UDC 629.3.025.2(045)
DOI: 10.18372/2306-1472.73.12170

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SIMULATION OF PRECISION ATTITUDE AND HEADING REFERENCE SYSTEM PERTURBED BY ENVIRONMENTAL DISTURBANCES

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Abstract

Purpose: *The paper focuses on problems of simulation of perturbed robust precision attitude and heading reference systems, which can be applied in navigation of marine vehicles. The main goal is to create the mathematical model adapted to simulation of the perturbed system and models of the environmental disturbances.* ***Methods:*** *To solve the given problem the methods of the robust control system theory, filtration theory and probability theory are used.* ***Results:*** *The model of the perturbed attitude and heading reference system created by means of Simulink is given. The expression for the disturbance moment is proposed. Analysis of possible environmental disturbances for a system of the considered type has been done. Models of environmental disturbances based on the filtration theory are obtained. Comparison of two approaches to development of models of environmental disturbances is carried out.* ***Conclusions:*** *The results of simulation of the precision attitude and heading reference system taking into consideration environmental disturbances are represented. Obtained results can be useful for design of precision navigation systems of the moving vehicles.*

Keywords*:* attitude and heading reference system; environmental disturbances; perturbed system; robust systems; simulation.

1. Introduction

Now processes attending operation of the vehicles are sufficiently complicated. To provide the high accuracy of navigation information measurements it is necessary to use the high precision attitude and heading reference systems. Synthesis of such systems is implemented in conditions of uncertainties caused by both inaccuracies of the mathematical description of the real system and influence of the internal and external disturbances. Mainly, the systems designed for operation at the marine vehicles are subjected to influence of the disturbances caused by the sea irregular waves. The modern approach to these systems design is creation of the robust systems able to operate in conditions of both the parametrical structured and the external coordinate disturbances. Simulation of the system perturbed by environmental disturbances is the important stage of the robust gimballed precision attitude and heading reference systems.

2. Analysis of the latest researches and publications

This paper completes the series of papers dealt with development of the precision attitude and heading reference systems [1-3]. The papers [1, 2] describe basic features of creation of the mathematical models of such systems. The mathematical descriptions of the high-precision attitude and heading reference systems with biaxial and triaxial platforms are given in these papers. Features of robust design of such systems are analyzed in the paper [3]. Both robust parametrical optimization and robust structural synthesis of the precision attitude and heading reference system are given in this paper. Features of stochastic precision attitude and heading reference system are given in the paper [4]. The general achievements of gimballed stabilization systems design and gimballed precision navigation systems are represented in papers [5, 6]. Features of simulation of robust systems by means of MatLab are described in the textbook [7].

3. Research tasks

The main goal of the research is to represent features of environmental disturbances typical for application of precision attitude and heading reference system, create mathematical models of these disturbances and carry out simulation of attitude and heading reference system in complex conditions of real operation.

4. Mathematical model of stochastic precision attitude and heading reference system

The studied gimballed system for the marine vehicle navigation represents the indicated or indirect inertially stabilized platform by its principle of operation. In such systems the gyro devices do not implement direct stabilization of some object, for example, the platform but represent indicator units, which control by servo-drive providing an object's stabilization [8].

In such systems compensation of disturbances that act on the stabilized platform are implemented due to moments created the actuators mounted at the gimbals axes. Control by the stabilization contours is carried out by the gyro devices signals which are corrected by signals of accelerometers and additional devices using the complex control laws.

In the studied system the control loops can be divided into navigation and stabilization ones that allows to carry out their simulation independently [3]. In this case, division of control functions takes place: the stabilization engines provide agreement of the stabilization object position with the position of the gyro device and the torques sensors implement correction of the rotor motion by the computing device based on information of accelerometers and data about the Earth and object motion [8].

If consider influence of rolling and pitching small, it is possible to neglect by the mutual influence of the gimbals. In this case, stabilization of the navigation devices by one channel can be researched only. This simplifies essentially simulation of the system.

If the dynamically tuned gyro is used as the sensor of the system, it is necessary to take into consideration that it operates in the indicated mode. Such mode does not require the high stability of the transfer coefficient but its sensitivity is very important. It means that the large slope of the statistic characteristic curve is close to zero and the low threshold of sensitivity must be provided. These factors lead to small angles of the rotor turns and correspondingly to the small operation range [8]. Therefore to improve stabilization accuracy it is necessary to use units which sufficiently increase the stabilization contour gain. Moreover, taking into consideration the operation principle of the dynamically tuned gyro it is necessary to use the selective filter. The structural scheme of the selective filter is given in Fig. 1.

