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Volodymyr Todchuk**STABILITY OF CYLINDRICAL SHELLS**

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Abstract

Purpose: Obtain more precise formulas for the theoretical axial critical load of a hinged cylindrical shell; find the cause of large differences between calculated and experimental critical loads. **Methods:** The energy criterion of stability and the relations of the general linear theory of thin-walled shells are used.

Results: New formulas for the dependence of the critical load on the mechanical and geometric characteristics of the shell and the parameters of wave formation are obtained. The values of the critical loads calculated from these formulas are close to the experimental data. A greater dependence of critical loads on: the ratio of the radius to the thickness of the shell is revealed; the ratio of length to radius; boundary conditions. **Discussion:** Cylindrical shells are widely used in engineering structures. The loss of stability of the shell can lead to the destruction of the structure. To estimate the bearing capacity of engineering structures, exact formulas are needed to calculate the critical loads of shells under axial compression. Such formulas have not yet been obtained. The reason for the large discrepancies between the theoretical and experimental values of the axial critical loads of cylindrical shells was not found. In this paper, an attempt is made to solve this problem. In contrast to the conventional approach, it is assumed here that when the shell is buckled, the distance between its ends does not change. This approach allowed obtaining formulas of axial critical loads that more accurately describe the process of loss of stability of a cylindrical shell under axial compression. Analysis of the obtained results allows us to conclude that the obtained formulas can be used for real calculations of critical loads of cylindrical shells, and the proposed approach can be used to continue studies of the stability of thin-walled structures.

Keywords: bending; critical load; displacement; energy; experiment; stability

1. Introduction

Cylindrical shells are widely used in engineering structures. The loss of stability of the shell can lead to the destruction of the structure. To estimate the bearing capacity of engineering structures, exact formulas are needed to calculate the critical loads of shells under axial compression. Such formulas have not yet been obtained. The reason for the large discrepancies between the theoretical and experimental values of the axial critical loads of cylindrical shells was not found. In this paper, an attempt is made to solve this problem.

2. Analysis of the latest research and publications

The first results of the study of the stability of structures were obtained by L. Euler [1], Brian [2], Lorentz [3] and S.P.Timoshenko [4].

Euler obtained the formula for the critical force of a rod hinged at its ends.

Brian first solved the problem of the stability of a hinged plate, compressed in one direction.

Lorentz and S.P.Timoshenko in a linear formulation based on the static criterion of L. Euler considered the stability of a hingedly supported circular cylindrical shell under axial compression. The value of the critical load (called the upper critical load) obtained in these studies was not confirmed experimentally. The critical loads observed in the experiments are much smaller than the upper critical loads.

All further development of the theory of shell stability was aimed at identifying the causes of this discrepancy. However, the problem is not solved and requires further research.

The most complete and detailed directions for the studies of the stability of shells, plates, and rods are presented in [5, 6, 7, 8].

In [9 -12], the stability of a hinged cylinder was studied for axial compression, taking into account the change in the external load during buckling, and it was shown that taking this change significantly affects the value of its critical load. In this work, these studies are continued.

The difference between the classical and the proposed approaches is as follows:

- the classical approach assumes that the transition from a rectilinear to a curved form of equilibrium occurs without a change in the value of the critical compression force N^* , i.e. with a constant length L of the shells. In this case, the ends of the shell receive some displacement in the axial direction, and the force $N^* = const$ completes the additional work $\Delta A \neq 0$ on these displacements;

- the proposed approach assumes that when the shell bulges out its edges remain in place, and the forming shells are extended. This is possible when the compressive forces decrease and become equal to $N_* - N_1$. The additional work of these forces is equal to zero ($\Delta A = 0$), since there are no displacements of the ends. The shell loses its stability not because of the work of the external load with additional displacements of its ends (this is accepted, for example, in the classical solution of the problem), but because of the redistribution of internal compression energy accumulated in the subcritical state.

