

## A HYBRID APPROACH TO THE OPTIMAL AERONAUTICAL ENGINEERING MAINTENANCE PERIODICITY DETERMINATION

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### **Abstract**

**Purpose:** The goal of this research is to investigate the possibility of the objectively existing aeronautical engineering maintenance optimal periodicity determination in the different from the entirely probabilistic methods way. In this paper there is a scientifically proven explanation for the mentioned above periodicity optimization with the help of specially introduced hybrid optional-probabilistic approach functions distributions. **Methods:** The described hybrid approach proposes to combine the probabilistic method, applied up to effectiveness functions determination, with following compilation a variational-optimal functional, as well as consider functioning of an aeronautical engineering system, with possible degrading failures and restorations from the damaged into upstate, on the multi-optimal basis. **Results:** It allows obtaining the wanted optimal periodicities sidestepping the related states probabilities determination and their further extremization. The optional objective effectiveness functions in such a case are the roots of the characteristic equation for the corresponding states probabilities Erlang differential equations system, which relates with the set of the considered operational options. **Discussion:** The revolutionary points here are in the methods bringing the described results. The methods are also applicable to the roots for the Laplace transformations matrix parameter. The preliminary considered case discussed in the previous paper happened to be a particular case of the presented research which means a step of generalization and evolution of entropy extremization principles. The conducted computer simulation proves that the roots are the self-measured special hybrid optional-probabilistic functions.

**Keywords:** airworthiness; aeronautical engineering; damage intensity; degrading failure; entropy extremization principle; extremal; failure intensity; hybrid functions distribution; maintenance periodicity; optimization; option; restoration intensity; variational problem.

### **1. Introduction**

Developing the scheduled maintenance timetable for aircraft airworthiness support one has to take into consideration a lot of a variety of accompanying issues.

It is obvious that optimization of the maintenance periodicity with respect to economical and safety affairs is always an actual problem.

Many fields of aviation industry, such as aeronautical engineering operation, maintenance, and repair described in books [1-3], function in conditions considered in monographs [4, 5], papers [6-8], and works [9, 10].

#### **Problem formulation**

It is absolutely not that an easy thing to find the aeronautical engineering scheduled maintenance optimal periodicity with the application of the new

approach proposed in the framework of subjective analysis developed in the previous monograph [4] and papers on the matter [8, 11-15].

### **2. Analysis of the latest researches and publications**

In order to keep aircraft worthy to fly, as a response to the requirements reflected upon in works [1-5], many different supporting, preventive, and restoring techniques and measures developed in publications [6-10] are used. It is significantly important to center upon optimal scheduled maintenance periodicity for aeronautical engineering taking into account the unsolved part of the general problem of the kind of conceptual ideology of subjective preferences functions optimal distributions started in monograph [4].

In actual fact, the subjective entropy paradigm, used in papers [8, 11-15], needs to be further developed and generalized.

### 1. Task setting

The task setting for the presented paper is to obtain the objectively existing optimal aeronautical engineering maintenance periodicity in the different from the purely probabilistic, hybrid, way.

## 2. Main material

There is a system with a degrading failure, suppose. Methods of the optimal scheduled maintenance periodicity determination are described in works [1, 11]. It corresponds with the graph illustrated in Figure 1.

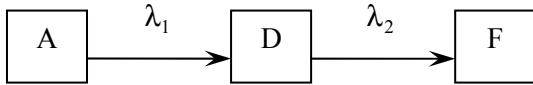


Fig. 1. Graph of states of a system with a degrading failure

Here, in Fig. 1, “A” designates the up state of the system; “D” – damage; “F” – failure. The corresponding with the system states transitions intensities are depicted as  $\lambda_1$  and  $\lambda_2$ ,

### 1. Problem setting

Now, let us consider a system with possible restoration. The related graph is represented in Figure 2.

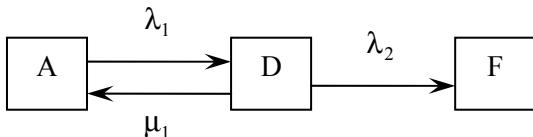


Fig. 2. Graph of states of a system with a degrading failure and possible restoration

Here, in Fig. 2, more general modelling will be done with a step implying a possible return of the system from the damaged state “D” into the normal state of “A”. This transition is carried out with the parameter of  $\mu_1$  illustrated on the graph, see Fig. 2.

The other (optional, alternativeness) way of the problem solving implies not the entire probabilistic but a hybrid, combined with the functions analogous to subjective preferences, subjective entropy extremization principle (SEEP), approach, likewise in works [4, 8, 11-15]. The corresponding intensities

of  $\lambda_1$ ,  $\lambda_2$ , and  $\mu_1$  for the considered problem setting can be represented as certain parameters of the options (multi-alternativeness).

