АЕРОПОРТИ ТА ЇХ ІНФРАСТРУКТУРА

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PARAMETRIC INVESTIGATION OF ACOUSTIC RADIATION BY A BEAM UNDER LOAD AND ACTUATOR FORCES

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The objective of this paper is to investigate the parametric characteristics of oscillating beam using an analytical theory of beam vibration. Analytical investigation of the bending oscillations of a finite elastic beam is considered for criteria based on minimal acoustic radiation. The solution of the task is defined by solving of Helmholtz equation and inhomogeneous differential equation for beam bending vibration with harmonic time dependence. For calculation of the acoustic field a model of a plane piston, which is set in an infinite rigid baffle, is used.

Introduction

The sound radiation from a vibrating beam is of practical importance and has been investigated extensively over many years. First of all, these works can be applied to optimum control of the oscillations of elastic beams [1; 2]. For a beam, which is set in an infinite baffle, the radiated sound field can be calculated by integral approach [2]. There are two methods are used usually to determine the acoustics radiation from vibrating beams.

First is based on integration of the far field acoustic intensity over hemisphere enclosing the beam.

Second method allows integrating the acoustic intensity over the surface of the vibrating beam. In this study for parametric investigation of acoustical characte-ristics of vibrating beams the second method is used.

Description of the method of analysis

Mathematical formalization of noise emission by a beam includes a model of the objects and the noise evaluation criterion.

The mathematical model for bending vibrations of elastic beam is represented by differential equations like following:

$$\rho_s(x)S(x)\frac{\partial^2 u(x,t)}{\partial t^2}+ \frac{\partial^2}{\partial x^2}\left[E(x)I(x)\frac{\partial^2 u(x,t)}{\partial x^2}\right]=F(x,t), \quad (1)$$

where $\rho_s(x)$ is a beam material density; $E(x)$ is an Young's modulus; $I(x)$ is a moment of inertia for the cross section of the beam $S(x)$; $F(x,t)$ is a load per unit length of the beam.

For harmonic vibration the displacement of beam oscillation $u(x,t)$ is represented as follows:

$$u(x,t)=u(x)\exp(-i\omega t).$$

Also it is supposed [2] that function $F(x,t)$ can be represented in form:

$$F(x,t)=F(x)\exp(i\omega t).$$

For the constant beam density, moment of inertia, cross section, Young's modulus equation (1) can be written in form of biharmonic equation (harmonic vibrations of the beam):

$$\frac{d^4u(x)}{dx^4}-k_f^4u(x)=\frac{F(x)}{EI}, \quad (2)$$

where

$$k_f^4=\frac{\omega^2\rho_s S}{EI},$$

and ω is an angular frequency, a load per unit of beam length is defined like

$$F(x)=\overline{F}_F\exp(i\varphi_F)\delta(x-x_F)+ +\sum_{j=1}^{M}F_j\exp(i\varphi_j)\delta(x-x_j),$$

where x_F, x_j are points of load and actuator forces' location respectively; $\delta(x)$ is a Dirac function; F_j, φ_j are a amplitude and phase of jth actuator force respectively.

The differential equation (2) is solved in a closed form for boundary conditions corresponding to the simply supported beam.

For harmonic waves a Helmholtz equation is given by

$$\Delta p + k^2 p = 0 \qquad (3)$$

and boundary condition are used in form

$$\frac{\partial p}{\partial z} = \rho\omega^2 u(x) \quad \text{for } z=0.$$

General solution of the equations (2), (3) and prescribed boundary conditions can be obtained in form of the following expansion (for simply supported beam) [2]:

$$u(x) = -\frac{2}{\rho_s SL}\sum_{n=1}^{\infty}\frac{\sin\left(\frac{n\pi x}{L}\right)}{(\omega^2-\omega_n^2)}\times$$

$$\left[\overline{F}_F \exp(i\varphi_F)\sin\left(\frac{n\pi x_F}{L}\right) + \sum_{j=1}^{M} F_j \exp(i\varphi_j)\sin\left(\frac{n\pi x_j}{L}\right)\right], \qquad (4)$$

where modal frequencies of the beam oscillations are defined as:

$$\omega_n = \sqrt{\frac{IE}{\rho_s S}\left(\frac{n\pi}{L}\right)^2}.$$

For solving the Helmholtz equation and for the calculation of the acoustic field around the beam a model of plane piston, which is set in an infinite rigid baffle, may be proposed. For the vibrating plane piston following conditions must be fulfilled on the beam

$$\frac{\partial p}{\partial z} = \rho\omega^2 u(x)$$

and on the baffle

$$(z=0)\ \frac{\partial p}{\partial z} = 0.$$

In this case exact solution can be presented for acoustic pressure in a form [2]

$$p(x,y,0) = -\frac{\rho\omega^2}{2\pi}\int_0^L\int_0^b \frac{\exp(ikr)}{r}u(x_0)dx_0dy_0,$$

$$r = \sqrt{(x-x_0)^2+(y-y_0)^2},$$

where b is the beam width.

Acoustical power of the beam in this case is specified by

$$W = \frac{1}{2}\mathrm{Re}\int_0^L\int_0^b p(x,y,0)w^*(x)dxdy,$$

where $w(x) = -i\omega u(x)$.

Program structure and input/output parameters

The above mentioned solution was realized in computer program. Using this program the parametrical investigation of forces location, their value and phase influence on sound radiation characteristics was performed. The flowchart of the program operation has the following view (fig. 1).

Thus sound radiation becomes a function of this parameters and frequency. In order to find the best set of actuator parameters their influence is summarized in the frequency range under investigation and total sound power level is defined, then its minimum must be found.

Next steps are used for calculations:

– frequency step is equal to 5 Hz;

– force location step is equal to 0,1 of relative beam length (the beam is considered of a unit length);

– force value step is equal to 0,1 N;

– phase step is equal to π/6;

– pressure array steps (along both coordinates x and y) are varied in accordance with beam dimensions.

One can see that the solution (4) enters in form of the expression for the transverse motion of the beam. The influence of actuator location is defined by $\sin\left(\frac{n\pi x_j}{L}\right)$, which represents a sinusoidal passing through zero in mode nodes.

Therefore the actuator doesn't influence the beam acoustic radiation on the given mode if it is located in the node. And influence is increasing if the shortest distance to the mode node increases.

The force influence on transverse motion of the beam is proportional to its value.

