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# ROBUST AUTOPILOTS BASED ON THE FUZZY MODEL REFERENCE LEARNING CONTROL

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Fuzzy learning algorithm of unmanned aerial vehicle is considered in this paper. It allows real time self-tuning of parameters of the controller's membership functions. The primary structure of the fuzzy controller is synthesized via "crisp" prototype based on the robust  $H_2/H_{\infty}$  -optimization. It is shown that obtained control algorithm possesses high level of performance and robustness.

Розглянуто алгоритм нечіткого управління з самонавчанням безпілотним літальним апаратом, що дозволяє адаптувати параметри функцій приналежності регулятора в реальному часі. Структура нечіткого регулятора без контуру адаптації синтезується за допомогою "чіткого" прототипу, що дозується на результатах робастної  $H_2/H_{\infty}$ -оптимізації. Показано, що отриманий алгоритм управління має високі показники якості та робастності.

#### Introduction

During recent years, fuzzy control became a good challenge to the classical control approaches [1], especially in the cases, when controlled plant and external disturbances possess uncertainty, timevarving dynamic characteristics. incomplete measurement of the state space vector components etc. However, the design procedure of the fuzzy controller, pursuing the proper choice of many parameters such as the membership functions, inference strategy and so on, is not formalized to the level of computer programs particularly in a cases of high-order models of plant and exogenous disturbances.

Also, the fuzzy controller constructed for nominal plant may later perform inadequate performance if some disturbance or structure changes in the process were occurred.

To overcome these difficulties adaptive and optimization methods were proposed in the literature [2-6].

There are two approaches for solving this problem. The first one is based on the procedures of adaptation and learning of the fuzzy controller using some reference model [2; 3].

However, the application of such procedure for high-order plants with incomplete measurement of the state space vector is connected with some difficulties caused by finding initial values of all aforementioned parameters of fuzzy controller, which guarantee stability of the closed-loop system. The second approach is based on the combination of "crisp" and "fuzzy" design procedures [4; 5; 6].

In this case some known classical methods of synthesis of the robust (or optimal) controller are applied at the first stage of the design procedure. Due to the incomplete measurement of the highorder state vector resulting controller possesses dynamic output feedback, which can be represented as the parallel coupling of static output feedback (some gain matrix) and dynamic feedback consisting of differentiators and integrators.

It is recommended in [4; 5] to remain dynamic feedback the same as it was produced by synthesis of the "crisp" controller, but to replace the static output feedback with fuzzy controller. The example of successful application of this approach to the robust control of the small unmanned aerial vehicle (UAV) is represented in [6].

The ultimate goal of this paper is the usage of combination of these two approaches in order to receive the hybrid "fuzzy-crisp" structure of fuzzy robust control of the small UAV with the possibility of the fuzzy controller learning. It is supposed that this combination will join the advantages of both aforementioned approaches.

The first of all it is necessary to consider briefly the procedure of the synthesis of "crisp" robust controller, which is used for determination of the dynamic feedback of the "combined" controller and simultaneously for creation of the reference model.

# Brief description of the "crisp" controller design procedure

This procedure is based on the robust parametric  $H_2/H_{\infty}$  – optimization of the control system with given structure.

Description of this procedure can be facilitated using case study of the small UAV longitudinal motion control in an altitude-hold mode [7].

The standard structure of this control system [8] is shown in fig. 1.

The navigation system provides the longitudinal channel of autopilot with the signals of three sensors: altitude h, pitch angle  $\vartheta$  and rate q for attitude stabilization. The digital control law has the following form:

$$\begin{split} \delta e(z) &= W_{a}(z)[W_{h}(z), K_{\vartheta}, K_{q}][h, \vartheta, q]^{T}, \\ W_{a}(z) &= 1 + \frac{T_{d\vartheta q}}{T \cdot z}(z - 1), \end{split} \tag{1} \\ W_{h}(z) &= K_{h} + \frac{T_{dh}}{T \cdot z}(z - 1), \end{split}$$

where  $W_a(z)$ ,  $W_h(z)$  are the dynamic compensators (PD-controllers) for the attitude  $\vartheta$ ,q and altitude h loops respectively;

 $K_{\vartheta}$ ,  $K_{q}$ ,  $K_{h}$  are pitch angle, rate and altitude gains respectively;

T is the sampling period (0,02 s);

 $T_{d\theta q}$ ,  $T_{dh}$  are time constants of the first difference elements in these loops;

 $\delta e$  is the deflection of elevator.

