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INVESTIGATION OF SOUND RADIATION BY CANTILEVER BEAM

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The research investigates vibration of a cantilever beam loaded with actuators and load. The equation of motion is based on Bernoulli-Euler-Timoshenko theory of beams with corresponding boundary conditions. This equation is solved by the method of Green functions. A feature of this investigation is taking into account the influence of concentrated force parameters, namely the value, location of actuator and phase difference between load and actuator. In the work represented the contribution of actuator parameters on total sound power level as well as on each mode separately.

Introduction

The sound radiation from a vibrating beam is of practical importance [1; 2]. A direct calculation of radiated power for a beam is possible for some simplest task [3]. In general case a radiated power of the vibrating beam can be obtained for model of plane piston, which is set in an infinite baffle [2]. In this study it is used for parametric investigation of acoustical characteristics of the cantilever beams.

Vibration of finite cantilever beam

Let us consider the sound radiation of the cantilever beam of finite length *L,* which includes next boundary conditions

$$u(0) = \frac{\partial u(0)}{\partial x} = 0;$$

$$\frac{\partial^2 u(L)}{\partial x^2} = \frac{\partial^3 u(L)}{\partial x^3} = 0 . \quad (1)$$

The solution of boundary problem (1) is defined by solving the Helmholtz equation and equation of transverse motion of beam (for harmonic waves)

$$\Delta p + k^2 p = 0 ; \ \frac{d^4 u}{dx^4} - k_f^4 u = 0 ;$$

$$z = 0; \ \frac{\partial p}{\partial z} = \rho\omega^2 u(x) . \quad (2)$$

Seeking solution of transverse motion of the beam $u(x)$ for homogeneous equation of the beam motion in form

$$u(x) = A\cos k_b \frac{x}{L} + B\sin k_b \frac{x}{L} +$$

$$+ Cchk_b \frac{x}{L} + Dshk_b \frac{x}{L} , \quad (3)$$

where $k_b = Lk_f$,

the four unknown constant *A, B, C, D* of solution was determined from condition (1):

$A + C = 0, \ B + D = 0$.

In general case a solution for (3) can be obtained in form (by setting the forcing function equal to zero)

$$u(x) = \sum_{n=1}^{\infty} A_n \left[\left(\cos k_{bn} \frac{x}{L} - chk_{bn} \frac{x}{L} \right) + \frac{\sin k_{bn} - shk_{bn}}{\cos k_{bn} + chk_{bn}} \left(\sin k_{bn} \frac{x}{L} - shk_{bn} \frac{x}{L} \right) \right]$$

where k_{bn} must be determined from characteristic equation:

$$\cos k_b chk_b = -1 . \quad (4)$$

The numerical calculation of characteristic equation (4) gives:

$k_{b1} = 1{,}875;$

$k_{b2} = 4{,}694;$

$k_{bn} = (n - 0{,}5)\pi,$

for $n = 3,4,\ldots$.

Solution for 3D acoustic field for surface of beam can be written as

$$p(x, y, 0) = -\frac{\rho\omega^2}{2\pi} \int_0^L \int_0^b \frac{\exp(ikr)}{r} u(x_0) dx_0 dy_0 ,$$

where

$$r = \sqrt{(x - x_0)^2 + (y - y_0)^2} .$$

Acoustic power of an oscillating beam is defined as:

$$W = \frac{1}{2} \mathrm{Re} \int_0^L \int_0^b p(x, y, 0) w^*(x) dxdy ,$$

where $w(x) = -i\omega u(x)$.

Sound power level can be calculated from the formula:

$$L_W = 10\lg \frac{W}{W_0} ,$$

where

$W_0 = 10^{-12}$ *Wt.*

Forced vibration of finite cantilever beam

Let we use equation (2) for cantilever beam subjected to the point harmonic time force F:

$$\Delta p + k^2 p = 0;$$

$$\frac{d^4 u}{dx^4} - k_f^4 u = \frac{F}{EI};$$

$$z = 0;$$

$$\frac{\partial p}{\partial z} = \rho\omega^2 u(x), \quad (5)$$

$$F = \sum_{j=1}^{M} F_j \exp(i\varphi_j)\delta(x - x_j),$$

where φ_j is a phase of j-th load in point x_j; $\delta(x - x_j)$ is a Dirac function.

In order to solve the second equation in set (5) a method of Green function can be used.

We consider a concentrated force applied at the point $x = x_0$, $0 < x_0 < L$.