Therefore, we may use the apparatus of preferences functions [11, 15].

### 2. Problem solution

The optimal value of the wanted periodicity can be obtained with the use of SEEP [4, 8, 11-15].

Supposedly, the optimized entropy functional has the view of [11]:

$$\Phi_h = -\sum_{i=1}^3 h_i(F_i) \ln h_i(F_i) + \beta \sum_{i=1}^3 h_i(F_i) F_i(\lambda_{1,2}, \mu_1) + \gamma \left[ \sum_{i=1}^3 h_i(F_i) - 1 \right], \quad (1)$$

where  $h_i(\cdot)$  – hybrid functions, obtained as partially based on probabilistic concept and preferences functions paradigm;  $F_i(\cdot)$  – effectiveness functions, related with corresponding options;  $\beta$  – system’s optimization internal parameter;  $\gamma$  – normalizing function (coefficient).

For the system shown in Fig. 2, the corresponding system of differential equations by Erlang, for probabilities of states determination, will have the view of

$$\left. \begin{aligned} \frac{dP_A}{dt} &= -\lambda_1 P_A + \mu_1 P_D; \\ \frac{dP_D}{dt} &= \lambda_1 P_A - (\lambda_2 + \mu_1) P_D; \\ \frac{dP_F}{dt} &= \lambda_2 P_D. \end{aligned} \right\} \quad (2)$$

Here,  $P_A$ ,  $P_D$ ,  $P_F$  – probabilities of corresponding states;  $t$  – time.

The characteristic equation for system (2) will be

$$\begin{vmatrix} -\lambda_1 - k & \mu_1 & 0 \\ \lambda_1 & -(\lambda_2 + \mu_1) - k & 0 \\ 0 & \lambda_2 & 0 - k \end{vmatrix} = 0, \quad (3)$$

where  $k$  – parameter to be determined for finding the probabilities of  $P_A$ ,  $P_D$ ,  $P_F$  of the system corresponding states.

**But from now on we do not need to determine the probabilities of  $P_A$ ,  $P_D$ ,  $P_F$ , being used roots of the determinant (3) as the options corresponding effectiveness functions  $F_i(\cdot)$ , although.**

In regards to the system's optimization internal parameter  $\beta$  we accept provisional expression of

$$\beta = -t_p^*, \quad (4)$$

where  $t_p^*$  – wanted optimal aeronautical engineering scheduled maintenance periodicity.

Concerning hybrid functions  $h_i(\cdot)$  we assume their relation with the options corresponding effectiveness functions  $F_i(\cdot)$ .

Then, compiling the functional (1), with all above mentioned suppositions, we obtain an expression that needs to be optimized:

$$\Phi_h = -\sum_{i=1}^3 h_i(x, k_i) \ln h_i(x, k_i) - t_p^* \sum_{i=1}^3 h_i(x, k_i) k_i + \gamma \left[ \sum_{i=1}^3 h_i(x, k_i) - 1 \right], \quad (5)$$

where  $x$  – parameter, unknown, uncertain multiplier in type of the Lagrange one;  $k_i$  – roots of the characteristic equation (3) for the corresponding Erlang differential equations system (2).

These roots namely are:

$$k_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}, \quad k_3 = 0, \quad (6)$$

where  $b = -(\lambda_1 + \lambda_2 + \mu_1)$ ,  $a = -1$ ,  $c = -\lambda_1 \lambda_2$  – corresponding coefficients of the quadratic equation obtained on the basis of the characteristic equation (3) determinant envelopment.

Since  $k_3 = 0$  in the sequence of roots (6) and we deem

$$h_i(x, k_i) = xk_i, \quad (7)$$

therefore

$$h_3 = 0, \quad (8)$$

and the problem becomes a two-optimal.

Anyway, applying the law of subjective conservatism introduced in paper [15] we get the sought after optimal periodicity:

$$t_p^* = \frac{\ln k_1 - \ln k_2}{k_2 - k_1}. \quad (9)$$

### 3. Computer simulation

Now, let us simulate the process, represented with the graph shown in Fig. 2 and modeled with the

mathematical expressions of (1)-(9), with the help of the computer calculation on the MathCad platform.

The data are as follows:  $\lambda_1 = 5 \cdot 10^{-3} \text{ h}^{-1}$ ;  $\lambda_2 = 1 \cdot 10^{-3} \text{ h}^{-1}$ ;  $\mu_1 = 1 \cdot 10^{-3} \text{ h}^{-1}$ ;  $t = 0 \dots 1.5 \cdot 10^3 \text{ h}$ .