The influence of actuator location

The load force of the value 1 *N* and actuator of 0,6 *N* are applied to the beam (fig. 2).

The curves, depicted in the figure, represent the shapes of the beam oscillations corresponding to different modes. In calculations just four first natural frequencies are used, because, as it can be seen from the figures below, other modes of oscillation can be considered insignificant.

The result of numerical investigation is represented in fig. 3, *a*.

This surface looks like to have the vertical planes. They represent the modes of beam oscillation. Locating the force at 0,5 of relative beam length (in centre of the beam) the minimum for the first mode of beam oscillations may be achieved, at 0,7 of relative beam length the minimum of second mode occurs, and the third one has the minimum at points 0,2 and 0,8 of relative beam length. Summarizing the sound spectrum the most efficient total noise reduction achieved if actuator is located at 0,7 (fig. 3, *b*).

Vertical axis of the graph corresponds to the total sound power level (SPL) of the beam oscillated just by single load (initial total SPL = 125,78 dB). So noise reduction of the received results may be defined by comparison with initial beam loading.

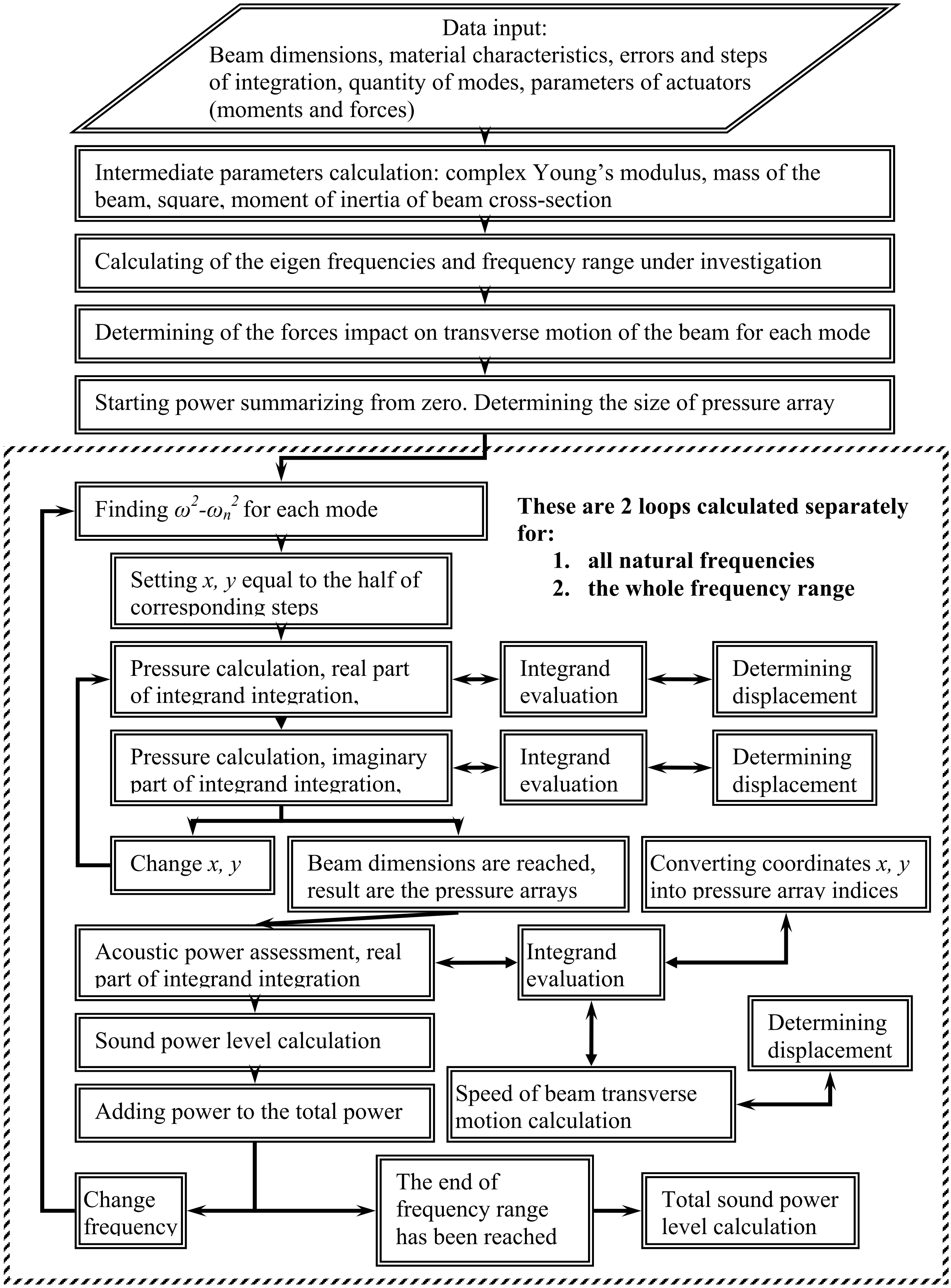


Fig. 1. Flowchart of the program

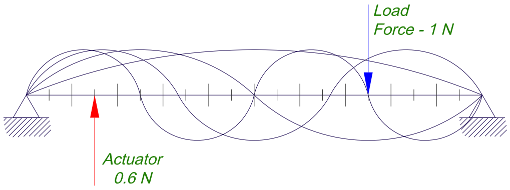
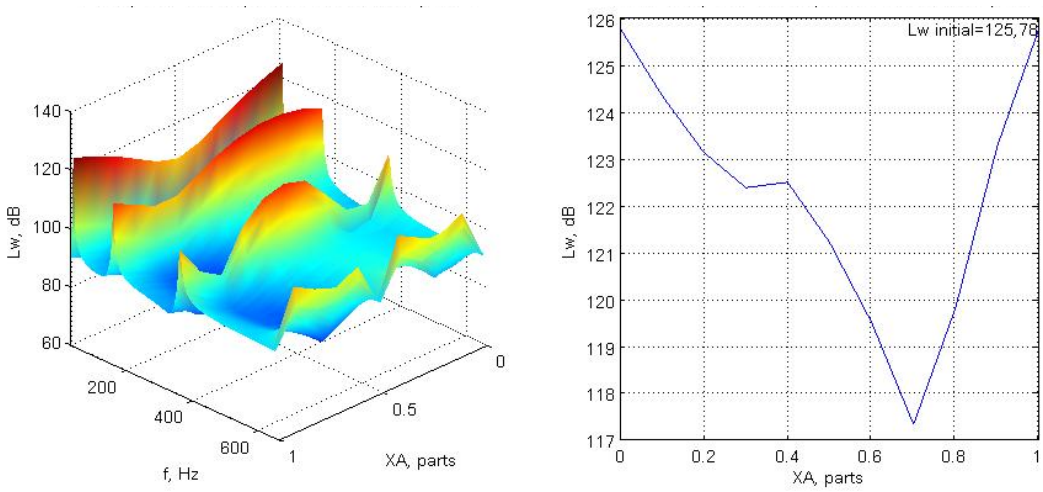


