positions and orientations of the subsystem sensor in space (the kinematics position) mathematically can be described by means of uniform transformations of dimensionality 4X4. These transformations are used for decision as direct so and inverse problem of the kinematics [2].

Statement of the problem

1. Solve direct problem of the kinematics:
– define kinematics position of the subsystem sensor main measuring module IMS Sigma 2030;
– get the corresponding to matrix of the position of the main measuring module.

Given:
– a vector of the displacement component

$$\theta_i = [\theta_1 \theta_2 \theta_3 ... \theta_n]^T$$

IMS with six degree of freedom;
– a parameters d_i, θ_i, a_i, and α_i, that form the minimum set, necessary kinematics for description to configuration each link the main measuring module IMS.

2. Solve inverse problem of the kinematics: define for the main measuring module IMS with six degree of freedom and six angles in join (the joint) θ_i where

$i = 1, 2, 3, 4, 5, 6.$

Given:
– a vectors n, o, a;
– a matrix T_0^n.

Decisions of the problem

For decision of the problem use such coordinate system.

1. The base reference coordinate system, which presents itself right-side cartesian square-wave coordinate system with six degree of freedom (x, y, z and three Euler corners θ, φ, ψ).

2. The coordinate system object, which presents itself right-side cartesian square-wave coordinate system with six degree of freedom (u, v, w and three Euler by corners θ, φ, ψ).

This coordinate system connected with base and moves together with her.

3. The corners in join θ_i that present itself angular coordinate system, which describes the position a sensor in joint in coordinate system link.

The corners become known if given an ensemble coordinate system link.

The algorithm of the choice coordinate system section

This procedure wholly to fix and describes the coordinate system a section joint with transformations Denavita – Hartenberga [2] for given mechanism IMS with six degree to mobility's. The nearby link can be bound one of one kinematics and described by means of matrixes of the size 4×4 uniform transformations. The first coordinate system connected with base, marked as uniform system $[x_0 y_0 z_0]^T$ in base. Begin coordinates of this system is accepted for zero join section main measuring module IMS.

1. Define the base coordinates $[x_0 y_0 z_0]$ so that axis z_0 complied the first join section with axis of the moving.

2. Execute the following points to sequences for each i=1,2, ..., 6.

3. Direct all axes z_i parallel axes of the onward moving the address $(i+1)$ join.

4. Assign begin i coordinate system or on section of the axes z_i and z_{i-1}, or in point of the section of the axis z_i with the general normal to axes z_i and z_{i-1}.

5. Assign the axis x_i in each i join or in correspondences to with correlation

$x_i = \pm(z_{i-1}\, X\, \mathrm{z})_i$,

or along the general normal to axes z_{i-1} and z_i if they parallel.

6. Assign the axis beside y_i, with correlation

$y_i = (z_i\, X\, x_i)$

for termination right-side coordinate system.

7. Find d_i as interval $(i-1)$ behind from begin coordinates $(i-1)$ coordinate system to point of the section of the axes z_{i-1} and x_i.

8. Find a_i as interval behind from point of the section of the axes z_{i-1} with the general normal to axes z_{i-1} and z_i to beginning i coordinate system.

9. Find θ_i as corner of the tumbling from axis x_{i-1} to axis x_i.

10. Find α_i, as corner of the tumbling from axis z_{i-1} to axis z_i comparatively x_i. For IMS Sigma d_i i, a_i and α_i (see table) there is constant, but θ_i, changes at tumbling link and for axis i joint.

The parameters of the join for IMS Sigma 2030 with six degree mobility's

Join	θ_i	α_i	a_i^*	d_i^*	$C\alpha_i$	$S\alpha_i$
1	θ_1	0	0	d_1	–1	0
2	θ_2	60°	0	d_2	1	0
3	θ_3	90°	a_3	0	0	1
4	θ_4	0	0	d_4	1	0
5	θ_5	–90°	a_5	0	0	–1
6	θ_6	0	0	d_6	1	0

* a_3 = 800 ml;
a_5 =620 ml;
d_1 = 211ml;
d_2 = 68 ml;
d_4 = 68 ml;
d_6 =131 ml

Thereby, a_i, and α_i is usually unchangeable and hang from design IMS Sigma. In this instance in IMS Sigma 2030 is used 6 degrees of freedom.