The solution of boundary problem (1) is defined by solving the equation of transverse motion of beam and include the Green's function approach (for harmonic waves)

$$\frac{d^4 G}{dx^4} - k_f^4 G = \delta(x - x_0). \quad (6)$$

The exact Green function $G(x, x_0)$, as a solution of equation (6), can be found for the regions $x < x_0$ and for $x > x_0$ in following forms accordingly:

$$G_-(x, x_0) = A_- \cos k_b \frac{x}{L} + B_- \sin k_b \frac{x}{L} +$$

$$+ C_- ch k_b \frac{x}{L} + D_- sh k_b \frac{x}{L}, \; x < x_0;$$

$$G_+(x, x_0) = A_+ \cos k_b \frac{x}{L} + B_+ \sin k_b \frac{x}{L} +$$

$$+ C_+ ch k_b \frac{x}{L} + D_+ sh k_b \frac{x}{L}, \; x > x_0,$$

where $k_b = L k_f$.

In the vicinity of x_0 ($|x_0| < \varepsilon$, where ε is an arbitrary small volume) for continuity conditions like following:

$$G_-(x_0 - \varepsilon, x_0) = G_+(x_0 + \varepsilon, x_0);$$

$$\frac{\partial G_-(x_0 - \varepsilon, x_0)}{\partial x} = \frac{\partial G_+(x_0 + \varepsilon, x_0)}{\partial x};$$

$$\frac{\partial^2 G_-(x_0 - \varepsilon, x_0)}{\partial x^2} = \frac{\partial^2 G_+(x_0 + \varepsilon, x_0)}{\partial x^2};$$

$$\frac{\partial^3 G_+(x_+ + \varepsilon, x_0)}{\partial x^3} - \frac{\partial^3 G_-(x_- - x_0, x_0)}{\partial x^3} = 1$$

and for boundary conditions (1) solution for 8 constants A_-, B_-, C_-, D_-, A_+, B_+, C_+, D_+ may be obtained.

Thus, the results for Green functions will be represented in form

$$G_-(x, x_0) = -\frac{L^3}{4k_b^3[1 + \cos(k_b) ch(k_b)]} \times$$

$$\times \{ch[k_b(-\frac{x_0}{L} + 1)]\sin[k_b(-\frac{x}{L} + 1)] -$$

$$- sh[k_b(-\frac{x_0}{L} + 1)]\cos[k_b(-\frac{x}{L} + 1)] -$$

$$- \sin(k_b) ch[k_b(-\frac{x + x_0}{L} + 1)] +$$

$$+ \cos(k_b) sh[k_b(\frac{x - x_0}{L} + 1)] +$$

$$+ \sin[k_b(-\frac{x_0}{L} + 1)] ch[k_b(-\frac{x}{L} + 1)] -$$

$$- \cos[k_b(-\frac{x_0}{L} + 1)] sh[k_b(-\frac{x}{L} + 1)] +$$

$$+ sh(k_b)\cos[k_b(-\frac{x + x_0}{L} + 1)] + \cos(k_b \frac{x}{L}) sh(k_b \frac{x_0}{L}) -$$

$$- \sin(\frac{k_b x}{L}) ch(\frac{k_b x_0}{L}) - ch(\frac{k_b x}{L})\sin(\frac{k_b x_0}{L}) +$$

$$- sh(k_b \frac{x}{L})\cos(k_b \frac{x_0}{L}) + \sin[k_b(\frac{-x + x_0}{L})] -$$

$$- sh[k_b(\frac{-x + x_0}{L})] - ch(k_b)\sin[k_b(\frac{x - x_0}{L} + 1)]\}; \quad (7)$$

$$G_+(x, x_0) = -\frac{L^3}{4k_b^3[1 + \cos(k_b) ch(k_b)]} \times$$

$$\times \{\cos(k_b) sh[k_b(\frac{-x + x_0}{L} + 1)] +$$

$$+ ch[k_b(-\frac{x_0}{L} + 1)]\sin[k_b(-\frac{x}{L} + 1)] -$$

$$- sh[k_b(-\frac{x_0}{L} + 1)]\cos[k_b(-\frac{x}{L} + 1)] -$$

$$- \sin(k_b) ch[k_b(-\frac{x + x_0}{L} + 1)] +$$

$$+ \sin[k_b(-\frac{x_0}{L} + 1)] ch[k_b(-\frac{x}{L} + 1)] -$$

$$- \cos[k_b(-\frac{x_0}{L} + 1)] sh[k_b(-\frac{x}{L} + 1)] +$$

$$+ sh(k_b)\cos[k_b(-\frac{x + x_0}{L} + 1)] +$$

$$+ \cos(k_b \frac{x}{L}) sh(k_b \frac{x_0}{L}) - \sin(k_b \frac{x}{L}) ch(k_b \frac{x_0}{L}) -$$

$$- ch(k_b)\sin[k_b(\frac{-x + x_0}{L} + 1)] -$$

$$- ch(k_b \frac{x}{L})\sin(k_b \frac{x_0}{L}) + sh(k_b \frac{x}{L})\cos(k_b \frac{x_0}{L}) -$$

$$- \sin[k_b(\frac{-x + x_0}{L})] + sh[k_b(\frac{-x + x_0}{L})]\}.$$

Influence of actuator location

The solution of second equation in (5) can be written as

$$u(x) = \frac{1}{EI}\sum_{j=1}^{M} F_j \exp(i\varphi_j)\cdot G_j(x, x_j), \qquad (8)$$

where a Green function $G_j(x, x_j)$ for cantilever beam is given by above solutions (7).

