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APPLICATION OF ELASTIC-PLASTIC MODEL IN STRESS-STRAIN STATES ESTIMATION OF SOIL MASS

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The scientific-methodical analysis of a soil mass modelling method by the elastic-plastic model that is used in research findings of heterogeneous and anisotropic material soil mass for their strength and stabilization by the example of system "Plaxis".

Виконано науково-методичний аналіз методу моделювання ґрунтових масивів за допомогою пружно-пластичної моделі, що застосовується під час проведення досліджень напружено-деформованого стану неоднорідних та анізотропних матеріальних середовищ для забезпечення їх міцності та стійкості, на прикладі обчислювального комплексу "Plaxis".

The introduction and the problem statement

The stress-strain state researches of heterogeneous and anisotropic soil mass are connected to using of the general algorithms for the problem solving of the elasticity, plasticity, and creep theories, and effective numerical methods of their computer realization in which practical application of soil mass mathematical models is the important and actual problem of the soil mechanics.

One of actual engineering questions of industrial and civil, road and aviation objects building is stability maintenance of soil mass at interaction with constructions.

It is known from theoretical researches, the soil strength and soil stability problems are partial problems of the general theory of limit equilibrium. The limit equilibrium of a soil in the given elementary domain is corresponding to such stress state that some additional influence can break this balance.

Such a stress state is characterized else by the equality the shear strength in an elementary domain (a final element) to limiting value for the given soil. As a rule, it takes place in the second phase of a stress state at continuous development of limit equilibrium zones, when it is necessary to apply the nonlinearly deformed solid theory consideration of geometrical nonlinearity – using Koshi-Green's tensor of finite deformations, and physical nonlinearity – correlations of the plasticity theory using elasticity tensor for elastic-plastic deformation [1; 2].

The numerical solution of soil mass stability problems is carried out using of different models on the basis of the finite element method (FEM) on the moment scheme. The target setting is assumed the discrete modelling of essentially heterogeneous soil layers taking into consideration the solid shots that are modelling the pavement and construction elements in the foundation analysis and design.

In a soil layers that are boundary with solid disseminations it is necessary to form boundary discrete layers of model elements (densening of net domain), where stress concentration take place, and as consequence, there is a research necessity of half-space model in the first limit state by the destruction criterion (shear deformations development) using correlations of nonlinear soil mechanics [3].

So, there is a research necessity of heterogeneous soil half-space in view of geometrical and physical nonlinearity in a target setting, and for input FEM-correlations is used the nonlinear elasticity and plasticity theory with application of different approaches to displacement, stress and deformations modelling. [4].

The basic models line is developed for FEM-schemes calculations using, that allows to solve a problem in view of a soil real behavior. Realization of the given scheme should take into account laws of soil behavior and reactions of models to different influences types that are rather complicated questions in theoretical researches of soil half-space.

The Mohr-Coulomb model (perfect-plasticity)

Plasticity is associated with the development of irreversible strains.

In order to evaluate whether or not plasticity occurs in a calculation, a yield function, f, is introduced as a function of stress and strain.

A yield function can often be presented as a surface in principal stress space.

A perfectly-plastic model is a constitutive model with a fixed yield surface, i.e. a yield surface that is fully defined by model parameters and not affected by (plastic) straining.

For stress states represented by points within the yield surface, the behavior is purely elastic and all strains are reversible [5; 6].

Elastic perfectly-plastic behaviour

The basic principle of elasto-plasticity is that strains and strain rates are decomposed into an elastic part and a plastic part (fig. 1):

$$\underline{\varepsilon} = \underline{\varepsilon}^e + \underline{\varepsilon}^p,$$

$$\underline{\dot{\varepsilon}} = \underline{\dot{\varepsilon}}^e + \underline{\dot{\varepsilon}}^p. \qquad (1)$$

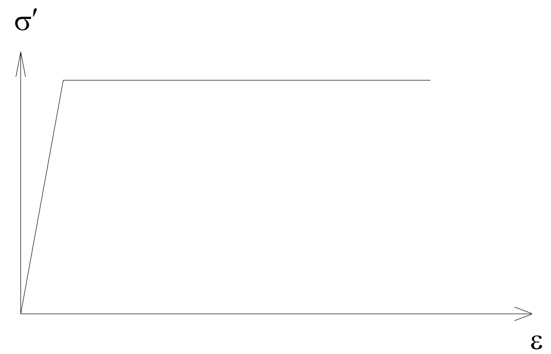


Fig. 1. Basic idea of an elastic perfectly plastic model

To relate the stress rates to the elastic strain rates is used the Hooke's law.