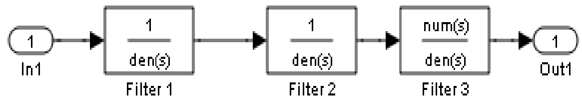


Fig. 1. Selective filter of the attitude and heading reference system (Simulink model)

For the studied system the basic disturbance moment acting at the input of the platform can be described by the expressions

$$M_{db1} = M_0 + M_{fr}\text{sign}(\sin\omega t) + M_s \sin\omega t\,, \quad (1)$$

$$M_{db2} = M_0 + M_{fr}\text{sign}(\sin\omega t) + M_{irr} \quad (2)$$

here M_0 is the constant disturbance moment caused by the construction defects; M_{fr} is the moment of dry friction; M_s is the amplitude of the moment due to sea regularities; ω is the frequency of the sea regularities; M_{irr} is the moment caused by the irregular sea.

Simulink model of disturbances (1), (2) are given in Fig. 2.

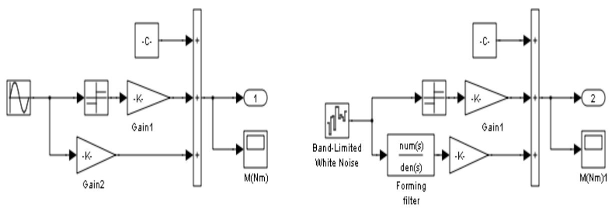


Fig. 2. Simulink model of determinate and random disturbance moments

Simulink model of perturbed precision attitude and heading reference system is shown in Fig. 3.

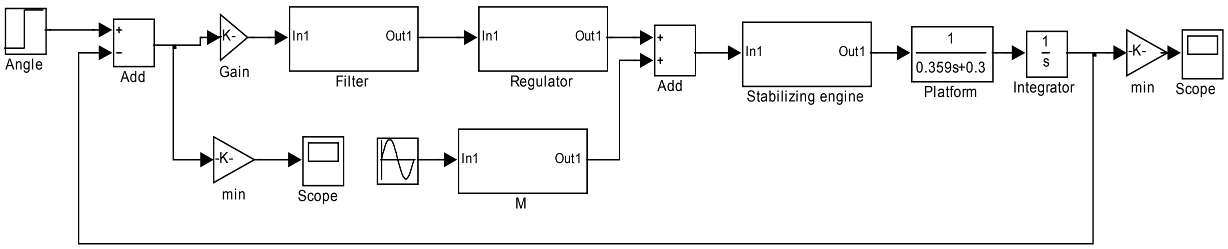


Fig. 3. Simulink model of the perturbed system

5. Characteristic of environmental disturbances

It should be noted that marine precision attitude and heading reference systems are subjected to influence of sea waves, wind and flows. The most important disturbances such as wind and sea waves are interconnected and correlated. Sea waves arise as result of the wind rippling by the speed and direction on the sea surface. The free waves, which are spread by inertia, are observed out zone of wind action or after its stopping.

Characteristics of both regular and irregular waves are given in [9]. Regular waves represent harmonic waves, which are spread on the sea surface. In the general case, the irregular sea is three-dimensional. The two-dimensional correlation function, which defines correlation between wave ordinates in different points of the sea at some time interval, can be accepted as the first approximation of the three-dimensional random process.

To solve some navigation problems of marine vehicles is possible using a simplified approach, for which the sea irregular waves are believed to be two-dimensional. The ordinate of the wave profile $\zeta(t)$ is considered to be a function of two variables such as the linear coordinate of the sea surface point and the time interval. The correlation function of wave profile ordinate at the given sea point $K_{\zeta}(t)$ or appropriate spectral density $S_{\zeta}(\omega)$ are characteristics of the function $\zeta(t)$ [10].

In accordance with theoretical and experimental researches represented in [11], irregularities of the sea surface can be considered as a time-invariant ergodic process with the zero mathematical expectation as the first approximation. This process is distributed in accordance with the normal distribution law. Waves are believed to spread in the same direction. It means that the spectral density depends on frequency only.

There are some known expressions of the spectral density, which can be used for mathematical description of irregular sea waves, for example, Neumann and Derbyshire filters. It should be noted that low frequencies are not taken into consideration in these filters. The bandwidth of the marine vehicles lies in the region of low frequencies at the same time.