The thrust control in this UAV is absent [9]. In accordance with control law (1) vector of the autopilot's adjustable parameters  $\vec{P}_n$ , which has to be determined from optimization procedure, has the following components:

$$\vec{P}_n = [K_h, K_g, K_q, T_{dh}, T_{dgq}].$$

Actuator "Act" can be described with the transfer function

$$W_{act}(s) = \frac{1}{T_{ac}s+1}.$$

In this case  $T_{ac} = 0.5$  s.

The purpose of the optimization procedure is sustaining the performance and robustness levels of the closed loop system with the same control law for "nominal" plant (true airspeed V=70 m/s) and parametrically disturbed plant (V=55,5 m/s).

The state space description of these plants is represented with the standard equations:

$$\begin{aligned} X &= AX + Bu + w; \\ Y - CX \end{aligned}$$
(2)

with following state and control matrices A and B respectively:

$$\mathbf{A} = \begin{bmatrix} -0.0345 & 6 & -9.78 & 0 & 0 \\ -0.0041 & -1.76 & 0 & 0.99 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0.0033 & -25.7 & 0 & -2.19 & 0 \\ 0 & -69.4 & 69.4 & 0 & 0 \end{bmatrix};$$

$$\mathbf{B} = \begin{bmatrix} 0.36 & -0.16 & 0 & -31.1 & 0 \end{bmatrix}^{\mathrm{T}};$$
$$\mathbf{A}_{\mathbf{p}} = \begin{bmatrix} -0.0273 & 6 & -9.78 & 0 & 0 \\ -0.0064 & -1.76 & 0 & 1 & 0 \\ 39 & 0 & 0 & 0 & 1 & 0 \\ 0.0036 & -16.1 & 0 & -1.73 & 0 \\ 0 & -55.6 & 55.6 & 0 & 0 \end{bmatrix};$$
$$\mathbf{B}_{\mathbf{p}} = \begin{bmatrix} 0.26 & 0.12 & 0 & -100 & 0 \end{bmatrix}^{\mathrm{T}}$$

 $\mathbf{B}_{\mathbf{p}} = [0.36 \ -0.13 \ 0 \ -19.9 \ 0]^{1}$ .

Variable w in (2) denotes the perturbation – turbulent wind, which can be described with standard Dryden model [8], state vector is equal to  $X = [V, \alpha, \vartheta, q, h]$ , where V is the true airspeed,  $\alpha$  is the angle of attack.

In accordance with block diagram depicted in the fig. 1, observation matrix C is equal to



Fig. 1. Block diagram of the UAV control system in the altitude-hold mode

The performance-robustness index, which has to be minimized with optimization procedure, would have the following form:

$$J = \lambda_{od} J_{od} + \lambda_{os} J_{os} + \lambda_{pd} J_{pd} + \lambda_{ps} J_{ps} + \lambda_{\infty} T_{\infty} + \lambda_{p\infty} T_{p\infty}.$$

where  $\lambda_{0d}$ ,  $\lambda_{0s}$  are for corresponding weight factors, while the same symbols with subscripts 'p' stand for the same values of perturbed system;  $J_{0d}$ ,  $J_{0s}$  are respectively for deterministic and stochastic partial performance indices (PPI) of nominal system.