Kinematics position in space subsystems sensor is expressed by matrix of the size 4×4, in which enters the vector p positions begin coordinate system of the probe, matrix of the tumbling R size 3×3, which consists of three vectors-column n, o, a, which are a vector to normal, orientation and approach.

If coordinate system Denavita – Hartenberga for each link described, that possible get the matrix of the uniform conversion in i coordinate system. Accordingly fig. 1 is seen that point r_i, described in i coordinate system, can be described in $(i-1)$ and coordinate system as r_{i-1} by means of consequent performing the following transformations.

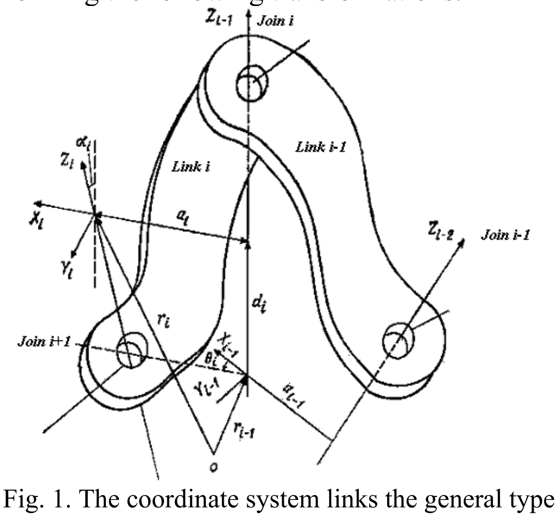

Fig. 1. The coordinate system links the general type

The algorithm of the joining *i* coordinate system and *i*-1 coordinate system

1. The tumbling for axis z_i on corner θ_i that axises x_{i-1} and x_i become parallel: $R(x_{i-1}, \theta_i)$.
2. Carrying along axis z_{i-1} on distance d_i to will combine the axises x_{i-1} and x_i: $T(z_{i-1}, d_i)$.
3. Carrying along axis x_i on distance a_i coordinate system began for joining two: $T(x_i, a_i)$.
4. The tumbling for axis x_i on corner α_i for full superposition two coordinate systems: $R(x_i, a_i)$.

Thereby, full transformation, which links i that section with $i-1$ or i joint with $i-1$, is of the form of

$$T_{i-1}^{i} = A_{i-1}^{i} = \begin{bmatrix} n_{xi} & o_{xi} & a_{xi} & p_{xi} \\ n_{yi} & o_{yi} & a_{yi} & p_{yi} \\ n_{zi} & o_{zi} & a_{zi} & p_{zi} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

or

$$A_{i-1}^{i} = \begin{bmatrix} C\theta_i & -S\theta_i & 0 & 0 \\ S\theta_i & C\theta_i & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix} *$$

$$* \begin{bmatrix} 1 & 0 & 0 & a_i \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & C\alpha_i & -S\alpha_i & 0 \\ 0 & S\alpha_i & C\alpha_i & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} =$$

$$= \begin{bmatrix} C\theta_i & -C\alpha_i S\theta_i & S\alpha_i S\theta_i & a_i C\theta_i \\ S\theta_i & C\alpha_i C\theta_i & -S\alpha_i C\theta_i & a_i S\theta_i \\ 0 & S\alpha_i & C\alpha_i & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}. \quad (1)$$

We shall hereinafter lower the index $i-1$ and name the matrix A_i, simply matrix A.