The results of the computer simulation are illustrated in Figure 3.

In Figure 3 the designations of  $y_1$ ,  $y_0$ , and  $y_2$  stand for the probabilities of corresponding states  $P_D$ ,  $P_A$ ,  $P_F$  of the Erlang differential equations system (2). The value of the optimal maintenance periodicity yielded by the equation of (9) is denoted here as  $T_{\text{Opt}}$ . It equals approximately 378.311 h and delivers the maximal value to the probability of the damaged state "D" (see Figure 2):  $P_D \approx 0.5949$  (see Figure 3).

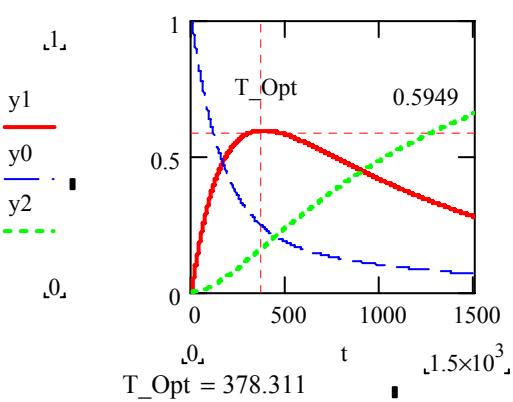


Fig. 3. Optimal periodicity of scheduled maintenance found on the basis of the hybrid optional-probabilistic approach

### 4. Discussion

The presented herein hybrid problem is a two-optimal because the state of "F" – failure, is not an option since it has no exit.

The hybridization elements combinations here are also the probabilistic approach in effectiveness functions  $F_i(\cdot)$  of functional (1) building and then compiling them into the variational-optimal functional (5).

It is applicable to the roots for the Laplace transformations matrix parameter as well when solving system (2).

It cannot be just coincidence or occasional thing that the roots  $k_i$  should be appointed the optional effectiveness functions  $F_i(\cdot)$  and the extremal options hybrid functions  $h_i(\cdot)$  are related linearly to them.

The previously considered in paper [11] case, discussed therein before this research, has happened to be a partial case of the problem described with formulae (1)-(9). Indeed, denoting the value of  $\mu_1 = 0$  in expressions (2) and (3), or immediately in the sequence of roots (6), we come to the presented in paper [11] situation (compare also Figures 1 and 2). The roots analogous to the ones of the systems of (2) and (3), and sequence (6), become as follows:  $k_1 = -\lambda_1$ ,  $k_2 = -\lambda_2$ , and  $k_3 = 0$ , which yields

through the equation (9) application the result obtained earlier in paper [11] and demonstrated for comparing convenience in Figure 4. In Figure 4  $x_1$  designates the probability of the damaged state "D"; the maximal value  $P_D \approx 0.66874$  in case described with the graph shown in Figure 1. The optimal periodicity value  $t_{opt} \approx 402.359$  h in the case considered in paper [11] has been found by equation (9).

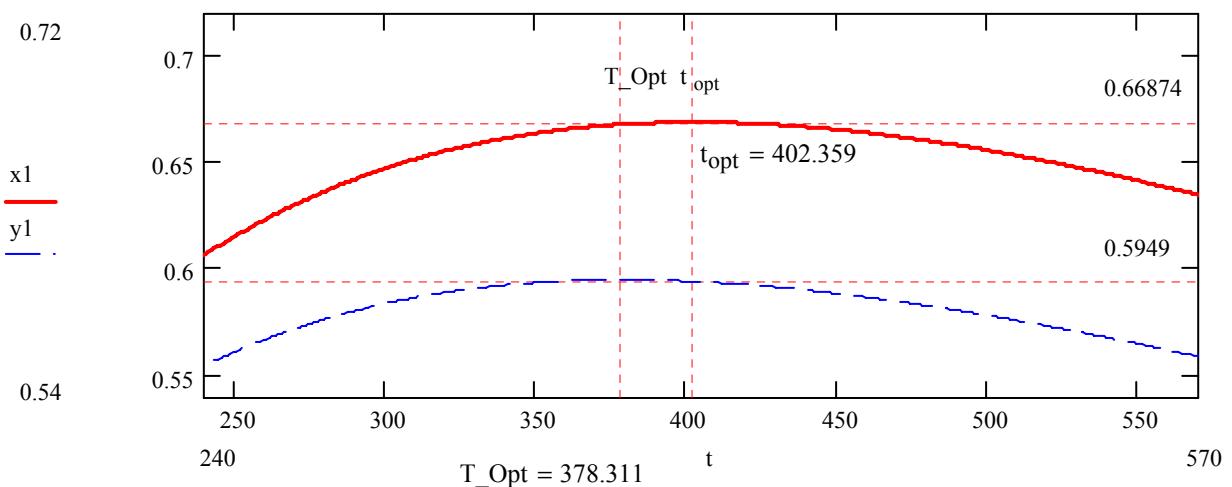


Fig. 4. Optimal periodicities of scheduled maintenances found on the basis of the hybrid optional-probabilistic approach

### 3. Conclusions

At the presented problem setting it is discovered an optional-probabilistic hybrid way of the optimal value of aeronautical engineering maintenance periodicity determination. The maximal values of the considered states probabilities are found avoiding the probabilities determinations and without their extremizations.