Symbols  $T_{\infty}(T_{p\infty})$  and  $\lambda_{\infty}(\lambda_{p\infty})$  denote  $H_{\infty}$ -norms for complementary sensitivity functions of the closed loop systems (measures of robustness [10]) and weight factors for the nominal and perturbed systems respectively. Deterministic PPI's were calculated as the H<sub>2</sub>-norms of the nominal and perturbed closed loop systems, meanwhile the stochastic PPI's were calculated as the H<sub>2</sub>-norms of the series connection of the Dryden filters [8] with the nominal and perturbed closed loop systems. This procedure is described in details in [7] along with the considerations concerning the choice of weight factors  $\lambda_{0d}$ ,  $\lambda_{0s}$ ,  $\lambda_{pd}$ ,  $\lambda_{ps}$ ,  $\lambda_{\infty}$ ,  $\lambda_{p\infty}$ . Running this procedure with various values of these weight factors [7] gives eventually the following numerical values for vector of adjustable parameters of autopilot:  $\vec{\mathbf{C}}_{\mathbf{n}} = [-9, 2, -1, -0, 05, 0, 14, 0, 008]$ .

The closed loop system having the controller with parameters defined by this vector  $\vec{C}_n$  can be considered as the reference model. In order to comply with the standard block diagram for application of learning approach proposed in [2; 3], we can consider the attitude stabilization loop including elements  $K_{\vartheta}, K_q, W_a(z)$  (fig. 1) as the inner loop for the stability and performance augmentation.

So we can apply the principles of fuzzy learning control to the generalized plant including actuator, UAV and the inner attitude stabilization loop.

## Fuzzy model reference learning control

The functional diagram of the fuzzy model reference learning control (FMRLC), which is shown in fig. 2, is similar to the structure of known "crisp" model reference adaptive control [2; 3].

It has four main parts: the generalized plant (GP), which is described in the previous chapter, the fuzzy controller (FC), the reference model (RM) and the learning mechanism, which uses the fuzzy inverse model (FIM).

The goal of this paper is to synthesize fuzzy controller and to adjust its membership functions in order to withstand to the action of parametric disturbances in the controlled plant. The term learning is used instead of adaptive only to distinguish the adjusting of the fuzzy controller's membership functions in the control process form a simple adaptive conventional control. In the first case the tuning processes include joint variations of input and output membership functions along with action of the inference mechanism. In the second case the adaptation process is reduced to the variations of the simple numerically adjusted parameters of "crisp" controller.

The expected performance of the overall closed loop system is specified in the reference model. Next, we describe aforementioned components of the FMRLC more in detail.

#### **Fuzzy controller**

The input of the FC is the error e(kT) between the reference input r(kT) and system output h(kT) – altitude of UAV.

The FC output u(kT) is the reference value for the inner Attitude Control Loop (ACL).

In accordance with control law (1) and fuzzy control ideology described in [2; 3], FC uses two input variables – error and change of error (the first difference):

$$e(kT) = r(kT) - h(kT),$$
  

$$c(kT) = \frac{e(kT) - e(kT-1)}{T}.$$



Fig. 2. Functional diagram of FMRLC

In FC each universe of discourse for each plant input is normalized to the interval [-1 +1] by means of constant scaling factors. The gains  $g_e, g_c$  and  $g_u$ were used to normalize the universe of discourse for the error e(kT) and change in error c(kT), and

controller output u(kT) respectively.

We apply inference mechanism (IM) of Mamdani type, which is recommended for aircraft usage in [4; 5]. It can be expressed in the form of IF-THEN rules as follows:

If 
$$\tilde{e}$$
 is  $\tilde{E}_i$  and  $\tilde{c}$  is  $\tilde{C}_j$ , then  $\tilde{u}$  is  $\tilde{U}_i$ , (3)

where  $\tilde{e}$  and  $\tilde{c}$  denote the linguistic variables associated with e and c and implemented in the Blocks of Input Membership Functions BIMF<sub>1</sub> and BIMF<sub>2</sub> respectively.

Linguistic variable  $\tilde{u}$  is associated with the controller output *u* and implemented in the Block of Output Membership Functions BOMF<sub>1</sub>.

The following fuzzy relation represents the fuzzy implication made by IM

$$R^{l} = \left(\widetilde{E}_{i} \times \widetilde{C}_{j}\right) \widetilde{U}_{ij}, \ \left(i \in [1,m], j \in [1,n], l \in [1,m \times n]\right),$$

where m, n are total amounts of membership functions in BIMF<sub>1</sub> and BIMF<sub>2</sub> respectively.