Justification of the proposed approach

In accordance with the general theorem of mechanics, the total potential energy of any system has a stationary value $U = const$, when this system is in equilibrium. And the beginning of possible displacements asserts that if the system is in equilibrium, then the work of all forces on any infinitesimal possible displacements is zero, i.e. $\Delta U = \Delta V - \Delta A = 0$. Proceeding from this, if $\Delta A = 0$, then the increment of the strain energy of the shell $\Delta V = 0$. This means that the transition to a new form of equilibrium is carried out without the expenditure of energy, and hence the potential energy of the system remains constant and equal to the energy of the subcritical state. Since the state of the dimensionless equilibrium of the system is considered, the condition $\delta \Delta U = 0$ is used to determine the critical forces. As a result, we have $U = const, \Delta U = 0, \delta \Delta U$

$= 0$. Based on this, we conclude: the Lezhen - Dirichlet principle underlying the energy method, is fully observed in the proposed approach.

The immobility of the ends can be provided in two ways: a) the shell has fixed hinged edges;

b) the shell has movable hinged edges, but the convergence of the ends due to bending is compensated by movements due to the elongation of the generators.

Case a) was considered in [9 - 12], where it was shown that taking into account changes in the external load during buckling of the shell significantly reduces the theoretical value of the critical load N^* .

In the present paper we consider case b), and for the analysis of the results of the studies as a whole we briefly describe case (a).

3. Research tasks

Obtain more precise formulas for the theoretical axial critical load of a hinged cylindrical shell; find the cause of large differences between calculated and experimental critical loads.

4. Methods

To solve the problem, the energy criterion of stability and the relations of the general linear theory of thin-walled shells are used.

5. The solution of the problem

The shell has movable hinged edges

A cylindrical shell of length L , radius R , with wall thickness h , is loaded along the edges by uniformly distributed compressive forces N (Fig.1)

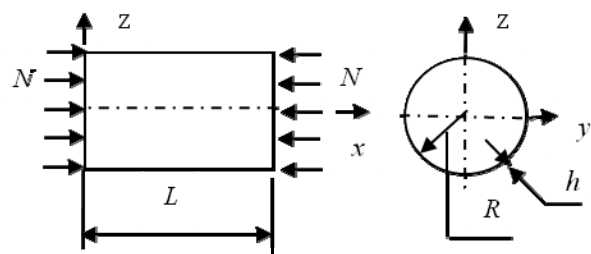


Fig. 1. Cylindrical shell with uniform axial compression

Initial assumptions: the shell is geometrically perfect and ideally elastic, the subcritical state is

momentless, the edges of the shell are supported on movable hinges.

In the case under consideration, the boundary conditions have the form: $u \neq 0, v = 0, w = 0, M_1 = 0$. Here u, v, w are the displacements of the points of the middle surface of the shell in the direction of the coordinates x, y, z ; M_1 is the bending moment.

We define the displacements corresponding to the boundary conditions

$$v = f_2 \sin \frac{m\pi x}{L} \sin \frac{ny}{R}; \quad w = f_3 \sin \frac{m\pi x}{L} \cos \frac{ny}{R}, \quad (1)$$

where f_2, f_3 are the displacement amplitudes in the direction of the y and z axes.

We find the displacement u from the condition

$$\int_0^L \int_0^L \left[\frac{\partial u}{\partial x} + \frac{1}{2} \left(\frac{\partial w}{\partial x} \right)^2 \right] dx dy = 0: \quad (2)$$

$$u = -\frac{1}{4} f_3^2 \left(\frac{m\pi}{L} \right)^2 \left(x + \frac{L}{2m\pi} \sin 2\frac{m\pi x}{L} \right) \cos^2 \frac{ny}{R}. \quad (3)$$

The problem is solved in a static formulation, where it is assumed that at each stage of the buckling process the shell equilibrium is preserved. This means that at each point of each face of the shell two equal and opposite forces are applied along one straight line: the external forces $N^* - NI$ and the internal forces $T^* - TI$, and the forces NI and TI slowly increase from zero to some finite value. Hence it follows that the force NI is numerically equal to the force TI at each point of each shell face. Under these conditions, the work done by the force NI is equal to half the product of the force to move the point of its application [13]. Thus, the additional work of the external compressive load

$$\Delta A = \frac{1}{2} \int_0^L \int_0^L \left(N_* - \frac{1}{2} N_1 \right) \left(\frac{\partial w}{\partial x} \right)^2 dx dy, \quad (4)$$

where $N_1 = T_1(0) = T_1(L)$.