Substitution of equality (1) into Hooke's law leads to:

$$\dot{\sigma}' = \underline{\underline{D}}^e \dot{\varepsilon}^e = \underline{\underline{D}}^e (\dot{\varepsilon} - \dot{\varepsilon}^p).$$

According to the classical theory of plasticity, plastic strain rates are proportional to the derivative of the yield function with respect to the stresses.

This means that the plastic strain rates can be represented as vectors perpendicular to the yield surface. This classical form of the theory is referred to as associated plasticity.

However, for Mohr-Coulomb type yield functions, the theory of associated plasticity leads to an overprediction of dilatancy.

Therefore, in addition to the yield function, a plastic potential function g is introduced.

The case $g \neq f$ is denoted as non-associated plasticity. In general, the plastic strain rates are written as:

$$\dot{\varepsilon}^p = \lambda \frac{\partial g}{\partial \underline{\sigma}'},$$

where λ is the plastic multiplier.

For purely elastic behaviour λ is zero, whereas in the case of plastic behaviour λ is positive:

$$\lambda = 0 \quad \text{for} \quad f{<}0 \quad \text{or} \quad \frac{\partial f^T}{\partial \underline{\sigma}'} \underline{\underline{D}}^e \underline{\dot{\varepsilon}} \le 0; \qquad (2)$$

$$\lambda = 0 \quad \text{for} \quad f{<}0 \quad \text{or} \quad \frac{\partial f^T}{\partial \underline{\sigma}'} \underline{\underline{D}}^e \underline{\dot{\varepsilon}} > 0. \qquad (3)$$

These equations may be used to obtain the following relationship between the effective stress rates and strain rates for elasto-plasticity:

$$\dot{\sigma}' = \left[\underline{\underline{D}}^e - \frac{\alpha}{d} \underline{\underline{D}}^e \frac{\partial g}{\partial \sigma'} \frac{\partial f^T}{\partial \sigma'} \underline{\underline{D}}^e \right] \underline{\dot{\varepsilon}},$$

where

$$d = \frac{\partial f^T}{\partial \underline{\sigma}'} \underline{\underline{D}}^e \frac{\partial g}{\partial \underline{\sigma}'}$$

The parameter α is used as a switch. If the material behaviour is elastic, as defined by equality (2) the value of α is equal to zero, whilst for plasticity, as defined by equality (3), the value of α is equal to unity.

The above theory of plasticity is restricted to smooth yield surfaces and does not cover a multi surface yield contour as present in the Mohr-Coulomb model.

For such a yield surface the theory of plasticity has been extended by Koiter and others to account for flow vertices involving two or more plastic potential functions:

$$\underline{\dot{\varepsilon}}^p = \lambda_1 \frac{\partial g_1}{\partial \underline{\sigma}'} + \lambda_2 \frac{\partial g_2}{\partial \underline{\sigma}'} + \dots$$

Similarly, several quasi independent yield functions (f_1, f_2 …) are used to determine the magnitude of the multipliers (λ_1, λ_2 ...).

Formulation of the Mohr-Coulomb model

The Mohr-Coulomb yield condition is an extension of Coulomb's friction law to general states of stress. In fact, this condition ensures that Coulomb's friction law is obeyed in any plane within a material element.

The full Mohr-Coulomb yield condition consists of six yield functions when formulated in terms of principal stresses:

$$f_{1a} = \frac{1}{2}(\sigma'_2 - \sigma'_3) + \frac{1}{2}(\sigma'_2 + \sigma'_3)\sin\varphi - c\cos\varphi \le 0;$$

$$f_{1b} = \frac{1}{2}(\sigma'_3 - \sigma'_2) + \frac{1}{2}(\sigma'_3 + \sigma'_2)\sin\varphi - c\cos\varphi \le 0;$$

$$f_{2a} = \frac{1}{2}(\sigma'_3 - \sigma'_1) + \frac{1}{2}(\sigma'_3 + \sigma'_1)\sin\varphi - c\cos\varphi \le 0;$$

