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STOCHASTIC METHODS OF STEEL CONSTRUCTIONS' CALCULATION OF THE SILO CAPACITIES AT THE SEISMIC STABILITY

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Abstract

The paper deals with the technique of assessment of the reliability function of the construction of vertical cylindrical silos under the seismic influence. The developed approach is based on the theory of the static seismic stability, which was worked out by V.V. Bolotin. The random character of geotectonic excitation and stochastic features of the material is considered. The introduced technique could be used both for direct stochastic calculations of constructions of the silo capacity and for the general normalization of seismic influences on buildings and constructions. The assessment of conditional probability of construction's failure is received in two variants: accordingly to the model of absolute maximums and on the basis of the exponential model, which was based on the Poisson classification. Mentioned convenient practical formulas to determine full reliability functions represent analytical complex expression, which combine different parameters: the proper frequency of the construction, the duration of intense phase of the earthquake, the parameter of repeatability of the earthquake, stochastic characteristics of the charging effect of the construction, useful life of the construction and others. The finiteness of the expounded material is the use of the described approach only for constructions, which works linearly. But, if the non-linearity of the construction is rather small we could use statistic linearization method, which is widely used in general theory of building construction's reliability and could be successfully used in this case.

Keywords: cylindrical silo; aerodynamics; membrane theory of shells; wind loads; internal forces; radial displacements

1. Introduction

The problem statement in general and its connection with important scientific or practical tasks. The natural and the man-made seismicity of the region is an important factor, which should be considered when calculating constructions of the silo capacities. This problem is topical, since every year the overall dimensions of the given constructions and the number of additional equipment, which is connected to it, grow in the industry of the silo manufacturing.

Analysis of recent research and publications. The main rules of the seismic stability design of silos were put in UNSS (Ukrainian national standardization system) N B EN 1998-4:2012 Eurocode 8 and ambiguously studied in research papers [1, 2, 3, 4]. But the analysis of the systems' reliability goes beyond the given normative document. The static theory of the seismic stability, based on the introducing an earthquake as a nonstationary random process, was firstly examined in the work of V.V. Bolotin [5]. Later this theory was developed in the work [6] and partially developed in works about theory of random vibrations, including nonlinear [7, 8]. Some aspects of the static theory of the seismic stability are partially represented in the fundamental monograph [9] and work [10].

Defining of the unsolved aspects of the problem. One of the main problems of the static theory of the seismic stability is the calculation of the expected useful life of the construction and the development of methods of design of such constructions, which probability of ruining will be not more than preassigned value during the established useful life. Since, it is universally recognized that seismic influences have clearly defined random character, then methods of probability theory and random processes theory should be put in the basis of stochastic methods of constructions' calculation at the seismic stability.

The purposes formulation of article. On the basis of static theory of seismic stability, which was formed and worked out by V.V. Bolotin, in the given paper authors give stochastic approach to the assessment of the reliability function of building constructions, which were affected by seismic random loads. In addition, the experience of the proper researches in the area of constructions' reliability is considered [11, 12].

2. General problem solving method

In the given active zone earthquakes is forming a stream of random events, which are caused by slow geotectonic processes in the heterogeneous and the heterogeneously strained crus. Every earthquake is characterized by series of random parameters, such as coordinates of the epicenter, the depth of the focus occurrence, released energy and others. On the given area seismic quakes are the result of seismic waves, which come to the area from the epicenter. These quakes are also forming stream of random events E, which characterized by random macro seismic parameters: intensity, maximum acceleration of the basis, duration of the earthquake, parameters of its spectral analysis. According to V.V. Bolotin [6], random events E we will divide into classes $\Phi_1, \Phi_2, ..., \Phi_j, ...\Phi_m$, which differ, for example, by the level of intensity of seismic influences. Let's assume that events of every class j is forming at the period of time $[0, T]$ independent fixed streams E with intensity λ_j . Probability of the event $E \in \Phi_j$ at the given period of time k times is defined by using $Q_k\left(\Phi_j \middle| T\right)$. Let $P_U\left(\Phi_j\right)$ is the probability that the breakdown (construction's failure) will not happen when the event $E \in \Phi_j$ appear. Then, neglecting the influence of damages, which are caused by previous events, and considering the independence of streams of events of different classes, let's calculate the reliability function as probability that the emergency situation will never appear at the period of time $[0, T]$ [6]

$$P_U(T) = \prod_{j=1}^{m} \left[Q_0(\Phi_j; T) + \sum_{k=1}^{\infty} P_U^k(\Phi_j) Q_k(\Phi_j; T) \right], \quad (1)$$

where $Q_0(\Phi_j; T)$ is the probability that the event $E \in \mathit{\Phi}_j$ will never appear at the period of time $[0, T]$.