The change in the energy of deformation of the shell with loss of stability

$$\Delta V = \frac{Eh}{2(1-\nu^2)} \int_0^L \int_0^L \left\{ \begin{aligned} &\varepsilon_1^2 + 2\nu\varepsilon_1\varepsilon_2 + \varepsilon_2^2 + \\ &\frac{1-\nu}{2}\varepsilon_{12}^2 + \frac{h^2}{12}[\chi_1^2 + \\ &2\nu\chi_1\chi_2 + \chi_2^2 + \\ &2(1-\nu)\chi_{12}^2] \end{aligned} \right\} dx dy, \quad (5)$$

where $\varepsilon_1 = \frac{\partial u}{\partial x}$; $\varepsilon_2 = \frac{\partial v}{\partial y} + \frac{w}{R}$; $\varepsilon_{12} = \frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}$;

$$\chi_1 = -\frac{\partial^2 w}{\partial x^2}; \quad \chi_2 = -\frac{\partial^2 w}{\partial y^2} + \frac{1}{R} \frac{\partial v}{\partial y};$$

$$\chi_{12} = -\frac{\partial^2 w}{\partial x \partial y} + \frac{1}{R} \frac{\partial v}{\partial x};$$

E, ν – Young's modulus and the Poisson's ratio of the material.

From (4), (5) and the condition $\Delta A = 0$, we

obtain: $N_1 = -\frac{Eh}{2(1-\nu^2)} f_3^2 \left(\frac{m\pi}{L} \right)^2 \cos^2 \frac{ny}{R}$;

$$\Delta A = \left(\frac{Eh}{1-\nu^2} \frac{3}{16} f_3^4 \frac{\lambda^4}{R^2} + N_* f_3^2 \lambda^2 \right) \frac{\pi L}{4R}, \quad (6)$$

$$\frac{3}{16} f_3^2 \frac{\lambda^2}{R^2} = -\frac{1-\nu^2}{Eh} N_*, \quad \text{where } \lambda = \frac{m\pi R}{L};$$

$$\Delta V = \frac{Eh}{1-\nu^2} \left\{ \begin{aligned} &\frac{3}{16} f_3^4 \frac{\lambda^2}{R^2} a_4 + f_2^2 a_1 + \\ &2f_2 f_3 a_2 + f_3^2 a_3 \end{aligned} \right\} \frac{\pi L}{4R}, \quad \text{where}$$

$$a_1 = \frac{1-\nu}{2} \lambda^2 + n^2 + \frac{1}{12} \left(\frac{h}{R} \right)^2 \left[2(1-\nu)\lambda^2 + n^2 \right]; \quad (7)$$

$$a_2 = n \left[1 + \frac{1}{12} \left(\frac{h}{R} \right)^2 \left[(2-\nu)\lambda^2 + n^2 \right] \right];$$

$$a_3 = 1 + \frac{1}{12} \left(\frac{h}{R} \right)^2 (\lambda^2 + n^2)^2;$$

$$a_4 = \frac{3}{4} \lambda^2 + \frac{1-\nu}{6} \left(\frac{2\pi m}{3} - \frac{3}{4} \right) n^2$$

and we find the change in the potential energy

$$\Delta U = \Delta V - \Delta A: \quad \Delta U = \frac{Eh}{1-\nu^2} \left\{ \begin{aligned} &f_2^2 a_1 + 2f_2 f_3 a_2 + \\ &f_3^2 \left(a_3 - \frac{1-\nu^2}{Eh} N_* a_4 \right) \end{aligned} \right\} \frac{\pi L}{4R}. \quad (8)$$

From the condition $\frac{\partial \Delta U}{\partial f_2} = 0$; $\frac{\partial \Delta U}{\partial f_3} = 0$ we

obtain a system of two equations $f_2 a_1 + f_3 a_2 = 0$;

$$f_2 a_2 + f_3 \left(a_3 - \frac{1-\nu}{Eh} N_* a_4 \right) = 0. \quad (9)$$

The system of equations (9) has a nonzero solution under the condition that the determinant of the coefficients of the parameters f_2 и f_3

$$\begin{vmatrix} a_1 & a_2 \\ a_2 & \left(a_3 - \frac{1-\nu^2}{Eh} N_* a_4 \right) \end{vmatrix} = 0. \quad (10)$$

From (10) we obtain the critical compression

force $N_* = \frac{Eh}{1-\nu^2} \frac{1}{a_4} \left(a_3 - \frac{a_2^2}{a_1} \right)$; $\bar{N}_* = \frac{N_*}{N_*^e}$, (11)

where $N_*^e = Eh^2 / R \sqrt{3(1-\nu^2)}$.