$$f_{2b} = \frac{1}{2}(\sigma'_1 - \sigma'_3) + \frac{1}{2}(\sigma'_1 + \sigma'_3)\sin\varphi - c\cos\varphi \le 0;$$

$$f_{3a} = \frac{1}{2}(\sigma'_1 - \sigma'_2) + \frac{1}{2}(\sigma'_1 + \sigma'_2)\sin\varphi - c\cos\varphi \le 0;$$

$$f_{3b} = \frac{1}{2}(\sigma'_2 - \sigma'_1) + \frac{1}{2}(\sigma'_2 + \sigma'_1)\sin\varphi - c\cos\varphi \le 0.$$

The two plastic model parameters appearing in the yield functions are the well-known friction angle φ and the cohesion c.
These yield functions together represent a hexagonal cone in principal stress space as shown in fig. 2.

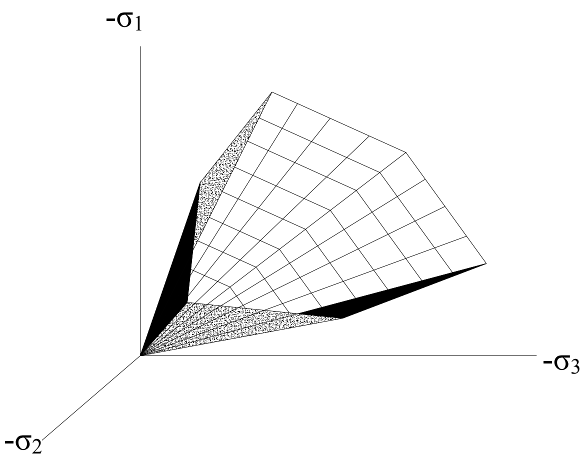


Fig. 2. The Mohr-Coulomb yield surface in principal stress space (c = 0)

In addition to the yield functions, six plastic potential functions are defined for the Mohr-Coulomb model:

$$g_{1a} = \frac{1}{2}(\sigma'_2 - \sigma'_3) + \frac{1}{2}(\sigma'_2 + \sigma'_3)\sin\psi ;$$

$$g_{1b} = \frac{1}{2}(\sigma'_3 - \sigma'_2) + \frac{1}{2}(\sigma'_3 + \sigma'_2)\sin\psi ;$$

$$g_{2a} = \frac{1}{2}(\sigma'_3 - \sigma'_1) + \frac{1}{2}(\sigma'_3 + \sigma'_1)\sin\psi ;$$

$$g_{2b} = \frac{1}{2}(\sigma'_1 - \sigma'_3) + \frac{1}{2}(\sigma'_1 + \sigma'_3)\sin\psi ;$$

$$g_{3a} = \frac{1}{2}(\sigma'_1 - \sigma'_2) + \frac{1}{2}(\sigma'_1 + \sigma'_2)\sin\psi ;$$

$$g_{3b} = \frac{1}{2}(\sigma'_2 - \sigma'_1) + \frac{1}{2}(\sigma'_2 + \sigma'_1)\sin\psi .$$

The plastic potential functions contain a third plasticity parameter, the dilatancy angle ψ. This parameter is required to model positive plastic volumetric strain increments (dilatancy) as actually observed for dense soils. A discussion of all of the model parameters used in the Mohr-Coulomb model is given at the end of this section.
When implementing the Mohr-Coulomb model for general stress states, special treatment is required for the intersection of two yield surfaces.
Some programs use a smooth transition from one yield surface to another, i.e. the rounding-off of the corners. In system “Plaxis”, however, the exact form of the full Mohr-Coulomb model is implemented, using a sharp transition from one yield surface to another.

For $c > 0$, the standard Mohr-Coulomb criterion allows for tension.
In fact, allowable tensile stresses increase with cohesion. In reality, soil can sustain none or only very small tensile stresses.
This behaviour can be included in a “Plaxis” analysis by specifying a tension cut-off. In this case, Mohr circles with positive principal stresses are not allowed. The tension cut-off introduces three additional yield functions, defined as:

$$f_4 = \sigma'_1 - \sigma_t \le 0 ,$$

$$f_5 = \sigma'_2 - \sigma_t \le 0 ,$$

$$f_6 = \sigma'_3 - \sigma_t \le 0 .$$

When this tension cut-off procedure is used, the allowable tensile stress, σ_t is, by default, taken equal to zero.
For these three yield functions an associated flow rule is adopted.
For stress states within the yield surface, the behaviour is elastic and obeys Hooke's law for isotropic linear elasticity.
Hence, besides the plasticity parameters c, φ, and ψ input is required on the elastic Young's modulus E and Poisson's ratio ν.