Function (7) satisfies to both equations (5) and boundary condition for cantilever beam.

Parametric investigation of sound radiation of the cantilever beam is done using the formula (8).

The load force of value 1 N is located at 0,75 of relative beam length and actuator of value 0,6 N has a variable location. They have an opposite phase (fig. 1).

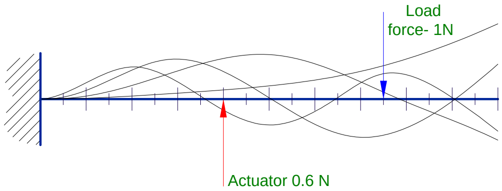

Fig. 1. Scheme of the load and actuator location (actuator location is variable)

As it can be seen from the fig. 2 the sound power level (SPL or L_w in corresponding figures) for the first mode decreases if actuator location becomes near to the load location.

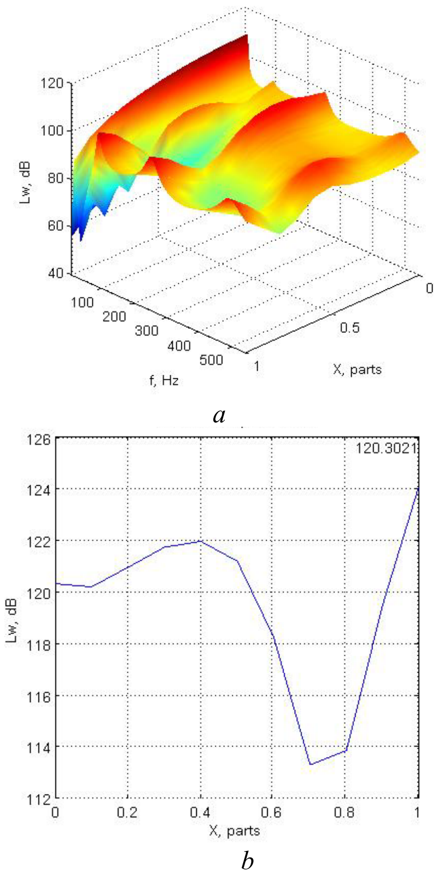

Fig. 2. Sound spectrum (*a*) and total SPL (*b*) for varied actuator location on the beam

The results of the researches show that locations 0,2 and 0,7 of relative beam length correspond to the second mode minima. The third mode has the minimum in 0,7 of relative beam length, locations 0,2 and 0,8 produce the fourth mode minima. After summation along the frequency range the total SPL minimum corresponds to 0,7.

Influence of actuator force value

Next investigation case considered for analysis of actuator value.

The locations of actuator and load are fixed and equal 0,75, 0,9 correspondingly (fig. 3).

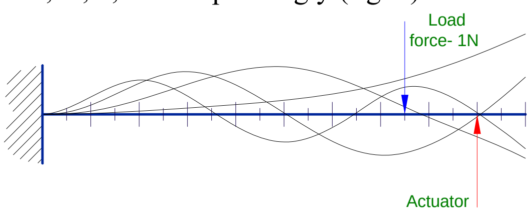

Fig. 3. Scheme of the load and actuator location (actuator value is variable)

According to our model the minimum SPL can be achieved at 0,8 N (fig. 4).

Second and third modes are increased with the actuator value increase, while the SPL of fourth mode is decreased.

Nevertheless after summation total SPL has a minimum corresponding to 0,5 N.

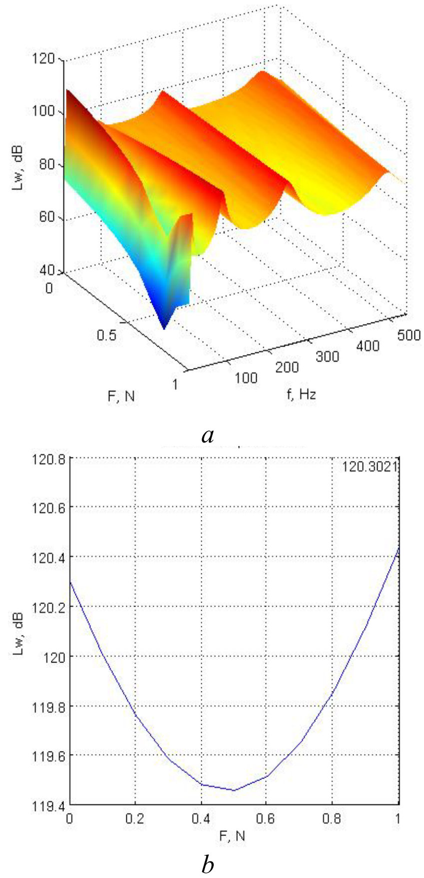

Fig. 4. Sound spectrum (*a*) and total SPL (*b*) for varied actuator value of the cantilever beam