If the stream of random events E is Poisson's, then the probability $Q_k\left(\Phi_j \middle| T\right)$ of appearing of k events at the period of time $[0, T]$ could be expressed by formula [13, 14]

$$Q_k(\Phi_j; T) = \frac{(\lambda_j T)^k}{k! \exp(-\lambda_j T)}. \quad (2)$$

When substituting (2) into (1) after the series of algebraic transformations, we will receive more concise expression

$$P_U(T) = \exp\left[-\sum_{j=1}^{m} \lambda_j T Q_U(\Phi_j) \right], \quad (3)$$

where $Q_U(\Phi_j) = 1 - P_U(\Phi_j)$ is the probability that the breakdown (construction's failure) will happen when the event $E \in \Phi_j$ appear.

If $\lambda_j T Q_U(\Phi_j) << 1$ when all j and all T, then the expression (3) results in simple formula for the reliability function

$$P_U(T) \approx 1 - \sum_{j=1}^{m} \lambda_j T Q_U(\Phi_j) \ . \quad (4)$$

Since, the supporting ability of the construction in designed stochastic calculations at the seismic stability is set with an accuracy of random value $\tilde{R}$, then full reliability function of the construction is denoted by using the method of conditional reliability functions [2, 4] in the form of

$$\begin{aligned} P(T) &= \int_L^{\infty} P_U(R \mid T) f_R(R) dR = \\ &= \int_L^{\infty} f_R(R) dR - \int_L^{\infty} f_R(R) \sum_{j=1}^{m} \lambda_j T Q_U(\Phi_j \mid R) dR. \end{aligned} \quad (5)$$

where $f_R(\bullet)$ is the density of distribution of the supporting capability of the construction; $Q_U(\Phi_j \mid R)$ is the conditional probability of the construction's failure, i.e. the probability of failure, calculated in assumption that the supporting capability of the construction is determined and defined by the value R; L is the left border of integrity (certainly $L = 0$, but in some cases, for example, for the model of the absolute maximums of loads $L > 0$).

In the next reasoning we will take $\tilde{q}_j(t)$ as one-dimensional random processes, which have identical physical content and act as a unique parameter, with accuracy to which is given the seismic influence. The example of this is the acceleration $\tilde{a}_j(t)$ from earthquakes at some area or proportional to it efforts, moments or strains in the construction $\tilde{S}_j(t)$. In addition, the class of seismic influence $\mathbf{\Phi}_j$ characterizes focal, while acceleration (and accordingly internal efforts $\tilde{S}_j(t)$) is considered in calculation, regardless of the origin of the earthquake. In most of the practically important cases seismic influences $\tilde{q}_j(t)$ (and correspondingly to them internal efforts) could be introduced in the form of fixed ergodic random processes with the medium duration τ_E [6].

The assessment of conditional probability of construction's failure in the formula (5) could occur in two ways. The first approach, which was offered by V.V. Bolotin and entitled as "the model of absolute maximums", consist identification of medium amount of the positive intersections $N_+(\Phi_j \mid R;\tau_E)$ of the random process of the internal effort $\tilde{S}_j(t)$ of some equation R with probable intersections of this level $Q_U(\Phi_j \mid R;\tau_E)$, i. e. $Q_U(\Phi_j \mid R;\tau_E) \approx N_+(\Phi_j \mid R;\tau_E)$, when

$$N_+(\Phi_j \mid R;\tau_E) << 1. \qquad (6)$$

In addition, there is always an equation:

$$Q_U(\Phi_j \mid R;\tau_E) \le N_+(\Phi_j \mid R;\tau_E). \qquad (7)$$

Consequently, the formula (6) will always make for the probability $Q_U(\Phi_j \mid R;\tau_E)$ a strict assessment from above. How precise the given assessment will be, depends from frequency structure of the particular earthquake and how far the level R from the level of expectation of the process $\tilde{S}_j(t)$.