Description final link main measuring module that is to say subsystems sensors comparatively coordinate system link $n-1$ through uniform transformations is of the form of

$$T_{n-1}^{6} = A_n A_{n+1} \dots A_6^{n}. \quad (2)$$

Direct problem kinematics Sigma 2030

The kinematics manipulator displays moving the system in three-dimensional space depending on time disregarding power and moments, which generate such motion (fig. 2).

Adjusted the decision of the direct problem.

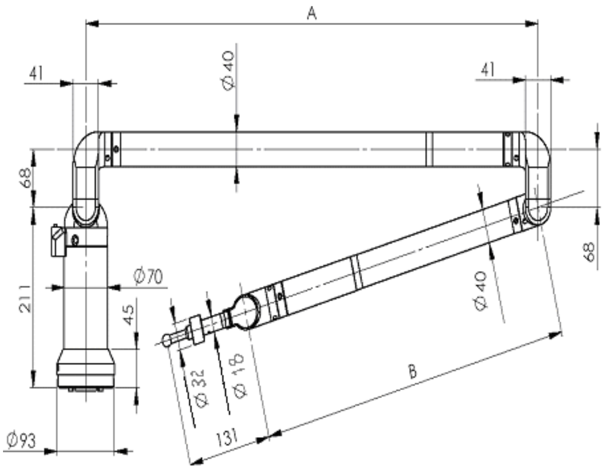

Fig. 2. Three-dimensional manipulator with six degree of freedom

Since positions and orientation of the subsystem sensor can be described three coordinates x, y, z and three Euler by corners θ, φ, ψ, base coordinates can be determined by means of fixed and rotating coordinate system.

In general coordinate system sixth link, connected with coordinate system of the base, the following transformation:

$$T_6 = A_1 A_2 A_3 A_4 A_5 A_6; \quad (3)$$

$$T_0^{n} = \begin{bmatrix} n_x & o_x & a_x & p_x \\ n_y & o_y & a_y & p_y \\ n_z & o_z & a_z & p_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

or

$$\begin{bmatrix} n & o & a & p \\ 0 & 0 & 0 & 1 \end{bmatrix} = \prod_{i=1}^{n} \begin{bmatrix} C\theta_i & -C\alpha_i S\theta_i & S\alpha_i S\theta_i & a_i C\theta_i \\ S\theta_i & C\alpha_i C\theta_i & -S\alpha_i C\theta_i & a_i S\theta_i \\ 0 & S\alpha_i & C\alpha_i & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}. \quad (4)$$

where n is a vector to normal;
o is a vector to orientation;
a is vector of the approach.

We shall consider IMS Sigma 2030 with six degree mobility's, axis of the referencing the probe complies with axis of the direction of the motion.

Using transformation Denavita – Hartenberga and using formula (1), shall get to matrix A_i for i join.

As a matter of convenience we shall choose the reduction:

$\cos \alpha_i = C\alpha_i$;
$\sin \alpha_i = S\alpha_i$;
$\cos \theta_i = C\theta_i$;
$\sin \theta_i = S\theta_i$:

$$A_1=\begin{bmatrix} C\theta_1 & S\theta_1 & 0 & 0 \\ S\theta_1 & -C\theta_1 & 0 & 0 \\ 0 & 0 & -1 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}; \qquad (5)$$

$$A_2=\begin{bmatrix} C\theta_2 & -S\theta_2 & 0 & 0 \\ S\theta_2 & C\theta_2 & 0 & 0 \\ 0 & 0 & 1 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}; \qquad (6)$$

$$A_3=\begin{bmatrix} C\theta_3 & 0 & S\theta_3 & a_3C\theta_3 \\ S\theta_3 & 0 & -C\theta_3 & a_3S\theta_3 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}; \qquad (7)$$

$$A_4=\begin{bmatrix} C\theta_4 & -S\theta_4 & 0 & 0 \\ S\theta_4 & C\theta_4 & 0 & 0 \\ 0 & 0 & 1 & d_4 \\ 0 & 0 & 0 & 1 \end{bmatrix}; \qquad (8)$$

$$A_5=\begin{bmatrix} C\theta_5 & 0 & -S\theta_5 & a_5C\theta_i \\ S\theta_5 & 0 & C\theta_5 & a_5S\theta_5 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}; \qquad (9)$$

$$A_6=\begin{bmatrix} C\theta_6 & -S\theta_6 & 0 & 0 \\ S\theta_6 & C\theta_6 & 0 & 0 \\ 0 & 0 & 1 & d_6 \\ 0 & 0 & 0 & 1 \end{bmatrix}. \qquad (10)$$