Fig. 2. Vibration displacement of the beam at the first four resonance frequencies



a *b*

Fig. 3. Sound spectrum (*a*) and total SPL (*b*) for varied actuator location on the beam

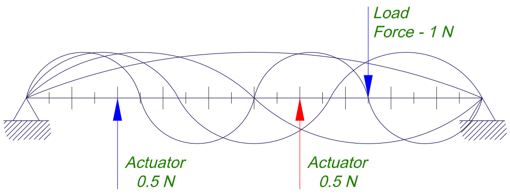
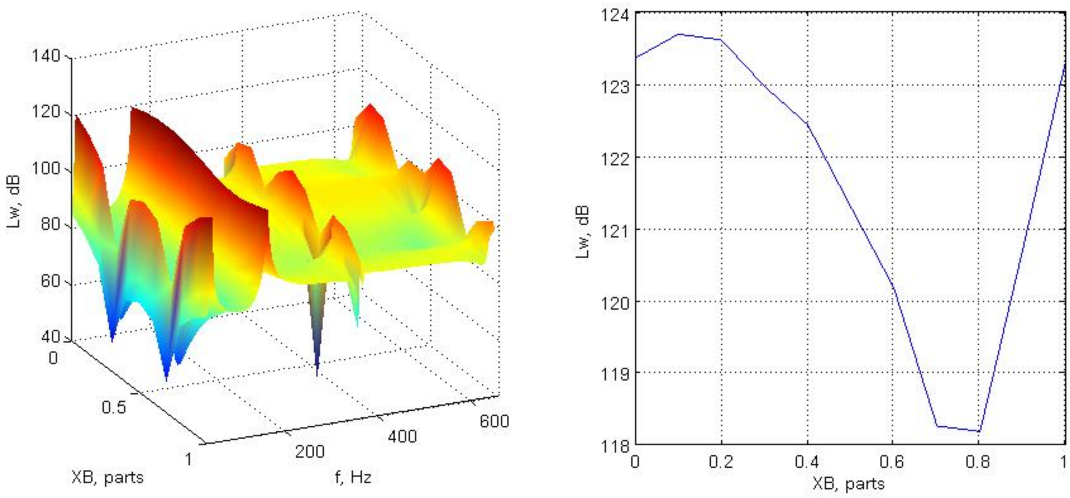


Fig. 4. Scheme of a load and actuator location (for two actuators)



a *b*

Fig. 5. Sound spectrum (*a*) and total SPL (*b*) for varied second actuator location on the beam

For the next investigated case two actuators are applied: one with the fixed position – 0,2 of relative beam length and value 0,5 *N* and second – with fixed value 0,5 N and variable location (fig. 4).

The first mode minimum is achieved by applying the second actuator at points 0,3 and 0,7 of relative beam length (fig. 5).

For the second mode SPL increasing is observed when actuator is located inside the first half of the beam and decreasing – inside the second half. The third and fourth modes can be neglected because of their small sound power contribution. As the result the minimum total SPL is achieved inside beam length range 0,7 – 0,8.

Influence of actuator force value

The actuator is fixed in point 0,5 of relative beam length and the task is to find its value, which produce the lowest total SPL of the beam (fig. 6). The actuator value at this location doesn't influence the SPL of second mode. As it can be seen in fig. 7 an efficient noise reduction for the first mode takes place for value 0,7 *N*. But due to slight SPL increasing of third mode the total sound power level minimum is achieved around 0,6 *N*. For the case shown in fig. 8 two actuators and one load are applied again. The load has the fixed parameters 0,75 of relative beam length and value 1 *N*. One of the actuators also has constant parameters: located at 0,2 of relative beam length and with value 0,5 *N*. Second actuator also has the fixed location, but is varied in value. The received spectrum is shown in fig. 9. The minimum of the first mode is achieved for value equal to 0,4 *N*. The second mode slightly increases with the actuator value increasing. The third and fourth modes can be neglected because of their small SPL.

The influence of actuator phase

For current case (shown in fig. 10) both – an actuator value and its location – are fixed, they correspond to the minimum taken from previous situation (graphs), and current task is to define: with what phase the actuator should be used to compensate the load.

The results in the fig. 11 show that this phase is equal to π. The major influence on total sound power level is given by first mode of beam oscillation. But the first mode cannot be compensated if load and actuator have the same phase. Thus for simplest case – one load and one actuator – the result is evident and equal to π . In all further investigations phase difference between load and actuator is taken to be equal to π .

The influence of actuator distribution

Now let us consider the different cases of actuators distribution along the beam. For uniform actuator value distribution, depicted in fig. 12, the location of the actuator group is varied.

The *x* coordinate on the both graphs (fig. 13) corresponds to the location of the central force from the actuator group.

The minimum of the first mode corresponds to locations 0,3 and 0,7 of relative beam length. The second mode increases when the actuator group is located inside the left half of the beam, and if it is located inside the right half of the beam the SPL little bit decrease. The third mode has minima at locations 0,2 and 0,8 of beam length. 4th mode does not give the significant contribution to the total SPL, where the minimum corresponds to location 0,7.

Comparing with first case (fig. 2 and 3) the distributed actuator has the same effectiveness as local one, their noise reduction is around 8 dB. In the case shown in fig. 14 a linear actuator value distribution is investigated. Again the location of actuator group is varied with aim to find its position corresponding to SPL minimum. As it can be seen from the fig. 15 minimum of SPL for all modes of beam oscillation corresponds to location 0,7 of relative beam length. In the case depicted on fig. 16 the reverse linear actuator value distribution is considered.