Therefore it is convenient to use Rachmanin and Firsov filters [11]. In this case, the correlation function of wave ordinates can be represented by the formula

$$K(\tau) = D_r e^{-\mu t} \cos \lambda t\,, \qquad (3)$$

here D_r is the variance of wave ordinates; μ is coefficient of the correlation function, which characterizes a level of sea irregularity; λ is the resonance frequency of the irregular sea spectrum. This frequency for rolling and pitching is equal to frequencies of the object characteristic oscillations.

In accordance with (2) the spectral density can be determined in the following way [11]

$$S_r(\omega) = \frac{2D_r\mu}{\pi} \frac{\omega^2 + \mu^2 + \lambda^2}{(\mu^2 + \lambda^2 + \omega^2)^2 - 4\lambda^2\omega^2}\,. \qquad (3)$$

In some cases, it is convenient to represent the correlation function of the sea irregular waves in the following form [11]

$$K(\tau) = D_r e^{-\mu t}\left(\cos \lambda t + \frac{\mu}{\lambda}\sin \lambda\tau\right). \qquad (4)$$

Then the expression for spectral density determination becomes

$$S_r(\omega) = \frac{2D_r\mu}{\pi} \frac{\mu^2 + \lambda^2}{(\mu^2 + \lambda^2 + \omega^2)^2 - 4\lambda^2\omega^2}\,. \qquad (5)$$

This dependence corresponds to real irregular sea in the region of spectrum maximum amplitudes on contrary to regions of the low and high frequencies. The correlation function (4) for $\mu \to 0$ becomes cosine function and the spectral density – δ-function at the frequency λ. It should be noted that λ characterizes frequency of the periodic process similar to the random process $\zeta(t)$. The parameter μ characterizes level of the function $\zeta(t)$ irregularities. Its kind depends on ratio between μ and λ. If the ratio μ/λ is small, the function $\zeta(t)$ is similar to the sine function with the slowly changed amplitudes and phase. If the ratio μ/λ increases, the periodicity decreases.

For the correlation function [11]

$$K(\tau) = D_r \sqrt{1+\left(\frac{\mu}{\lambda}\right)^2} e^{-\mu t}\left(\cos\lambda t + \frac{\mu}{\lambda}\operatorname{arctg}\frac{\mu}{\lambda}\right), \quad (6)$$

the expression for determination of the spectral density becomes

$$S_r(\omega) = \frac{2D_r\mu\omega^2}{\omega^4 + 2(\mu^2-\lambda^2)\omega^2 + (\mu^2+\lambda^2)^2}. \quad (7)$$

To determine the spectral density of the wave ordinates described by (6), (7), it is necessary to determine the variance and parameters μ and λ based on data about intensiveness of the irregular sea. Usually the wave ordinates are believed to be distributed by the normal law.

In this case, amplitude values will be distributed by the Rayleigh law. As maximum values are realizations of a random variable, the function of probability distribution $P_\zeta(\zeta)$ depends on width of the spectrum defined by the formula [9]

$$\Delta = \sqrt{1-\frac{T_g}{T_n}}, \quad (8)$$

here $T_n = 2\pi\sqrt{m_\zeta^0 / m_\zeta^2}$ is time of transition through zero; $T_g = 2\pi\sqrt{m_\zeta^2 / m_\zeta^4}$ is average period between wave crests; m_ζ^0, m_ζ^2, m_ζ^4 are statistical moments of orders 0, 2, 4. If $T_g << T_n$ and width of the spectrum (8) tends to one ($\Delta \to 1$), a realization of the random process is characterized by sufficient quantity of maximums and minimums.

In this case, the spectrum is believed to be broadband and the distribution law – normal. If $T_g \approx T_n$, then $\Delta \approx 0$. It means that the multiple maximums and minimums are absent in the realization. The spectrum is believed to be narrow-band. In this case, amplitude values of wave heights are ordered in accordance with the Rayleigh law, which depends on the parameter $B = \sqrt{m_\zeta^0}$. In these conditions the following expression takes place [9]

$$p_\zeta(\zeta) = (\zeta / m_\zeta^0)\exp(-\zeta^2 / 2m_\zeta^0). \quad (9)$$

The variance of values distributed by the Rayleigh law (9) is defined by the formula

$$D_r = \frac{4-\pi}{2}B^2.$$

If a vehicle moves with some speed, the observed frequencies differ from the same frequencies in the immovable reference frame. The spectrum of the wave, which is observed from the vehicle, is called the imaginary wave spectrum.