Figure 2 shows the results of minimizing expression (11) with integer parameters m and n at $\nu = 0.3$. (Fig.2)

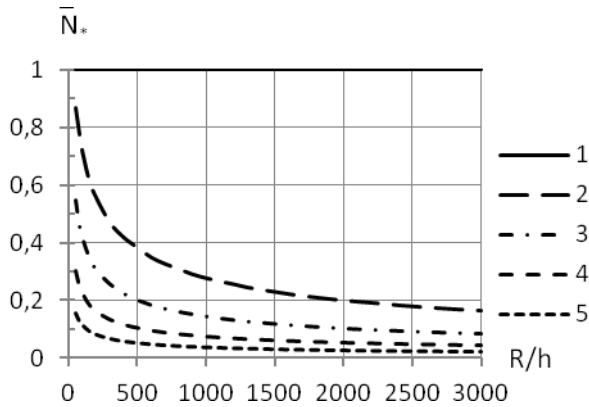


Fig.2. Dependence of a cylindrical shell supported on movable hinges on changes in the ratios L / R and R / h

1 - $\bar{N}_* = 1$; 2 - L/R = 1; 3 - L/R = 2; 4 - L/R = 4; 5 - L/R = 8

The shell has fixed hinged edges

According to [14], if the shell has hinged, immovably supported edges, then the boundary conditions have the form: $u=0, v=0, w=0, Ml = 0$.

The displacements corresponding to the boundary conditions in this case have the form

$$u = -\frac{1}{4} f_3 \frac{m\pi}{L} \sin 2 \frac{m\pi x}{L} \sin^2 \frac{ny}{R};$$

$$v = f_2 \sin \frac{m\pi x}{L} \cos \frac{ny}{R}; w = f_3 \sin \frac{m\pi x}{L} \sin \frac{ny}{R}. \quad (12)$$

Substituting (13) into (4), (5), similarly to the

above, we obtain $N_* = \frac{Eh}{1-\nu^2} \frac{1}{a_5} \left(a_3 - \frac{a_2^2}{a_1} \right)$; (13)

$$a_5 = \lambda^2 + \frac{1-\nu}{6} n^2.$$

Figure 3 shows the results of calculations using formula (13) (Fig.3).

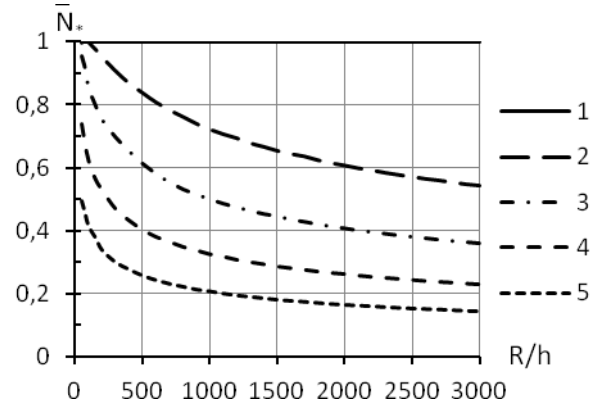


Fig.3. Dependence of a cylindrical shell supported on movable hinges on changes in the ratios L / R and R / h

1 - $\bar{N}_* = 1$; 2 - L/R = 1; 3 - L/R = 2; 4 - L/R = 4; 5 - L/R = 8

6. Results and discussion

Analysis of the results of theoretical calculations using formulas (11) and (13), which are presented in Fig. 2 and Fig. 3, shows:

1. Absolute and relative values of the axial critical loads of cylindrical shells depend strongly on the ratio of radius to thickness and on the ratio of the length to the radius of the shell.