The actuator group location is variable. The results of calculation shows that 0,8 of relative beam length corresponds to the reduction of sound power level for all natural mode (fig. 17). Also minima corresponding to 0,3 take place for first and third modes. Summarising the received spectrum we get minimum total sound power level corresponding to 0,8. Actuator group location was varied also for inverse triangular value distribution is shown in fig. 18. From the fig. 19 it can be seen that minimum of the first mode corresponds to locations 0,3 and 0,7 of relative beam length.

The second mode increases if locating the actuator group inside the left half of the beam, and if locating it inside right half of the beam the SPL little bit decreases. The actuator value doesn't influence the SPL of the third mode almost.

The total SPL reduction of acoustic radiation is the most efficient for location 0,7 and it is up to 6 dB.

In fig. 20 the triangular type of actuator distribution is shown. Fig. 21 is quite similar to the previous actuator value distribution (fig. 19). But minimum for the first mode is not so deep as it was in the previous case and more efficient noise reduction for the second and third modes leads to deeper reduction of the total SPL – up to 9 dB, becomes near to similar to the most efficient case in fig. 15.

In the case of parabolic actuator distribution, shown in fig. 22, the sound power level (fig. 23) appears to be very similar to case shown in fig. 18, 19. But in this case the reduction of total SPL reaches on 0,2 dB more. Another case of parabolic actuator distribution (inverse) is considered in fig. 24. As it can be seen from fig. 25 the SPL has not so deep minimum like it was in the previous case and it is very similar to SPL of triangular actuator distribution depicted in fig. 20, 21.

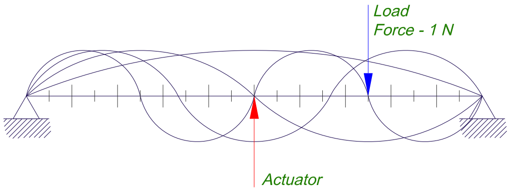
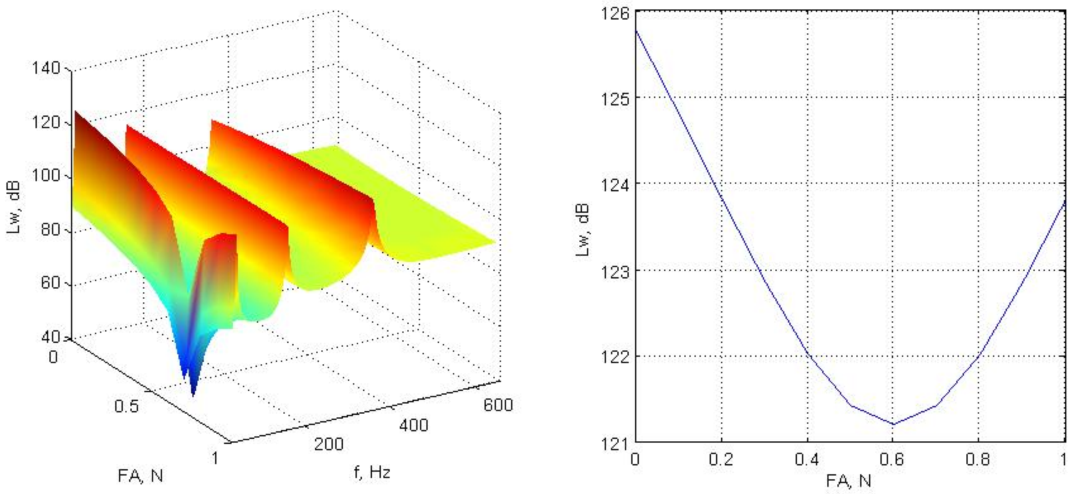


Fig. 6. Scheme of load and actuator location (one load and one fixed actuator)



a *b*

Fig. 7. Sound spectrum (*a*) and total SPL (*b*) for varied value of the actuator, fixed at location 0,5 of the beam length

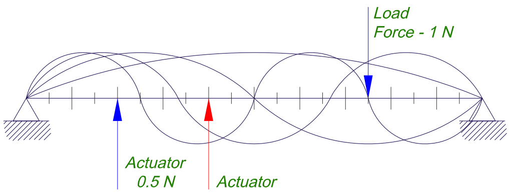
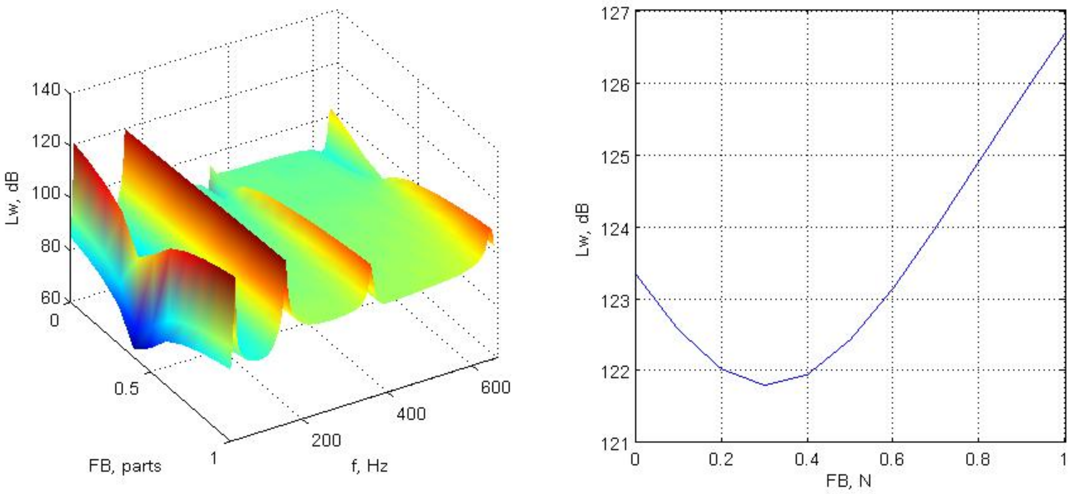


Fig. 8. Scheme of a load and actuator location (for two actuators)



a *b*

Fig. 9. Sound spectrum (*a*) and total SPL (*b*) for varied value of the two actuators, fixed at specified location