**In further researches** it should be taken into account more possible cases and made the related generalizations.

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### Гібридний підхід до визначення оптимальної періодичності технічного обслуговування авіаційної техніки

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**Мета:** Метою даної роботи є дослідити можливість визначення об'єктивно існуючої оптимальної періодичності технічного обслуговування авіаційної техніки іншим від повністю ймовірнісних методів шляхом. У даній статті міститься науково доведене пояснення для оптимізації вищезгаданої періодичності за допомогою розподілів спеціально введених гібридних функцій опціонно-ймовірнісного підходу. **Методи дослідження:** Описаний підхід пропонує скомбінувати ймовірнісний метод, залучений до визначення функцій ефективності, із подальшим складанням варіаційно-опціонного функціоналу, а також розглядати функціонування системи авіаційної техніки, із можливими поступовими відмовами та відновленнями з ушкодженого до працездатного стану, на основі багатоопційності. **Результати:** Це дозволяє отримати бажані оптимальні періодичності оминаючи визначення ймовірностей відповідних станів та їхню подальшу екстремізацію. Опціонні об'єктивні функції ефективності у такому разі – це корені характеристичного рівняння системи диференційних рівнянь Ерланга ймовірностей експлуатаційних опцій, яка розглядається. **Обговорення:** Поворотні пункти тут є у методах, що дають описані результати. Дані методи є також застосовуваними до коренів параметру матриці перетворень Лапласа. Попередньо розглянутий випадок, обговорений у статті, що передувала, виявився частинним випадком представленого дослідження, що означає крок в узагальненні та

еволюції ентропійних екстремізаційних принципів. Поведене комп'ютерне моделювання доводить, що означені корені є само-вимірними спеціальними гібридними опціонно-ймовірнісними функціями.

**Ключові слова:** авіаційна техніка; варіаційна задача; екстремаль; льотна придатність; опція; оптимізація; параметр потоку відновлень; параметр потоку відмов; параметр потоку ушкоджень; періодичність технічного обслуговування; поступова відмова; принцип екстремізації ентропії; розподіл гібридних функцій.

**А. В. Гончаренко**

**Гибридный подход к определению оптимальной периодичности технического обслуживания авиационной техники**

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**Цель:** Целью данной работы является исследовать возможность определения объективно существующей оптимальной периодичности технического обслуживания авиационной техники иным нежели полностью вероятностными методами путем. В данной статье содержится научно доказанное пояснение для оптимизации вышеупомянутой периодичности с помощью распределений специально введенных гибридных функций опционно-вероятностного подхода. **Методы исследования:** Описанный подход предлагает скомбинировать вероятностный метод, привлеченный к определению функций эффективности, с дальнейшим составлением вариационно-опционного функционала, а также рассматривать функционирование системы авиационной техники, с возможными постепенными отказами и восстановлениями из поврежденного в работоспособное состояние, на основе многоопционности. **Результаты:** Это позволяет получить желаемые оптимальные периодичности избегая определение вероятностей соответствующих состояний и их дальнейшую экстремизацию. Опционные объективные функции эффективности в таком случае – это корни характеристического уравнения системы дифференциальных уравнений Эрланга вероятностей соответствующих состояний, что соответствует множеству рассматриваемых эксплуатационных опций. **Обсуждение:** Поворотные пункты здесь в методах, дающих описанные результаты. Данные методы также являются применимыми к корням параметра матрицы преобразований Лапласа. Предварительно рассмотренный случай, обсуждавшийся в предыдущей статье, оказался частным случаем представленного исследования, что означает шаг в обобщении и эволюции энтропийных экстремизационных принципов. Проведенное компьютерное моделирование доказывает, что указанные корни являются само-измеренными специальными гибридными опционно-вероятностными функциями.

**Ключевые слова:** летная годность; авиационная техника; периодичность технического обслуживания; постепенный отказ; параметр потока повреждений; параметр потока восстановлений; параметр потока отказов; оптимизация; принцип экстремизации энтропии; опция; распределение гибридных функций; вариационная задача; экстремаль.

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