Basic parameters of the Mohr-Coulomb model

The Mohr-Coulomb model requires a total of five parameters, which are generally familiar to most geotechnical engineers and which can be obtained from basic tests on soil samples. These parameters with their standard units are listed below:
E is Young's modulus, kN/m^2;
ν is Poisson's ratio;
φ is Friction angle, °;
c is Cohesion, kN/m^2;
ψ is Dilatancy angle, °.

Young’s modulus E

“Plaxis” uses the Young's modulus as the basic stiffness modulus in the elastic model and the Mohr-Coulomb model, but some alternative stiffness module are displayed as well. A stiffness modulus has the dimension of stress.
The values of the stiffness parameter adopted in a calculation require special attention as many geomaterials show a non-linear behavior from the very beginning of loading.
In soil mechanics the initial slope is usually indicated as E_0 and the secant modulus at 50% strength is denoted as E_{50} (fig. 3).
For materials with a large linear elastic range it is realistic to use E_0, but for loading of soils one generally uses E_{50}. Considering unloading problems, as in the case of tunneling and excavations, one needs E_{ur} instead of E_{50}.

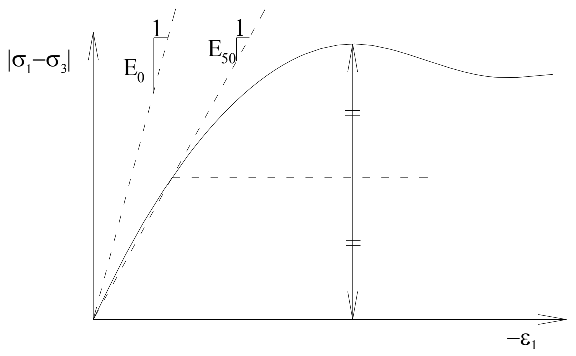


Fig. 3. Definition of E_0 and E_{50} for standard drained triaxial test results

For soils, both the unloading modulus, E_{ur}, and the first loading modulus, E_{50}, tend to increase with the confining pressure. Hence, deep soil layers tend to have greater stiffness than shallow layers.

Moreover, the observed stiffness depends on the stress path that is followed.

The stiffness is much higher for unloading and reloading than for primary loading. Also, the observed soil stiffness in terms of a Young's modulus may be lower for (drained) compression than for shearing.

Hence, when using a constant stiffness modulus to represent soil behaviour one should choose a value that is consistent with the stress level and the stress path development.

Note that some stress-dependency of soil behaviour is taken into account in the advanced models. For the Mohr-Coulomb model, there is a special option for the input of a stiffness increasing with depth.

Poisson's ratio ν

Standard drained triaxial tests may yield a significant rate of volume decrease at the very beginning of axial loading and, consequently, a low initial value of Poisson's ratio (ν_0).

For some cases, such as particular unloading problems, it may be realistic to use such a low initial value, but in general when using the Mohr-Coulomb model the use of a higher value is recommended.

The selection of a Poisson's ratio is particularly simple when the elastic model or Mohr-Coulomb model is used for gravity loading (increasing ΣM_{weight} from 0 to 1 in a plastic calculation).

For this type of loading "Plaxis" should give realistic ratios of

$K_o = \sigma_h / \sigma_v$.

As both models will give the well-known ratio of

$\sigma_h/\sigma_v = \nu/(1-\nu)$

for one-dimensional compression it is easy to select a Poisson's ratio that gives a realistic value of K_o. Hence, ν is evaluated by matching K_o.

This subject deals with initial stress distributions.

In many cases one will obtain ν values in the range between 0,3 and 0,4. In general, such values can also be used for loading conditions other than one-dimensional compression. For unloading conditions, however, it is more common to use values in the range between 0,15 and 0,25.

Cohesion *c*

The cohesive strength has the dimension of stress.

It can handle cohesionless sands ($c = 0$), but some options will not perform well.