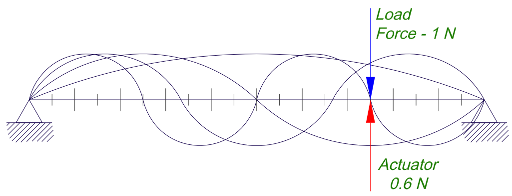
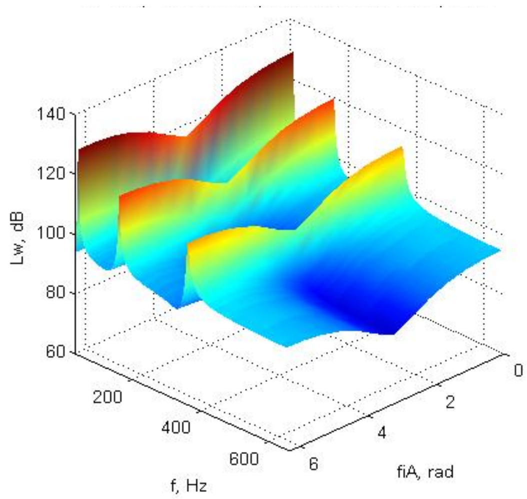
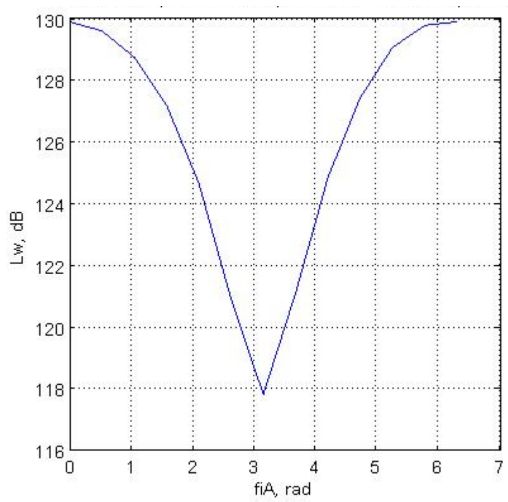


Fig. 10. Scheme of simplest load and actuator location



a



b

Fig. 11. Sound spectrum (*a*) and total SPL (*b*) for varied force phase of the actuator, fixed at the same location as a load

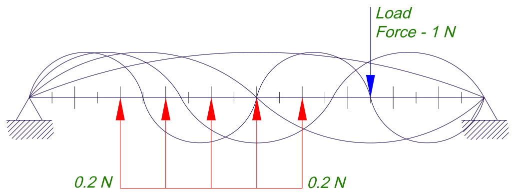
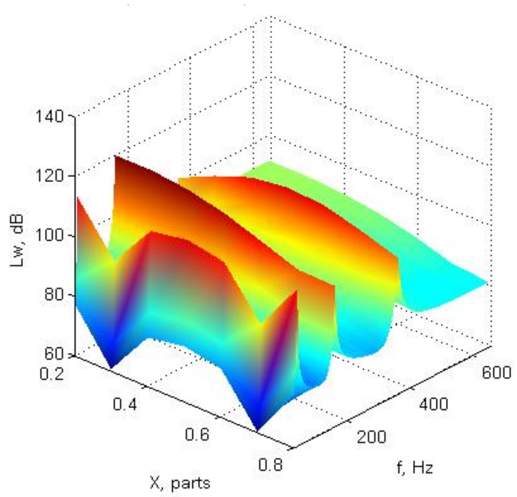
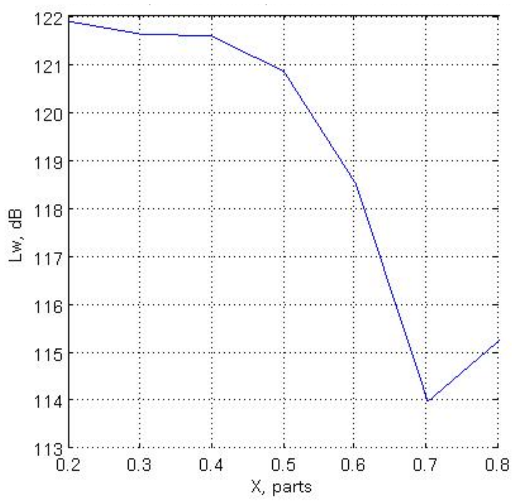


Fig. 12. Scheme of load and actuator for uniform actuator value distribution



a



b

Fig. 13. Sound spectrum (*a*) and total SPL (*b*) for varied location of the uniform actuator distribution

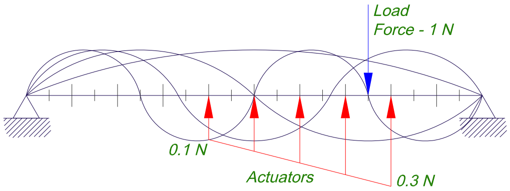


Fig. 14. Scheme of load and linear actuator value distribution

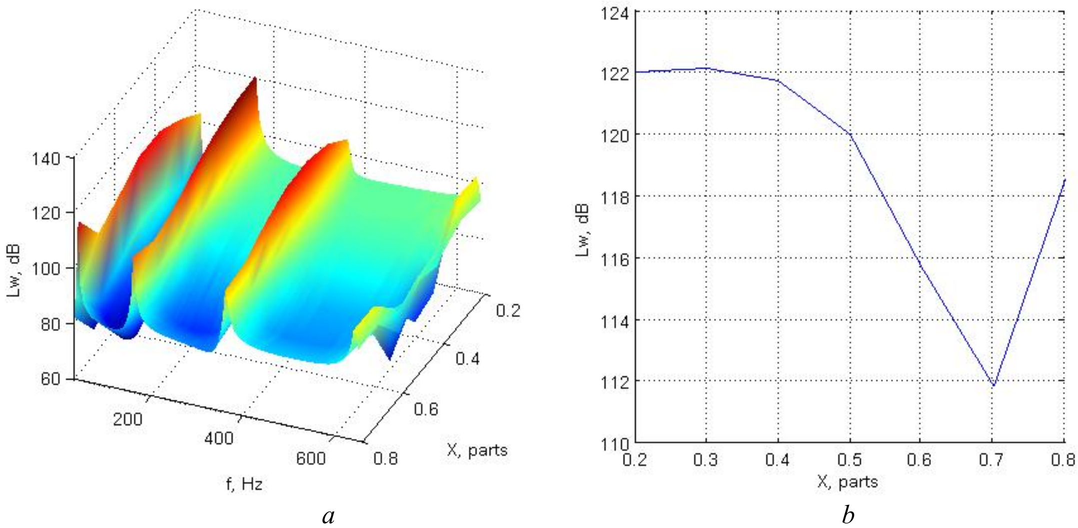


Fig. 15. Sound spectrum (*a*) and total SPL (*b*) for varied location of the linear actuator distribution