5. Simulation of environmental disturbances

Simulation of environmental disturbances is the important stage of the precision attitude and heading reference system simulation. For this it is necessary to give the white noise at the input of the forming filter. There are different approaches for filter implementation.

To obtain the transfer function of the forming filter using the first approach, the Wiener filtration theory is used. Factorization of the expression for irregular sea spectral density can be represented as product of stable and unstable multipliers

$$S_\zeta(\omega) = \left|W_f(j\omega)\right|^2 S_w(\omega),$$

here $S_w(\omega)$ is the white noise intensity.

To factorize the spectral density by the expression (3) it is convenient to represent it in the form

$$S_\zeta(\omega) = \frac{2D_r\mu}{\pi}\frac{\omega^2+\mu^2+\lambda^2}{\omega^4 + 2(\mu^2-\lambda^2)\omega^2 + (\mu^2+\lambda^2)^2}. \quad (10)$$

The factorization of the expression (10) can be carried out in the following way

$$S_\zeta(\omega)=\frac{2D_r\mu}{\pi}\frac{s^2+\mu^2+\lambda^2}{(j\omega)^4+2(\mu^2-\lambda^2)(j\omega)^2+(\mu^2+\lambda^2)^2}=$$

$$=\sqrt{\frac{2D_r\mu}{\pi}}\sqrt{\frac{2D_r\mu}{\pi}}\frac{(-j\omega+\sqrt{\mu^2+\lambda^2})(j\omega+\sqrt{\mu^2+\lambda^2})}{a_2^2(j\omega)^4+(2a_0a_2-a_1^2)(j\omega)^2+a_0^2}= \qquad (11)$$

$$=\sqrt{\frac{2D_r\mu}{\pi}}\sqrt{\frac{2D_r\mu}{\pi}}\frac{(-j\omega+\sqrt{\mu^2+\lambda^2})(j\omega+\sqrt{\mu^2+\lambda^2})}{(a_2(j\omega)^2+a_1j\omega+a_0)(a_2(j\omega)^2-a_1j\omega+a_0)},$$

here

$a_2^2=1;\ 2a_0a_2-a_1^2=2(\mu^2-\lambda^2);\ a_0^2=(\mu^2+\lambda^2)^2$,

and $a_2=1;\ a_1=2\lambda;\ a_0=\mu^2+\lambda^2$.

Finally, expression for the forming filter transfer function, which can be obtained based on the relation (11) becomes

$$W_f(j\omega)=\sqrt{\frac{2D_r\mu}{\pi}}\frac{(j\omega+\sqrt{\mu^2+\lambda^2})}{(j\omega)^2+2\lambda j\omega+\mu^2+\lambda^2}. \qquad (12)$$

It should be noted that for the studied system it is necessary to take into consideration disturbances by the angle slope. The spectral density of the angle slope α measured in direction of wave motion can be determined by the expression [11]

$$S_\alpha(\omega)=S'_\zeta(\omega)=k^2S_\zeta(\omega)=\frac{\omega^2}{g^2}S_\zeta(\omega). \qquad (13)$$

Using the expression (13), the forming filter transfer function (12) can be transformed to the form

$$W_f(j\omega)=\sqrt{\frac{2D_r\mu(\mu^2+\lambda^2)}{\pi}}\frac{j\omega}{g}\frac{(j\omega+\sqrt{\mu^2+\lambda^2})}{(j\omega)^2+2\lambda j\omega+\mu^2+\lambda^2}. \qquad (14)$$

Carrying out the comparative analysis of expressions (3), (5) it is possible to write the transfer function of the forming filter based on the expression (5) in the following way

$$W_f(j\omega)=2\sqrt{\frac{D_r\mu(\mu^2+\lambda^2)}{\pi}}\frac{j\omega}{g}\frac{\sqrt{\mu^2+\lambda^2}}{(j\omega)^2+2\lambda j\omega+\mu^2+\lambda^2}. \qquad (15)$$

The expressions (14), (15) take into consideration that variance of the angle slope is determined by the formula $D_\alpha=(\mu^2+\lambda^2)D_r$ [10].