Influence of actuator phase

The next factor we are going to research is a phase of the actuator and load.

The scheme of the forces applied to the beam is depicted in fig. 5.

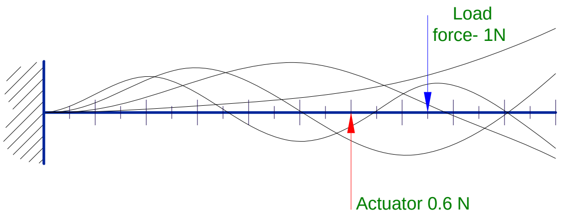

Fig. 5. Scheme of the load and actuator location (actuator phase is variable)

Actuator has both location and value fixed.

The result (fig. 6) shows that phase difference π provides minimal noise for the first three natural modes of cantilever beam oscillations, while the fourth mode have maximum corresponding to π.

On the total SPL graph the evident result, equal to π, is observed. In the majority of cases the same result should be get because the first mode gives the main contribution to the total SPL. The best way to compensate first mode is to apply load and actuator with the phase difference π. Also the situations are possible when the contribution of the first mode is not dominant (e.g. when the actuator is placed at the free beam end). In this case another result for optimum phase difference is possible.

Conclusion

An analytical model has been presented for vibrating cantilever beam. The method employs usage of Green function approach. Closed-form expressions have been developed for the cantilever beam structure subjected to the different loading conditions. Numerical results have been presented for the conditions of the different types of loading with the varied actuator parameters. It is hoped that the simplicity and ease of this technique will initiate a renewed interest in active optimal vibrating cantilever beam control.

This study represents a major step towards employing active methods for realistic flexible structure of cantilever beam.

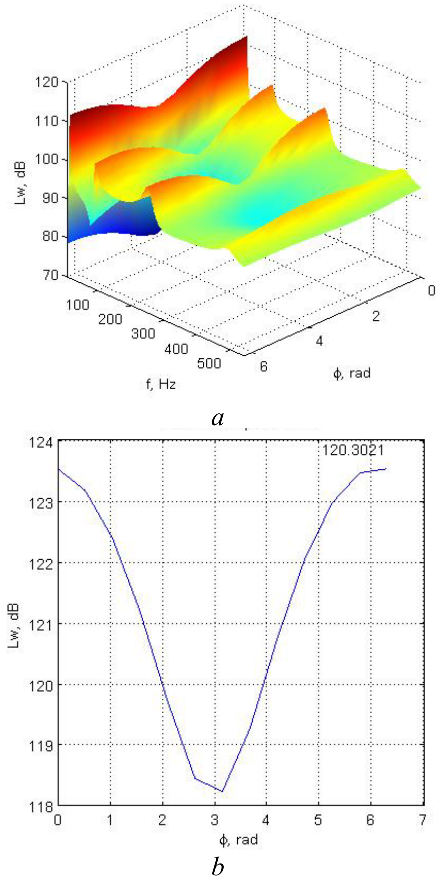

Fig. 6. Sound spectrum (*a*) and total SPL (*b*) for varied actuator phase of the cantilever beam

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Наведено результати дослідження вібрації консольної балки, що навантажена актуатором та навантаженням. Рівняння руху, яке базується на теорії балки Бернуллі–Ейлера–Тимошенко з відповідними крайовими умовами, вирішене методом функцій Гріна. Особливістю дослідження є врахування впливу параметрів концентрованої сили, а саме величини, розміщення актуатора та різниці фаз між навантаженням та актуатором. Розглянуто внесок параметрів актуатора на сумарний рівень звукової потужності і на кожну моду окремо.

Приведены результаты исследования вибрации консольной балки, нагруженной актуатором и нагрузкой. Уравнение движения, основанное на теории балки Бернулли–Ейлера–Тимошенко с соответствующими граничными условиями, решено методом функций Грина. Особенностью исследования является учёт влияния параметров концентрированной силы, а именно величины, размещения актуатора и разницы фаз между нагрузкой и актуатором. Рассмотрен вклад параметров актуатора как на суммарный уровень звуковой мощности, так и отдельно на каждую моду.