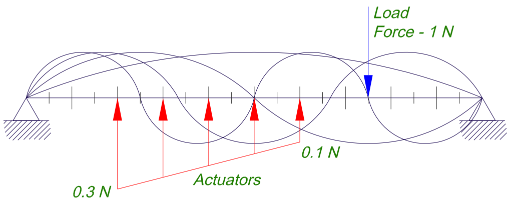


Fig. 16. Scheme of load and reverse linear actuator value distribution

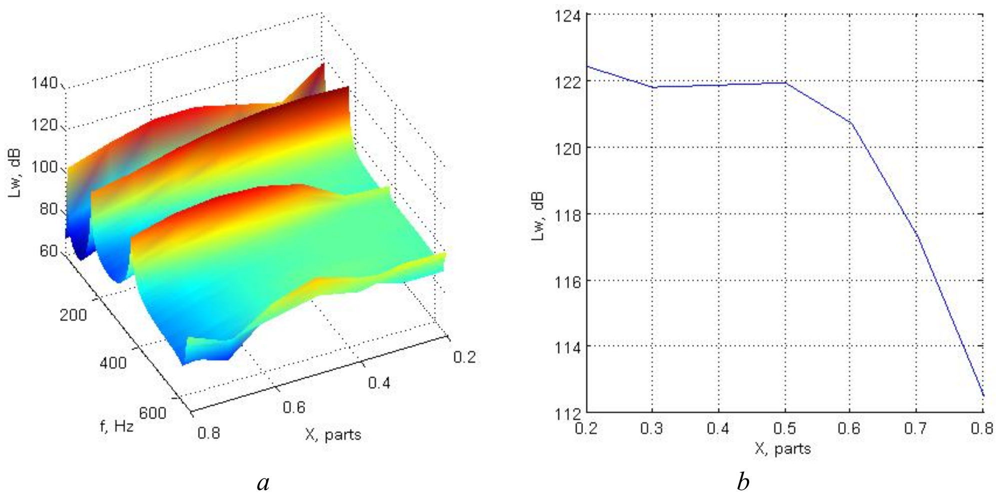


Fig. 17. Sound spectrum (*a*) and total SPL (*b*) for varied location of the reverse linear actuator distribution

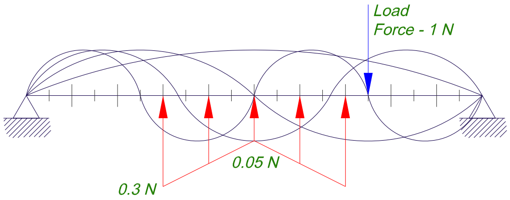
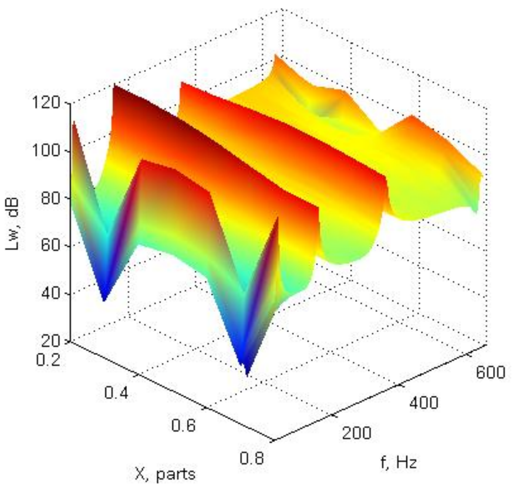
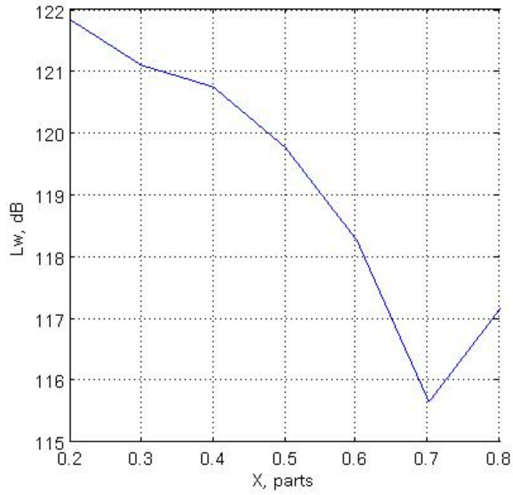


Fig. 18. Scheme of load and inverse triangular actuator group location





a *b*

Fig. 19. Sound spectrum (*a*) and total SPL (*b*) for varied location of the inverse triangular actuator distribution

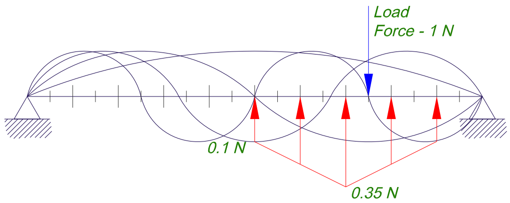
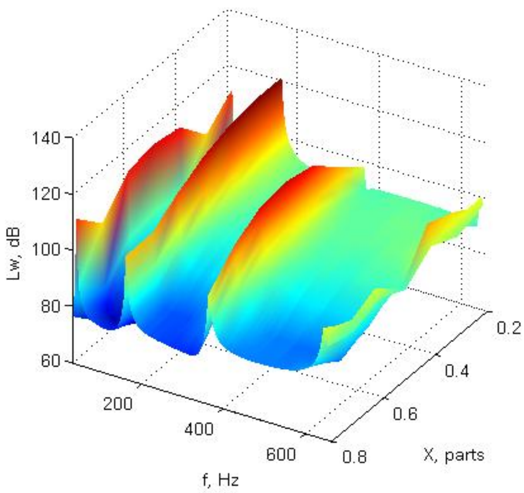
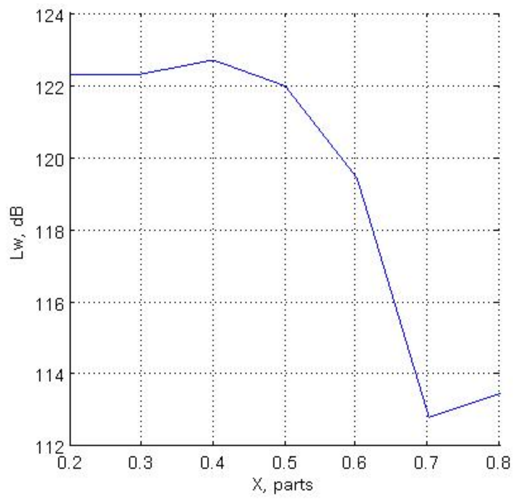