The forming filter (15) in the state space can be represented in the following way

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix}=\begin{bmatrix}0 & 1\\ -\mu^2-\lambda^2 & -2\lambda\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix}+\begin{bmatrix}0\\ k\end{bmatrix}\omega;$$

$$y=\begin{bmatrix}0 & 1\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix}, \qquad (16)$$

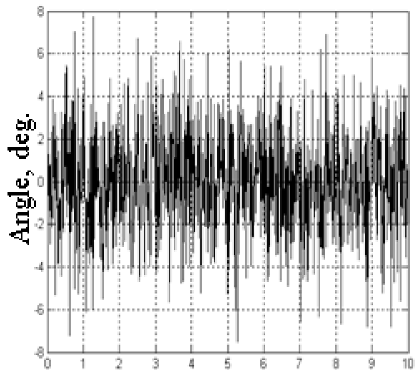
here $k=2\sqrt{\frac{D_r\mu}{\pi}}\frac{\mu^2+\lambda^2}{g}$.

Random processes with spectral densities (3), (5) can be simulated as result of passing random signal (the white noise) through forming filters (14), (15). Simulation results are given in Fig. 4.

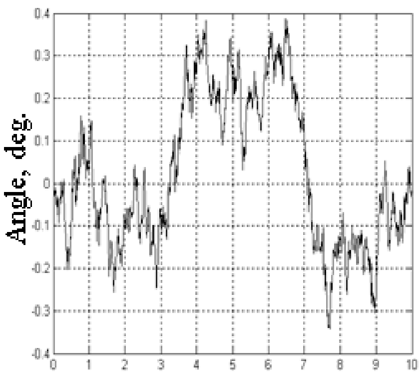
Simulation results show that random signal forming by means of the forming filter (14) allows to give the more rigid operations conditions.

It was obtained based on results of experimental researches that variance is related with the height of the wave of 3% provision by the relation [10]

$$D_r=D[\zeta(t)]=0{,}143(h_{3\%}/2)^2.$$



a



b

Fig. 4. Random signals: *a* – simulation based on the formula (14); *b* – simulation based on the formula (15)

The second approach foresees usage of the forming filter, which provides simulation of disturbances with the given spectral density as some approximation.

As in the previous case, the forming filter is determined using factorization. As a rule, the given structure is used. The most widespread structure is the filter of the second order [11]

$$W_f(j\omega)=\frac{2\xi\omega_n(j\omega)}{(j\omega)^2+2\xi\omega_n(j\omega)+\omega_n^2}. \quad (17)$$

For this filter

$$|W_f(j\omega)|^2=\frac{4(\xi\omega_n\omega)^2}{(\omega_n^2-\omega^2)^2+4(\xi\omega_n\omega)^2},$$

hence

$$\max_{\omega} S_\zeta = |W_f(j\omega)|^2\ S_w .$$

Parameters of the filter can be chosen in the following way [11].

1. The intrinsic frequency of the filter is defined by the formula

$$\omega_n \approx \arg\max_{\omega} S_\zeta(\omega).$$

2. Damping coefficient $0<\xi<1$ is chosen to provide correspondence between variance of the filter output and variance of the given spectrum.

Similar to expression(16), the forming filter (17) in the state space can be represented in the form

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix}=\begin{bmatrix}0 & 1\\ -\omega_n^2 & -2\xi\omega_n\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix}+\begin{bmatrix}0\\ 2\xi\omega_n\end{bmatrix}\omega;$$

$$y=\begin{bmatrix}0 & 1\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix}.$$

6. Simulation results

Expressions for determination of forming filters (3), (5) can be applied for forming disturbance moment (2).

Results of the perturbed system simulation are given in Fig. 5.