The second approach, which was entitled as "the exponential model", is based on the Poisson's classification and in the case of fixed random process determines probability $Q_U(\Phi_j \mid R;\tau_E)$ in the form of

$$Q_U(\Phi_j \mid R;\tau_E) \approx 1 - \exp[-N_+(\Phi_j \mid R;\tau_E)]. \qquad (8)$$

The precise of the assessment of the probability $Q_U(\Phi_j \mid R;\tau_E)$ depends from the type of the stream of ejections of the random process $\tilde{S}_j(t)$ over the level R. If the positive intersection of the process $\tilde{S}_j(t)$ of the level R could be interpreted as an event in the Poisson stream, then expression (8) gives the ability to find precise solution for conditional probability of failure $Q_U(\Phi_j \mid R;\tau_E)$. In the other case, the assessment will be made with some error, the value of which will be depended on how the stream of ejections is different from the Poisson's.

Let's substitute (6) and (8) into the formula (5) to receive full reliability function. We consider every case apart.

3. The model of absolute maximums

$$P(T) = \int_{S_{0,j}}^{\infty} f_R(R)dR - \int_{S_{0,j}}^{\infty} \sum_{j=1}^{m} \lambda_j T N_+(\Phi_j \mid R;\tau_E) f_R(R)dR, \qquad (9)$$

where $S_{0,j}$ is the characteristic maximum of the random process; $\tilde{S}_j(t)$ is the level to which the equation $N_+(\Phi_j \mid S_{0,j};\tau_E) = 1$ is satisfied.

The first constituent of the given formula determine the probability that supporting ability of the construction $\tilde{R}$ is higher than the level of the characteristic maximum of the random process of the internal effort $S_{0,j}$. The second constituent could be extended by writing down the expression for the medium amount of positive intersections as the random process $\tilde{S}_j(t)$ of the level R [10, 12]

$$N_+(\Phi_j \mid R;\tau_E) = \hat{S}_j \frac{\omega_{e,j}\tau_E}{\beta_{\omega,j}\sqrt{2\pi}} f_S(R) = \frac{f_S(R)}{f_S(S_{0,j})}, \qquad (10)$$

where $\omega_{e,j}, \beta_{\omega,j}, \hat{S}_j$ and $f_S(\bullet)$ are correspondingly effective frequency, the coefficient of widebending, the standard and the law of ordinate's distribution of the random process $\tilde{S}_j(t)$ from the Φ_j class of seismic influences (for simplicity it is received that the density of distribution of the random process $\tilde{S}_j(t)$ have the same analytical form of recording for all the classes of seismic influences).

If substituting (10) into (9), we will have

$$P(T) = \int_{S_{0,j}}^{\infty} f_R(R)dR - \int_{S_{0,j}}^{\infty} \sum_{j=1}^{m} [\frac{\lambda_j T f_S(R)}{f_S(S_{0,j})}] f_R(R)dR. \qquad (11)$$

The final formula for assessment of the reliability function of construction will be received by passing to standardized form and considering that value $\tilde{R}$ mostly obey the normal law of distribution

$$P(T)=\frac{1}{\sqrt{2\pi}}\int_{Z_{0,j}}^{\infty}\exp(-0.5Z^2)dZ-\frac{1}{\sqrt{2\pi}}\times \int_{Z_{0,j}}^{\infty}\sum_{j=1}^{m}\{\lambda_j T\frac{f_{nS}[E_j(Z)]}{f_{nS}(\gamma_{0,j})}\}\exp(-0.5Z^2)dZ, \quad (12)$$

where $f_{nS}(\bullet)$ and $\gamma_{0,j}$ are correspondingly standardized density of the distribution and standardized characteristic maximum of the random process $\tilde{S}_j(t)$; $Z_{0,j}=\gamma_{0,j}p_j^{-1}+p_j^{-1}V_{S,j}^{-1}-V_R^{-1}$ is an inferior boundary of integrity; $E_j(Z)=Zp_j+p_jV_R^{-1}-V_{S,j}^{-1}$ is a non-dimensional argument of the function $f_{nS}(\bullet)$; V_R and $V_{S,j}$ are coefficients of variations of the supporting ability and process $\tilde{S}_j(t)$; $p_j=\frac{\hat{R}}{\hat{S}_j}$ is the relation of standards.