Fig. 20. Scheme of load and triangular actuator distribution





a *b*

Fig. 21. Sound spectrum (*a*) and total SPL (*b*) for varied location of the triangular actuator distribution

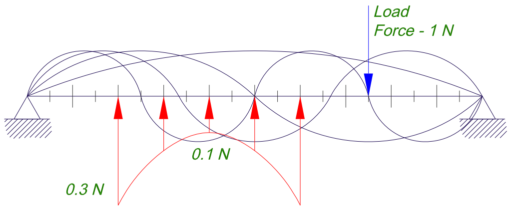
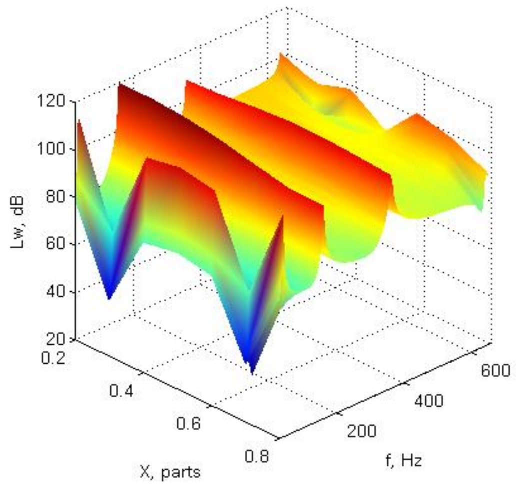
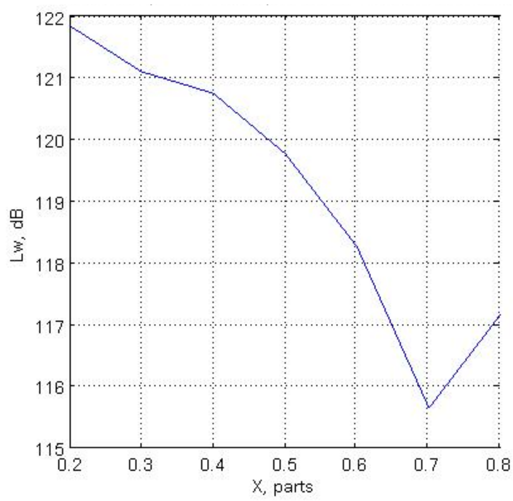


Fig. 22. Scheme of load and parabolic actuator value distribution



a



b

Fig. 23. Sound spectrum (*a*) and total SPL (*b*) for varied location of the parabolic actuator distribution

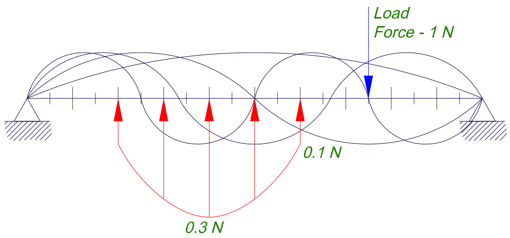
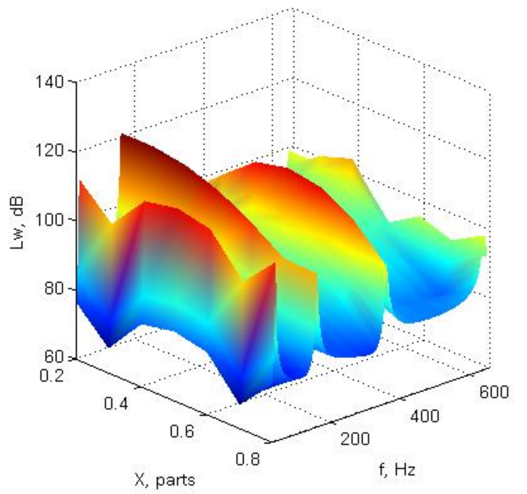
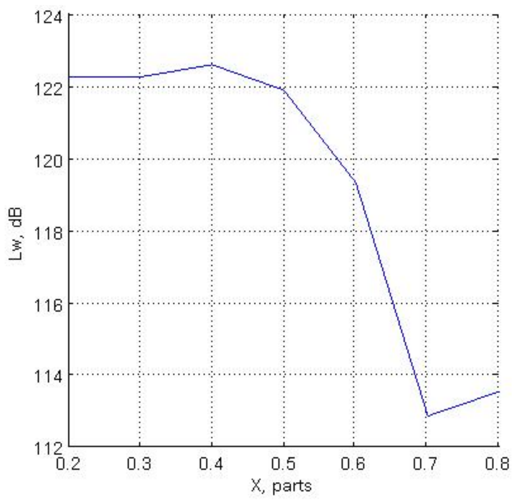


Fig. 24. Scheme of load and inverse parabolic actuator value distribution



a



b

Fig. 25. Sound spectrum (*a*) and total SPL (*b*) for varied location of the inverse parabolic actuator distribution

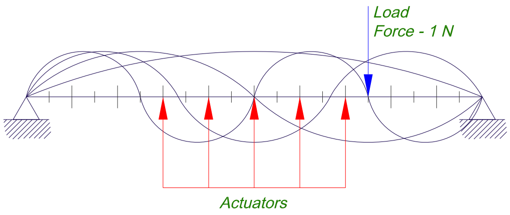
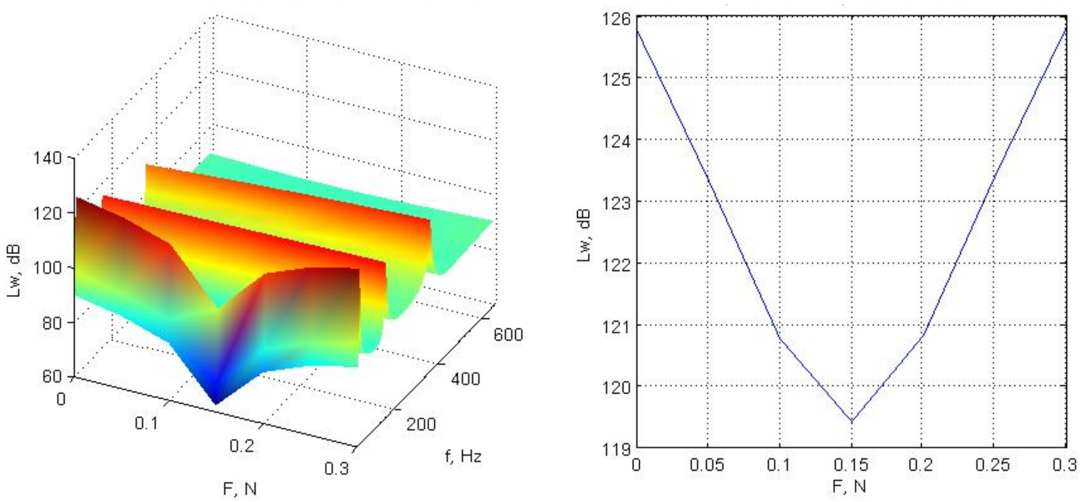