2. Critical loads strongly depend on boundary conditions.

3. Theoretical critical loads, calculated from formulas (11) and (13), are close to experimental data, which are systematized in [6]. However, for a more accurate assessment, it is necessary to take into account, in each specific case, the geometric and mechanical characteristics of the shell, and the conditions for attaching edges.

7. Conclusions

1. The obtained formulas can be used to calculate the axial critical loads of real shells.

2. The proposed approach can bring researchers closer to solving the problem of stability and bearing capacity of engineering structures in general.
3. The proposed approach should be used to solve problems of shell stability under other boundary conditions.

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Стійкість Циліндричних Оболонки

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Мета: Отримати більш точні формули теоретичної осової критичного навантаження шарнірно закріпленої циліндричної оболонки; знайти причину великих відмінностей між розрахунковими і експериментальними критичними навантаженнями. **Метод:** Використовується енергетичний критерій стійкості і співвідношення загальної лінійної теорії тонкостінних оболонки. **Результати:** Отримано нові формули залежності критичного навантаження від механічних і геометричних характеристик оболонки і параметрів хвилеутворення. Значення критичних навантажень, розраховані за цими формулами, близькі до експериментальних даних. Виявлено велика залежність критичних навантажень від: відносини радіуса до товщини оболонки; відношення довжини до радіусу; граничних умов. **Обговорення:** Циліндричні оболонки широко використовуються в інженерних конструкціях. Втрата стійкості оболонки може привести до руйнування конструкції. Для оцінки несучої здатності інженерних споруд, необхідні точні формули для обчислення критичних

навантажень оболонок при осьовому стисненні. Такі формули поки не отримані. Причину великих розбіжностей між теоретичними і експериментальними значеннями осьових критичних навантажень циліндричних оболонок не знайдено. У даній роботі зроблена спроба вирішити цю проблему. На відміну від традиційного підходу, тут передбачається, що при випинанні оболонки, відстань між її кінцями не змінюється. Такий підхід дозволив отримати формули осьових критичних навантажень які більш точно описують процес втрати стійкості циліндричної оболонки при осьовому стисненні. Аналіз отриманих результатів дозволяє зробити висновок, що отримані формули можна використовувати для реальних розрахунків критичних навантажень циліндричних оболонок, а запропонований підхід може бути використаний для продовження досліджень стійкості тонкостінних конструкцій.

Ключові слова: вигин; експеримент; енергія; зміщення; критичне навантаження; стійкість

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Устойчивость цилиндрических оболочек

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Цель: Получить более точные формулы теоретической осевой критической нагрузки шарнирно закрепленной цилиндрической оболочки; найти причину больших различий между расчетными и экспериментальными критическими нагрузками. **Метод:** Используется энергетический критерий устойчивости и соотношения общей линейной теории тонкостенных оболочек. **Результаты:** Получены новые формулы зависимости критической нагрузки от механических и геометрических характеристик оболочки и параметров волнообразования. Значения критических нагрузок, рассчитанные по этим формулам, близки к экспериментальным данным. Выявлена большая зависимость критических нагрузок от: отношения радиуса к толщине оболочки; отношения длины к радиусу; граничных условий. **Обсуждение:** Цилиндрические оболочки широко используются в инженерных конструкциях. Потеря устойчивости оболочки может привести к разрушению конструкции. Для оценки несущей способности инженерных сооружений, необходимы точные формулы для вычисления критических нагрузок оболочек при осевом сжатии. Такие формулы пока не получены. Причина больших расхождений между теоретическими и экспериментальными значениями осевых критических нагрузок цилиндрических оболочек не обнаружена. В данной работе предпринята попытка решить эту проблему. В отличие от обычного подхода, здесь предполагается, что при выпучивании оболочки, расстояние между ее концами не изменяется. Такой подход позволил получить формулы осевых критических нагрузок которые более точно описывают процесс потери устойчивости цилиндрической оболочки при осевом сжатии. Анализ полученных результатов позволяет сделать вывод, что полученные формулы можно использовать для реальных расчетов критических нагрузок цилиндрических оболочек, а предложенный подход может быть использован для продолжения исследований устойчивости тонкостенных конструкций.

Ключевые слова: изгиб; критическая нагрузка; смещение; устойчивость; эксперимент; энергия

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