The fuzzy decision mechanism for this control rule may be expressed by

$$R = \bigcup_{l=1}^{n \times m} R^l ,$$

where  $\bigcup$  denotes Zadeh's union operator. Therefore, the defuzzification mechanism DF<sub>1</sub> of the fuzzy controller produces control action, which is computed by the "center of gravity" method expressed as:

$$u_n(kT) = \frac{\sum_{j,\dots,k,l,\dots,m} A_n^{j,\dots,k,l,\dots,m}(kT) c_n^{j,\dots,k,l,\dots,m}(kT)}{\sum_{j,\dots,k,l,\dots,m} A_n^{j,\dots,k,l,\dots,m}(kT)}$$

where  $A_n^{j,\dots,k,l,\dots,m}(kT)$  and  $c_n^{j,\dots,k,l,\dots,m}(kT)$  are the area and center of area, respectively, of the membership function associated with  $U_n^{j,\dots,k,l,\dots,m}(kT)$ .

#### The reference model

The reference model generates the desired performance of the overall process. In general, the RM may be any type of dynamical system. The performance of the overall system is computed with respect to the RM output  $h_m(kT)$  by generating an error signal

$$h_e(kT) = h_m(kT) - h(kT).$$

In our case the RM is the closed loop control system with "crisp" controller, produced by the robust parametric  $H_2/H_{\infty}$ -optimization procedure described above. We only used its simple approximation by the second order model with the same rise time and small overshoot in order to facilitate its usage.

## The learning mechanism

As previously mentioned, the learning mechanism performs the function of modifying the knowledge base (membership functions) of the direct fuzzy controller so, that the closed loop system behaves like the reference model. These knowledge base modifications are made based on observing data from the controlled process, the reference model and the fuzzy controller. In accordance with fig. 2 the learning mechanism consists of two parts: a fuzzy inverse model (FIM) and a knowledge base modifier (KBM). FIM performs the function of mapping observed output signals: the error  $h_e(kT)$  and the change of error  $h_c(kT)$ , to the changes of the FC output membership function's parameters  $p = [p_1, \dots, p_r]^T$  in BOMF<sub>1</sub>, which are necessary to force  $y_e(kT)$  to zero. KBM performs the function of modifying the fuzzy controller's knowledge base to perform the needed change in the process inputs. More details of this process are discussed next.

# The fuzzy inverse model

A fuzzy system is used to represent the inverse plant dynamics. It is not necessary for fuzzy control to accurately characterize the inverse dynamics; only approximate representation is needed [2; 3]. As it is shown in the fig. 2, FIM simply maps  $h_e(kT)$  and  $h_e(kT)$  to the necessary changes in the plant "fuzzy" inputs, that is why it is called FIM. Hence, FIM is used to characterize how to change the plant "fuzzy" inputs to force the plant output h(kT) to  $h_m(kT)$  as close as possible. Likewise to the FC, the FIM shown in fig. 2 contains normalizing scaling factors, namely  $g_{he}$  ,  $g_{hc}$  and  $g_p$  for each universe of discourse. Selection of the normalizing gains can impact the overall performance of the system.

The knowledge base for the FIM is generated from fuzzy rules of the form:

if  $\tilde{h}_{e}$  is  $H_{e}^{j}$  and  $\tilde{h}_{c}$  is  $H_{c}^{k}$  then  $\widetilde{p}$  is  $P^{j,k}$ ,

where  $H_e^j$  and  $H_c^k$  denote the linguistic values of the error  $h_e(kT)$  and change in error  $h_c(kT)$ respectively.  $P^{j,k}$  denotes the consequent fuzzy set for this rule describing the necessary changes in the plant input. As was the case for the direct FC, the overall input changes for the direct FC are determined from the COG defuzzification method.