4. The exponential model

$$P(T)=\int_0^{\infty}\exp[-\sum_{j=1}^{m}\lambda_j TN_+(\Phi_j\,|\,R;\tau_E)]f_R(R)dR\,. \quad (13)$$

By substituting the formula (5) for $N_+(\Phi_j\,|\,R;\tau_E)$, we will receive:

$$P(T)=\int_0^{\infty}\exp\left[-\sum_{j=1}^{m}[\frac{\lambda_j Tf_S(R)}{f_S(S_{0,j})}]\right]f_R(R)dR\,. \quad (14)$$

The transition to standardized form makes the expression more convenient for the practical usage

$$P(T)=\frac{1}{\sqrt{2\pi}}\int_{V_R^{-1}}^{\infty}\exp[-\sum_{j=1}^{m}\frac{\lambda_j Tf_{nS}\{E_j(Z)\}}{f_{nS}(\gamma_{0,j})}]\exp(-\frac{Z^2}{2})dZ\,. \quad (15)$$

Apparently, formulas (12) and (15) were proposed by authors for the first time and are the further development of the general procedure of assessment of the reliability of the steel constructions' elements, which was worked out in works [5-7, 11, 12]. They allow considering time-and-frequency structure of geotectonic excitations, actual laws of the distribution of silo influences in constructions (efforts, strains) and solving the problem of constructions' reliability within this work.

5. An example of the calculation

As the example we will consider partial case $m=1$* of behavior of the capacity's construction (one-mass system). We will take the model of the earthquake, according to which the acceleration of the basis $\tilde{a}(t)$, at which the silo capacity is placed, is the segment of realization of the fixed normal random process with expectation, which is equal to zero and with standard $\hat{a}$ (Fig. 1).

The reaction of the construction at the action of the seismic load we will introduce in the form of differential equation (the linear system with one level of freedom is considered)

$$\ddot{u}(t)+2\beta\omega_0\dot{u}(t)+\omega_0^2u(t)=-\tilde{a}(t)\ , \quad (16)$$

where ω_0 is the proper frequency of construction's oscillation; β is the coefficient of damp; the operation of differentiation is shown by points.

Let's admit that the proper period of construction's oscillation, the main period of the quake, and also characteristic time of correlation of the process $\tilde{a}(t)$ is rather small, comparing to the duration of the intense phase of the earthquake τ_E. Let also the damp is rather small, so $\beta^2 << 1$

Then, in the expression (16) we could receive $a(t)+\ddot{u}(t)\approx-\omega_0^2u(t)$. We will introduce efforts, which appear in the construction, proportionally to the construction's acceleration using proportionality constant k – $S(t)=k\cdot\omega_0^2u(t)$.

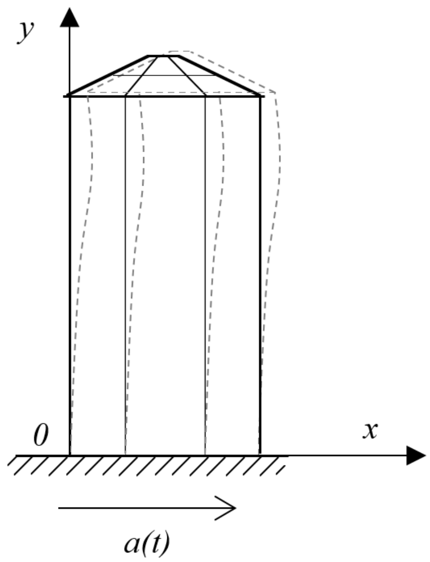

Fig. 1. The system of coordinates, which is moving with the basis

Within the given remarks the formula (10) will be in the form of (in standardized form)

$$N_+(\Phi_j \mid \gamma;\tau_E) = \exp[0.5(\gamma_0^2 - \gamma^2)] \,, \quad (17)$$

where standardized characteristic maximum is determined from the expression:

$$\gamma_0 = \sqrt{2\ln\left[\frac{\omega_0 \tau_E}{\pi \beta_\omega}\right]}. \quad (18)$$

Considering that the standard of acceleration of the construction is $\hat{a} = \omega_0^2 \hat{u}$ ($\hat{u}$ is the standard of the process $\tilde{u}(t)$), for formulas (12) and (15) correspondingly we will receive formula to assess the reliability function

$$P(T) = \frac{1}{\sqrt{2\pi}} \int_{Z_0}^{\infty} \exp(-0.5Z^2)dZ - \\ -\frac{\lambda T}{\sqrt{2\pi}} \int_{Z_0}^{\infty} \exp\{0.5[\gamma_0^2 - E^2(Z)]\}\exp(-0.5Z^2)dZ. \quad (19)$$

$$P(T) = \frac{1}{\sqrt{2\pi}} \int_{1/V_R}^{\infty} \exp[\lambda T \exp\{0.5\big[\gamma_0^2 - \\ -E^2(Z)]\}\big]\exp(-0.5Z^2)dZ, \quad (20)$$

where $Z_0 = \gamma_0 p^{-1} + p^{-1}V_S^{-1} - V_R^{-1}$, $E(Z) = Zp + pV_R^{-1} - V_S^{-1}$ relation of standards is found in the form of $p = \frac{\hat{R}}{\omega_0^2 \hat{u} k}$.