Fig. 26. Scheme for fixed locations and varied value of the actuator



a *b*

Fig. 27. Sound spectrum (*a*) and total SPL (*b*) for fixed location and varied value of the actuator distribution

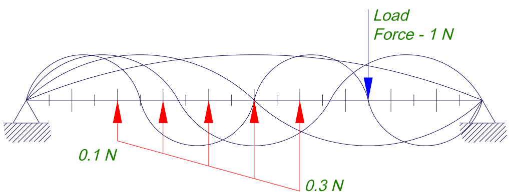
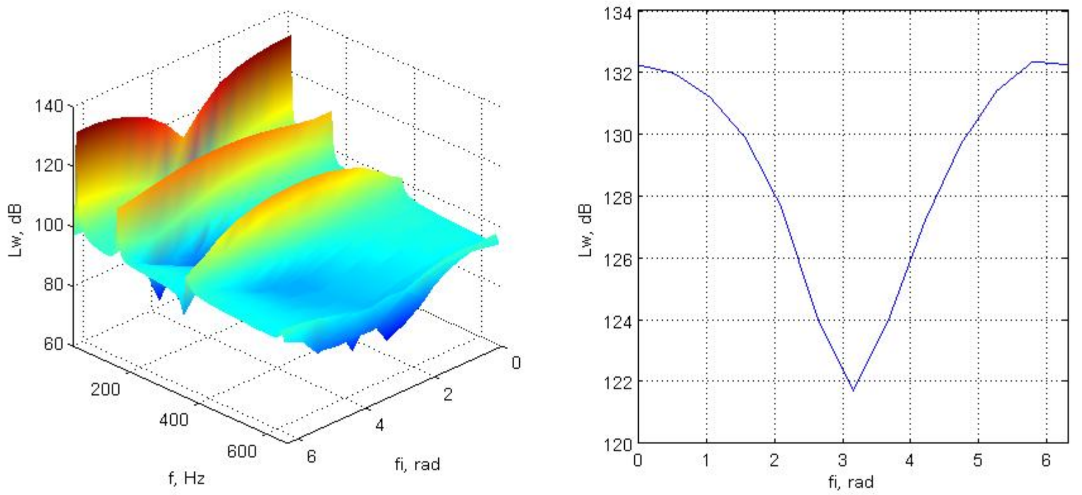


Fig. 28. Scheme for fixed actuator group location and value, but its phase is variable



a *b*

Fig. 29. Sound spectrum (*a*) and total SPL (*b*) for varied force phase of the actuator with fixed location and force value

The reduction of total SPL reaches 9 dB. And this minimum corresponds again to location 0,7 of relative beam length.

The results of actuator value distribution investigation shows that the best noise reduction corresponds to linear and reverse linear actuator value distribution. But for these cases some shifting (relatively to load location) of actuator group location is preferable.

The fig. 26 represents the case when locations of load and actuator are fixed, actuator force distribution is uniform (so each of this forces in actuator group has the same value) and actuator value is variable – from 0 *N* to 0,3 *N*.

Result shows that the first mode SPL minimum corresponds to 0,15 *N* (fig. 27). The second mode SPL is indifferent to the actuator value. And the third mode SPL slightly increases with the actuator force value increase. But nevertheless the total SPL is defined by first mode and corresponds to 0,15 *N*. For the case shown in fig. 28 actuator group location is fixed, it's value is constant, but the phase is variable. All unit actuators have the same phase.

Fig. 29 shows that the minimum of the first mode corresponds to phase difference equal to π (the same result as in fig. 11). But for other modes the maximum occurs for this case, because of location difference. Nevertheless, because of small contribution of their SPL, minimum of total sound power level corresponds to π again.

Conclusion

As beam acoustic radiation control is defined by actuator parameters, such as the delayed phase of actuator forces, their location and distribution law along a beam, all of them are analyzed.

For most of cases the results show that first four modes of simply supported beam oscillation are enough to be considered for the noise radiation investigation. Te influence of actuator parameters is different for each mode of beam oscillations.

For the simplest case of loading the minimum of sound power level can be achieved with delayed phase on a π between load and actuator.

The results show also the correlation between load and actuator values, specially the influence of actuator distribution on optimal sound power level of the beam was investigated.

The modeling approach can be used for forming and grounding of feedback control of the system with vibrating loads by means of optimal distributions of the actuators along the beam length.

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Параметричне дослідження акустичного випромінювання балкою під навантаженням та актуаторами типу сили

Досліджено параметричні характеристики коливань балки з використанням аналітичної теорії вібрації балки. Аналітичне дослідження згинальних коливань еластичної балки кінцевих розмірів розглянуто для критерію, що визначає мінімальне акустичне випромінювання. Розв'язок задачі визначено для рівняння Гельмгольца і неоднорідного диференційного рівняння гармонічних згинальних коливань балки. Для обчислення акустичного поля застосовано модель поршня, установленого на нескінченний екран.

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Параметрическое исследование акустического излучения балкой под нагрузкой и актуаторами типа силы

Исследованы параметрические характеристики колебаний балки с использованием аналитической теории вибрации балки. Аналитическое исследование изгибных колебаний эластичной балки конечных размеров рассмотрено для критерия, определяющего минимальное акустическое излучение. Решение задачи определено для уравнения Гельмгольца и неоднородного дифференциального уравнения гармонических изгибных колебаний балки. Для расчета акустического поля использована модель поршня, установленного на бесконечный экран.