Transformations T is of the form of, according to formula (2):

$$T_0^6 = A_1\,A_2\,A_3\,A_4\,A_5\,A_6 . \qquad (11)$$

Multiplying expressions for A_i (5)–(10) and compare to T_0^6 (4) shall get 12 equations for determination vector angles [3]:

$\theta_i = [\theta_1\ \theta_2\ \theta_3\ \theta_4\ \theta_5\ \theta_6]^T$.

The complex mathematical transformations can do by mathematical package MatLab [3].

$T=[(mC\theta_4C\theta - mS\theta_4S\theta_5)C\theta_6+(gS\theta3-kC\theta_3)\ C\alpha_4$ *
$*S\alpha_5S\theta_6$, $-(mC\theta_4\ C\theta_5 - mS\theta_4S\theta_5)S\theta_6+(gS\theta_3-kC\theta_3)$*
$*C\alpha_4S\alpha_5C\theta_6$,
$(-mC\theta_4S\theta_5-mS\theta_4\ S\theta_5)C\alpha_6$,
$(-mC\theta_4S\theta_5-mS\theta_4C\theta_5)d_6+mC\theta_4a_5C\theta_5- mS\theta_4a_5S\theta_5+$
$+(gS\theta_3-kC\theta_3)d_4+ga_3C\theta_3+ka_3S\theta_3]$
$[(vC\theta_4C\theta_5-vSC\theta_4S\theta_5)C\theta_6 +(wS\theta_3-qC\theta_3)C\alpha_4S\alpha_5$*
$*S\theta_3$,
$-(vC\theta_4C\theta_5-vS\theta_4S\theta_5)S\theta_6 +(wS\theta_3-qC\theta_3)C\alpha_4S\alpha_5C\theta_3$,
$(-vC\theta_4S\theta_5-vS\theta_4C\theta_5)C\alpha_6$,
$(-vC\theta_4S\theta_5-vS\theta_4C\theta_5)d_6+vC\theta_4a_5C\theta_5-vS\theta_4a_5S\theta_5+$
$+(wS\theta_3-qC\theta_3)d_4 + wa_3C\theta_3+qa_3S\theta_3]$
$[(S\alpha_3S\theta_4C\theta_5 +S\alpha_3C\theta_4\,S\theta_5)C\theta_6$,
$-(S\theta_3S\theta_4C\theta_5+S\alpha_3C\theta_4sS\theta_5)S\theta_6$,
$(-S\alpha_3S\theta_4S\theta_5+S\theta_3C\theta_4C\theta_5)C\alpha_6$,
$(-S\alpha_3S\theta_4S\theta_5+S\alpha_3C\theta_4C\theta_5)d_6+S\alpha_3S\theta_4a_5C\theta_5+S\alpha_3C\theta_4$ *
$*a_5S\theta_5-d_2+d_1]$
$[0 , 0 , 0 , 1]$
$q= -S\theta_1S\theta_2+C\theta_1C\theta_2$;
$w=S\theta_1C\theta_2-C\theta_1S\theta_2$;
$v= wC\theta_3+qS\theta_3$;
$k= -C\theta_1S\theta_2-S\theta_1C\theta_2$;
$g=C\theta_1C\theta_2+S\theta_1S\theta_2$;
$m=gC\theta_3+kS\theta_3$.