To avoid complications, non-experienced users are advised to enter at least a small value (use $c > 0{,}2$ kPa).

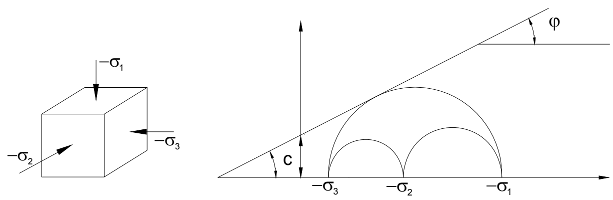
There is a special option for the input of layers in which the cohesion increases with depth.

Friction angle φ

The friction angle, φ, is entered in degrees. High friction angles, as sometimes obtained for dense sands, will substantially increase plastic computational effort.

The computing time increases more or less exponentially with the friction angle. Hence, high friction angles should be avoided when performing preliminary computations for a particular project.

The friction angle largely determines the shear strength as shown in fig. 4 by means of Mohr's stress circles.



a *b*

Fig. 4. Stress circles at yield; one touches Coulomb's envelope:
a is main stresses;
b is shear diagram

A more general representation of the yield criterion is shown in fig. 2. The Mohr-Coulomb failure criterion proves to be better for describing soil behaviour than the Drucker-Prager approximation, as the latter failure surface tends to be highly inaccurate for axisymmetric configurations.

Dilatancy angle ψ

The dilatancy angle, ψ, is specified in degrees. Apart from heavily over-consolidated layers, clay soils tend to show little dilatancy ($\psi \approx 0$).

The dilatancy of sand depends on both the density and on the friction angle. For quartz sands the order of magnitude is $\psi \approx \varphi - 30°$.

For φ-values of less than 30°, however, the angle of dilatancy is mostly zero.
A small negative value for ψ is only realistic for extremely loose sands.

Advanced parameters of the Mohr-Coulomb model

When using the Mohr-Coulomb model, may be enter some additional parameters for advanced modelling features.
The advanced features comprise the increase of stiffness and cohesive strength with depth and the use of a tension cut-off. In fact, the latter option is used by default.

Increase of stiffness E_{inc}

In real soils, the stiffness depends significantly on the stress level, which means that the stiffness generally increases with depth. When using the Mohr-Coulomb model, the stiffness is a constant value.
In order to account for the increase of the stiffness with depth the E_{inc}-vaiue may be used, which is the increase of the Young's modulus per unit of depth (expressed in the unit of stress per unit depth).
At the Level given by the y_{ref} parameter, the stiffness is equal to the reference Young's modulus, E_{ref} as entered in the tab sheet. The actual value of Young's modulus in the stress points is obtained from the reference value and E_{inc}.
Note that during calculations a stiffness increasing with depth does not change as a function of the stress state.

Increase of cohesion c_{inc}

An advanced option for the input of clay layers in which the cohesion increases with depth.
In order to account for the increase of the cohesion with depth the c_{inc}-value may be used, which is the increase of cohesion per unit of depth (expressed in the unit of stress per unit depth).
At the level given by the y_{ref} parameter, the cohesion is equal to the (reference) cohesion, c_{ref}, as entered in the tab sheet.
The actual value of cohesion in the stress points is obtained from the reference value and c_{inc}.

Tension cut-off

In some practical problems an area with tensile stresses may develop. According to the Coulomb envelope shown in fig. 4 this is allowed when the shear stress (radius of Mohr circle) is sufficiently small. However, the soil surface near a trench in clay sometimes shows tensile cracks. This indicates that soil may also fail in tension instead of in shear. Such behaviour can be included in “Plaxis” analysis by selecting the tension cut-off. In this case Mohr circles with positive principal stresses are not allowed. When selecting the tension cut-off the allowable tensile strength may be entered. For the Mohr-Coulomb model and the Hardening-Soil model the tension cut-off is, by default, selected with a tensile strength of zero.

Conclusions

Thus, the choice and application of this model depends on a target setting, parameters definition, is carried out by classical geotechnical methods of traditional soil mechanics, or demands application of special optimizing technique.
Adequate using of the considered soil behavior model, with parameters that are determined on the basis of the different approaches coordinated with a reliable and universal final element, is a basis for research operation for stress-strain state of soil half-space at the of real problems decision.

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