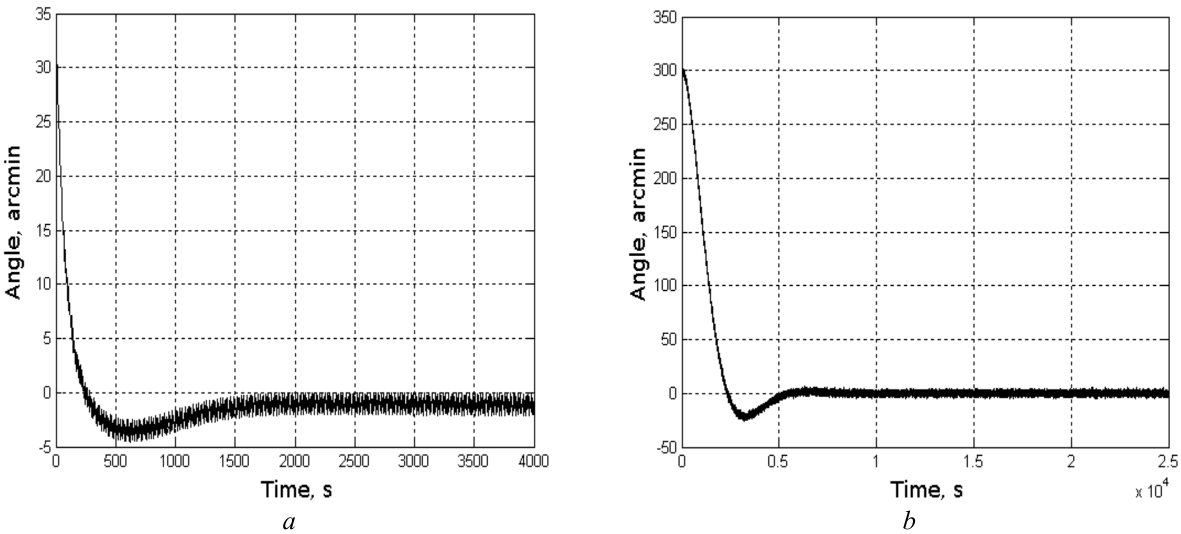


Fig. 5. Simulation of the perturbed system: transient processes by the pitch *(a)* and heading (*b*)

The represented results have been obtained for the case of the most complex operation conditions.

7. Conclusions

The mathematical model of the stabilizing system for the marine navigation complexes is created.

Characteristic of irregular sea waves is given. Analysis of mathematical descriptions of environmental disturbances is carried out. Expressions for determination of forming filters, which allow to simulate the perturbed motion of marine vehicles are obtained.

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О.А. Сущенко

Моделювання високоточної системи визначення просторової орієнтації під дією збурень зовнішнього середовища

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Мета: У статті розглянуто проблеми моделювання робастних високоточних систем визначення просторової орієнтації, які можуть використовуватися у навігації морських рухомих об’єктів. Головною метою є створення математичної моделі, пристосованої для моделювання збуреної системи, та моделей збурень зовнішнього середовища. **Методи дослідження:** Для розв’язання даної проблеми було використано теорію робастних систем управління, теорію фільтрації та теорію ймовірностей. **Результати:** Представлено модель збуреної системи визначення просторової орієнтації, створеної за допомогою засобів Simulink. Запропоновано вираз для визначення моменту збурення. Виконано аналіз можливих збурень зовнішнього середовища для системи досліджуваного типу. Отримано моделі збурень зовнішнього середовища на основі теорії фільтрації. Виконано порівняння двох підходів до розробки моделей зовнішнього середовища. **Висновки:** Представлено результати моделювання високоточної системи визначення просторової орієнтації рухомих об’єктів з урахуванням збурень зовнішнього середовища. Отримані результати можуть бути корисними для проектування високоточних навігаційних систем рухомих об’єктів.

Ключові слова: збурена система; моделі збурень; моделювання; робастні системи; система визначення просторової орієнтації.

О.А. Сущенко

Моделирование высокоточной системы определения пространственной ориентации под действием возмущений внешней среды

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Цель: В статье рассмотрены проблем моделирования робастных высокоточных систем определения пространственной ориентации, которые можно использовать в навигации морских подвижных объектов. Главной целью является создание математической модели, приспособленной к моделированию возмущенной системы. **Методы исследования:** Для решения данной проблемы были использованы теория робастных систем управления, теория фильтрации и теория вероятностей. **Результаты:** Представлена модель возмущенной системы определения пространственной ориентации, созданной при помощи средств Simulink. Предложено выражение для определения момента возмущения. Выполнен анализ возможных возмущений внешней среды для системы исследуемого типа. Получены модели возмущений внешней среды на основе теории фильтрации. Выполнено сравнение двух подходов к разработке моделей внешней среды. **Выводы:** Представлены результаты моделирования высокоточной системы определения пространственной ориентации подвижного объекта с учетом возмущений внешней среды. Полученные результаты могут быть полезными для проектирования высокоточных навигационных систем подвижных объектов.

Ключевые слова: возмущенная система; модели возмущений; моделирование; робастные системы; система определения пространственной ориентации.

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