At measurement large-scale details, for instance fuselage, possible that corners

$\theta_i=[30, 45, -60, -30, 0, 10^o]$

and shall find the matrix T_0^6 and its graphic scene (fig. 3):

$$T_0^6=\begin{bmatrix} 0{,}0340 & 0{,}1862 & -0{,}8317 & -654{,}3099 \\ 0{,}2532 & -0{,}6458 & 0{,}8895 & 736{,}0327 \\ 0{,}8290 & -0{,}5375 & -0{,}1543 & -775{,}7865 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

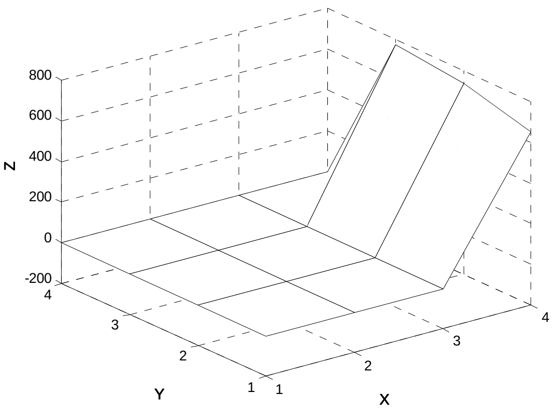

Fig. 3. The graphic scene of the matrix T_0^6

Inverse problem of the kinematics Sigma 2030

Usually most it is important to solve the inverse problem of the kinematics, since its locate the basis of operations in adjustment processes.

Six angles shall find in accordance with statement of the problem in join θ_i where and i = 1, 2, 3, 4, 5, 6.

Calling on certain mathematical transformations with expressions (6)–(11) shall find meaning an angles [4]:

$$\theta_1 = \operatorname{arctg}\left(\frac{p_y - d_6 a_y}{p_x - d_6 a_x}\right); \tag{12}$$

$$\theta_5 = \operatorname{arctg}\left(\frac{\pm d_6\left[1-\left(a_y\cos\theta_1 - a_x\sin\theta_1\right)^2\right]^{1/2}}{p_y\cos\theta_1 - p_x\sin\theta_1}\right); \tag{13}$$

$$\theta_6 = \operatorname{arctg}\left(\frac{o_x\sin\theta_1 - o_y\cos\theta_1}{n_y\cos\theta_1 - n_x\sin\theta_1}\right); \tag{14}$$

$$\theta_{234} = \operatorname{arctg}\left(\frac{-a_z}{a_x\cos\theta_1 + a_1\sin\theta_1}\right); \tag{15}$$

$$\theta_2 = -\operatorname{arctg}\left(\frac{\pm\left[1-(w/q)^2\right]^{1/2}}{w/q}\right) + \operatorname{arctg}\left(\frac{u}{t}\right); \tag{16}$$

$$q = (t^2 + u^2)^{1/2};$$

$$w = \frac{-a_3^2 + t^2 + u^2}{2a_5};$$

$$t = C_1 p_x + S_1 p_y + d_6 S_5 C_{234};$$

$$u = -p_z + d_1 - d_4 S_{234} + d_6 S_5 S_{234};$$

$$\theta_3 = \operatorname{arctg}\left(\frac{u - a_5\sin\theta_2}{t - a_5\cos\theta_2}\right) - \theta_2; \tag{17}$$

$$\theta_4 = \theta_{234} - \theta_2 - \theta_3 . \tag{18}$$

For formula (12)–(18) shall find meaning an angles θ_i:
$\theta_1 =$ 49,45°;
$\theta_2 =$ 215,11°;
$\theta_3 =$ –137,16°;
$\theta_4 =$ –77,77°;
$\theta_5 =$ –1,77°;
$\theta_6 =$ –75,48°.
This terminates the decisions of the inverse problem of the kinematics for IMS Sigma 2030.

Conclusions

Broughted results of the decision direct and inverse problem of the kinematics, is motivated use kinematics conversions in problem of the measurements angular and linear sizes of the complex geometric figures on the base of IMS Sigma.

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