## The knowledge base modifier

The knowledge base modifier performs the function of modifying the FC rule base to achieve better performance. Given the information about the necessary changes in the "fuzzy" input, which are represented by p(kT), to force the error  $h_{a}(kT)$  to zero, the knowledge base modifier change the FC rule-base so, that the previously computed control action u(kT - T) would be modified at the next step as follows:  $u(kT-T)+u_n(kT)$ , where  $u_n(kT)$  is the increment of the control action caused by vector  $p = [p_1, \dots, p_r]^T$ . By modifying the fuzzy controller's knowledge base, we may force the FC to produce a desired output, which we should put in at time kT - T to make  $h_{e}(kT)$  smaller. Then the next time we get similar values for the error and change in error, the input to the plant will be one that will reduce the error between the RM and the plant output.

Assume that we use symmetric output membership function for the FC, and let  $b_i$  denote the center of

the membership function associated with  $\tilde{U}_l$ . Knowledge base modification is performed by shifting centers  $b_l$  of the membership function of the output linguistic value  $\tilde{U}_l$ , which are associated with the fuzzy controller rules that contributed to previous control action u(kT - T). This is two-step process.

1. Find all FC rules (3), which satisfy the following condition:

 $\mu_l(e(kT-T),c(kT-T)) \succ 0$ ,

 $(l \in [1, m \times n])$ , defining the set of the output

membership functions with non-zero values  $\mu_l$ , and call it the as the "active set" of rules at time kT - T. We can characterize the active set by the indices of the input membership function of each rule that is activated.

2. Let  $b_l$  denote the center of *l*-th output membership function at time kT. For all rules in the active set, use

$$b_l(kT) = b_l(kT - T) + p_l(kT)$$

to modify the output membership function centers.

Rules that are not in the active set do not have their membership function modified.

The knowledge base modifier includes also the storage (fig. 2), which preserves the results of tuning of membership functions. These results could be used in order to avoid the on-line learning in each control scenario and to simplify fuzzy control.

# Design and implementation of fuzzy model reference learning control

The total design procedure for the FMRLC involves the following steps. The specification of a direct FC with consequent membership functions that can be tuned. This fuzzy controller can be chosen via conventional (heuristic) fuzzy control design techniques for the nominal plant.

Specifying the RM of control system which characterizes the desired system performance.

Specifying the fuzzy inverse model, which characterizes how the inputs to the plant should be changed so, that the desired performance is achieved.

Selection of the normalizing gains for the FC and the fuzzy inverse model.

So far as the selection of the normalizing gains for both the FC and the fuzzy inverse model can impact the overall performance, it is necessary to provide a procedure for choosing these parameters. Due to physical constraints for a given system, the range of values for the process inputs and outputs is generally known from a qualitative analysis of the process especially, when the crisp prototype of system is determined via some known procedure of control synthesis. As a result, we can determine the range of values or the universe of discourse for e(kT),

u(kT),  $h_e(kT)$  and p(kT).

Consequently,  $g_e$ ,  $g_c$ ,  $g_u$ ,  $g_{he}$ , and  $g_p$  are chosen so that the appropriate universes of discourse are mapped to [-1, 1]. They could be determined on the basis of the "crisp" prototype by iteratively applying inputs to r(kT), observing c(kT), and finding scaling factors to map the universes of discourse to [-1, 1]. The coefficient  $g_{hc}$  is left as a tuning parameter for the FMRLC. Recall that the scaling factor  $g_{hc}$  associated with the change in the desired output changes has the effect of providing "damping" to the controller modifications. Moreover, the "damping" effect is increased as the scaling factor  $g_{hc}$  is increased. A suitable selection of  $g_{hc}$  may be obtained by monitoring the response of the overall process with respect to the reference model response.

If undesirable oscillations exist between a given process and the associated reference model output response, it is likely that  $g_{hc}$  is too small and should be increased. Likewise, if a given value of  $g_{hc}$  is too large, the process will be unable to keep up with the reference model due to the resulting damping. Below a simple procedure is presented for selecting the gains [2; 3].