Formulas (19) and (20) in connection with the expression (18) have a special interest, since there is a multipurpose expression in their right parts, which combines different parameters: the proper frequency of the construction ω_0, the duration of the intense phase of the earthquake τ_E, the parameter of repeatability of the earthquake λ, probable characteristics of the charging effect of the construction, useful life of the construction T stochastic features of the construction's material. These parameters are characterizing the stochastic nature of geotectonic excitation, on the one hand, and determined and stochastic features of the construction, which is affected by seismic influences, on the other.

6. Conclusions

To summarize, we consider that modified in the paper approach to assess the reliability function of constructions, which are affected by seismic influences, not connected with significant mathematic and computational difficulties, and consequently it could be used directly at practice. The described technique could be used not only for direct stochastic calculations of constructions at the seismic stability, but also for creating simple by form normative calculations. The finiteness of the expounded material is the use of the described approach only for constructions, which works linearly. However, if the nonlinearity of constructions is rather small, then static linearization method, which is widely used in general theory of building construction's reliability, could be successfully used in this case.

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Н.О. Махінько

Імовірнісні методи розрахунку сталевих конструкцій силосних ємностей на сейсмостійкість

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Наводиться методика оцінки функції надійності конструкції вертикальних циліндричних силосів при дії сейсмічних впливів. Розроблений підхід ґрунтується на теорії статичної сейсмостійкості, розробленої В.В. Болотіним. Враховується випадковий характер геотектонічних збуджень та стохастичні властивості матеріалу. Запропонована методика може застосовуватися як для прямих імовірнісних розрахунків конструкцій силосної ємності так і для загального нормування сейсмічних впливів на будівлі та споруди. Оцінка умовної імовірності відмови конструкції отримана в двох варіантах – відповідно до моделі абсолютних максимумів та на основі експоненціальної моделі, яка базується на розподілі Пуассона. Зазначені зручні практичні формули для визначення повної функції надійності, які являють собою аналітичний комплексний вираз, що поєднує різні параметри: власну частоту конструкції, тривалість інтенсивної фази землетрусу, параметр повторюваності землетрусу, імовірнісні характеристики завантажувального ефекту конструкції, строк експлуатації конструкції та ін. Обмеженістю викладеного матеріалу є застосування описаного підходу лише для конструкцій, які працюють лінійно. Проте якщо нелінійність конструкцій досить невелика можливе використання методу статистичної лінеаризації.

Ключові слова: функція надійності; імовірність відмови; сейсмічний вплив; статистична теорія сейсмостійкості; випадковий процес

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Вероятностные методы расчета стальных конструкций силосных емкостей на сейсмостойкость

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Приводится методика оценки функции надежности конструкции вертикальных цилиндрических силосов при действии сейсмических воздействий. Разработанный подход основывается на теории статической сейсмостойкости, разработанной В.В. Болотиным. Учитывается случайный характер геотектонической возбуждений и стохастические свойства материала. Предложенная методика может применяться как для прямых вероятностных расчетов конструкций силосной емкости, так и для общего нормирования сейсмических воздействий на здания и сооружения. Оценка условной вероятности отказа конструкции получена в двух вариантах - в соответствии с моделью абсолютных максимумов и на основе экспоненциальной модели, основанной на распределении Пуассона. Указанные удобные практические формулы для определения полной функции надежности, которые представляют собой аналитическое комплексное выражение, объединяющее различные параметры: собственную частоту конструкции, продолжительность интенсивной фазы землетрясения, параметр повторяемости землетрясения, вероятностные характеристики загрузочного эффекта конструкции, срок эксплуатации конструкции и др. Ограниченностью изложенного материала является применение описанного подхода только для конструкций, работающих линейно. Однако если нелинейность конструкций достаточно небольшая возможно использование метода статистической линеаризации.

Ключевые слова: функция надежности; вероятность отказа; сейсмический влияние; статистическая теория сейсмостойкости; случайный процесс

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