1. Using observation of corresponding coordinates e,u,c in the "crisp" prototype, select the controller gains  $g_e$ ,  $g_u$ ,  $g_c$  so that each universe of discourse is mapped to the interval [-1, 1].

2. Choose the controller gain  $g_p$  to be the same as for the fuzzy controller output gain  $g_u$ . This will allow the p(kT) to take on values as large as the largest possible input u(kT).

3. Use simple aforementioned experiments with "crisp" prototype for choosing  $g_c$  to map the universes of discourse of c(kT) to [-1, 1].

4. Assign the numerical value 0 to the scaling factor  $g_{hc}$ .

Apply a step input to the process, which is of a magnitude that may be typical for the process during normal operation. Observe the process response and the reference model response. Three cases are possible as the outcomes of this step.

1. If there exist unacceptable oscillations in a given process output response about the reference model response, then increase  $g_{hc}$ . Then go to step 5.

2. If a given process output response is unable to "keep up" with the reference model response, and then decrease the associated element of  $g_{hc}$ . Go to step 5.

3. If the process response is acceptable with respect to the reference model response, then the controller design is completed.

#### Case study

The inputs to the FC are the altitude error and change in altitude error and the output is the pitch angle reference, which is compared with the actual pitch output of the UAV.

In this fuzzy controller design 11 fuzzy sets are defined for both controller inputs (using the structure of fig. 2) such that the membership functions are triangular shaped and uniformly distributed on appropriate universe of discourse as it is shown in fig. 3.

The normalizing controller gains for the error, change in error and the controller output are chosen to be  $g_e = 0.026$ ,  $g_c = 1.7$  and  $g_u = 0.3$ , respectively.

The fuzzy set for the controller output is also assumed to be triangular shaped with width of 0,4 on the normalized universe of discourse. The knowledge base array was initially chosen with all zero entries. The fuzzy controller sampling time period was chosen to be T = 100 ms.

The reference model for this plant was chosen to represent somewhat realistic performance specifications complied with the robust "crisp" prototype and is expressed by the following state space representation:

$$\begin{bmatrix} \dot{x}_{r1} \\ \dot{x}_{r2} \end{bmatrix} = \begin{bmatrix} -2 & -1,333 \\ 0,4 & 0 \end{bmatrix} \begin{bmatrix} x_{r1} \\ x_{r2} \end{bmatrix} + \begin{bmatrix} 0,01 \\ 0 \end{bmatrix} r;$$
  
$$h_m = \begin{bmatrix} 0 & 133 \end{bmatrix} \begin{bmatrix} x_{r1} \\ x_{r2} \end{bmatrix} + \begin{bmatrix} 0 \end{bmatrix} r, \qquad (4)$$

where  $x_{r1}, x_{r2}$  are the state variables of the reference model.



Model (4) provides the altitude step response shown in the fig. 4 with dotted lines. The input to the fuzzy inverse model includes the error and change in error between the reference model and the UAV's altitude output expressed as

$$h_e(kT) = h_m(kT) - h(kT)$$
  
and

$$h_{c} = \frac{h_{e}(kT) - h_{e}(kT - T)}{T}$$

respectively.

For these inputs, 11 fuzzy sets are defined with triangular membership functions, which are evenly distributed on the appropriate universe of discourse. The normalizing gains associated with  $h_e(kT)$   $h_c(kT)$  and p(kT) are chosen to be  $g_{he} = 0.01$ ,  $g_{hc} = 1.6$  and  $g_p = 0.13$  respectively.

The rule base of the fuzzy inverse model is proposed in [2; 3] and shown in the tab. 1. In tab. 1,  $E^{j}$  denotes the j-th fuzzy set associated with the error signal  $h_{e}$  and  $C^{k}$  denotes the k-th fuzzy set associated the change in error signal  $h_{c}$ .

Pitch angle		Error change of altitude C <sup>k</sup>										
۳,۲۹		-1	-0.8	-0.6	-0.4	-0.2	0	0.2	0.4	0.6	0.8	1
Altitude	-1	-1	-1	-1	-1	-1	-1	-0.8	-0.6	-0.4	-0.2	0
	-0.8	-1	-1	-1	-1	-1	-0.8	-0.6	-0.4	-0.2	0	0.2
	-0.6	-1	-1	-1	-1	-0.8	-0.6	-0.4	-0.2	0	0.2	0.4
	-0.4	-1	-1	-1	-0.8	-0.6	-0.4	-0.2	0	02	0.4	0.6
	-0.2	-1	-1	-0.8	-0.6	-0.4	-0.2	0	0.2	0.4	0.6	0.8
error	0	-1	-0.8	-0.6	-0.4	-0.2	0	0.2	0.4	0.6	0.8	1
Еj	0.2	-0.8	-0.6	-0.4	-0.2	0	0.2	0.4	0.6	0.8	1	1
	0.4	-0.6	-0.4	-0.2	0	0.2	0.4	0.6	0.8	1	1	1
	0.6	-0.4	-0.2	0	0.2	0.4	0.6	0.8	1	1	1	1
	0.8	-0.2	0	0.2	0.4	0.6	0.8	1	1	1	1	1
	1	0	0.2	0.4	0.6	0.8	1	1	1	1	1	1

Rule-base of the fuzzy inverse controller

The entries of the table represent the center values triangular membership functions with base widths 0,4 for fuzzy sets  $P^{j,k}$  on the normalized universe of discourse.

## **Simulation results**

The simulation results for the FMRLC of the UAV are shown in fig. 4, 5, 6, 7.

The comparison of the dynamic properties is represented in the tab. 2 for the step responses and tab. 3 for the response at the stochastic disturbance, which is the turbulent wind w in (2) represented by Dryden model [8].

These results could be summarized as follows.

1. Deflections of all UAV state variables are within tolerances, which are acceptable from the viewpoint of flight safety.

2. The step response performances of fuzzy system for nominal and parametrically perturbed plants are better than the same performances of the "crisp" robust prototype.

3. The r.m.s. of all state variables of fuzzy system with nominal and parametrically disturbed plant in a case of turbulent wind action are less, than the same values of "crisp" prototype.

Table 2

	Settling	Overshoot <i>h</i> , %	Maximal deflection						
System	time, s		<i>u</i> , m/s	$\alpha$ , deg	${\mathcal G}$ ,deg	q ,deg/s	$\delta\!e$ ,deg		
Fuzzy nominal	12	1,0	12	5,8	11	23	5		
Crisp nominal	15	10	14,2	8,85	17,1	27,6	8,1		
Fuzzy perturb	12	1,0	12,5	6,08	11,5	21,7	5,1		
Crisp perturb	16	5	17,8	9,2	17	28	8,2		

Comparison of the fuzzy and crisp systems' step responses

Table 3

Comparison	of the st	tate variables	' r.m.s. o	of the fu	izzy and	crisp systems
					•/	•

System	$\sigma_h, m$	$\sigma_u$ , m/s	$\sigma_{\alpha}$ , deg	σ <sub>,</sub> , deg	$\sigma_q$ , deg/s	$\sigma_{\partial e}$ , deg
Fuzzy nominal	0,1213	2,1942	0,9853	0,7326	2,4877	0,6362
Crisp nominal	3,593	7,1538	3,0791	4,4520	6,1054	2,0939
Fuzzy perturb	0,413	3,1068	1,1347	1,1257	1,8757	1,1033
Crisp perturb	5,878	11,9290	4,5465	5,9787	4,8389	3,1954

Table 1



Fig. 6. Altitude output and the reference model output



## Conclusions

This comparison shows that using the fuzzy autopilots it is possible to find such tuning of parameters of the membership functions, which provides the same level of robustness (accepted parameter uncertainty) and even some small improvement of performance for the deterministic disturbances and better suppression of the random errors under deterministic disturbances.

The learning mechanism in the FMRLC dynamically and continually updates the rule-base in the direct FC in response to process parameter variations and/or disturbances. In this way if unpredictable changes occur within the plant, the FMRLC can make on-line adjustments to a direct FC to maintain adequate performance levels.

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