$$\frac{F^2}{c_p\,gRG^2} + (gR - c_p)\frac{T}{p^2}$$

$$c_p \left(\frac{F}{\ldots}\right)^2 - c_p\, z^2$$

$$\tau_w = \ldots$$

$$\left(\int R(z)dz\right) \quad \frac{}{\partial x} \quad dz \Big| dz$$

$$\int R(z)dz \Big| \int g(z) e^{\int R(z)dz} dz + \text{const} \Big| \left(1 - \frac{y}{gR}\right)$$

$$\frac{gRG}{F} \Big| \frac{dz}{dx} \quad \frac{1}{gR} \Big| \frac{gRG}{} \Big) \int \left\{ w_x + \left\{ c_p e^{\int R(z)dz} \right\} \right\}$$

$$dz + \text{const} \Big| \left(\frac{2\lambda}{\omega D_h} Z \approx f_x \frac{1}{z} \right)$$

It is evident, that the above ODE variables are separable. One can notice that the solution has singularity when

$$c_p + \left(1 - \frac{c_p}{gR}\right)\left(\frac{gRG}{F}\right)^2 T = 0,$$

thence

$$G = \frac{PF}{\sqrt{(c_p - gR)gRT}}$$

presents the sonic flow rate.

Let's find out the link between pressure and temperature in every channel point. To do it, it is necessary to solve integrals in (6). The following integral

$$\int R(z)dz = \ln \left|z^2 + a^2\right|^{1-b} \tag{7}$$

where

$$b = \frac{1}{2}\frac{\frac{c_p}{gR}}{\left(1 - \frac{c_p}{gR} f_x\right)}, \tag{8}$$

and integral $\int Q(z) e^{\int R(z)dz} dz$ split into the following ones

$$I_3 = \left(\frac{gRG}{F}\right)^2 \int z^{2(b+1)}\left(z^2 + a^2\right)^{\frac{1+2b}{2}} dz;$$

$$I_4 = \frac{(w_x - f_x)}{\frac{2\lambda}{\omega D_h}} \int z^{2b}\left(z^2 + a^2\right)^{\frac{1+2b}{2}} dz,$$

where

$$a^2 = \frac{\left(w_x - \frac{c_p}{gR} f_x\right)}{\frac{c_p}{gR}\left(\frac{gRG}{F}\right)^2 \frac{2\lambda}{\omega D_h}}; \tag{9}$$

It is obvious from (8), (9) that two forms of integrals appear which depend on whether sign of a^2 is positive or negative, correspondingly

$$I_{31} = \int \frac{y^c}{\left(y^2 - 1\right)^2} dy; \quad I_{41} = \int \frac{y^c}{\left(y^2 - 1\right)} dy;$$

$$I_{32} = \int \frac{y^c}{\left(y^2 - 1\right)^2} dy; \quad I_{42} = \int \frac{y^c}{\left(y^2 - 1\right)} dy;$$

The integrals have binomial differentials as integral functions, where the following substitution formulae are used for each case

$$a^2 + z^2 = z^2 y^2 \text{ or } a^2 - z^2 = z^2 y^2, \quad a^2 > 0,$$

and
$$c = \frac{\frac{c_p}{gR} f_x}{\left(w_x - \frac{c_p}{gR} f_x\right)}.$$

As one can see, both integrals have singularity at $y = 1$ that occurs at $a^2 = 1$. This corresponds to the nonisentropic adiabatic flow case. Constant c is not guarantied to be rational. Thus, the binomial integrals can not be solved analytically. Let's expand the following function into a series

$$y^c = y_0^c + \frac{c}{1!} y_0^{c-1}(y - y_0) + \frac{c(c-1)}{2!} y_0^{c-2} \times$$

$$\times (y - y_0)^2 + \frac{c(c-1)(c-2)}{3!} y_0^{c-3}(y - y_0)^3 + K +$$

$$+ \frac{1}{n!} c(c-1)(c-2)K[c-(n-1)] y_0^{c-n}(y - y_0)^n +$$

$$+ K$$

According to d'Alembert criterion of the convergence radius of the above series is expressed as

$$r = \lim_{n\to\infty} \left| \frac{\frac{1}{(n+1)!} c(c-1)(c-2)K[c-n] y_0^{c-(n+1)}(y - y_0)^{n+1}}{\frac{1}{n!} c(c-1)(c-2)K[c-(n-1)] y_0^{c-n}(y - y_0)^n} \right|$$

$$= \lim_{n\to\infty} \left| \frac{(c-n)(y - y_0)}{(n+1) y_0} \right| = \left| \frac{y - y_0}{y_0} \right| < 1.$$

Because no restrictions were imposed on choice of y_0, i.e. it is, in a sense, arbitrary, then it means that there always can be found such y_0 from the physical problem statement which satisfies the convergence radius obtained above. The integrals I_{31} and I_{41} take the following forms

$$I_{31} = y_0^c \int \frac{dy}{\left(y^2 - 1\right)^2} + \frac{c}{1!} y_0^{c-1} \int \frac{y\,dy}{\left(y^2 - 1\right)^2} +$$

$$+ \frac{c(c-1)}{2!} \int \frac{y^2 dy}{\left(y^2 - 1\right)^2} + \frac{c(c-1)(c-2)}{2!}$$

$$\times y_0^{c-3}\int\frac{y^3dy}{\left(y^2-1\right)^2}+K+\frac{1}{n!}c(c-1)(c-2)K\times$$

$$\times[c-(n-1)]y_0^{c-n}\int\frac{y^n dy}{\left(y^2-1\right)^2}+P_n(y);$$

$$I_{41}=y_0^c\int\frac{dy}{\left(y^2-1\right)}+\frac{c}{1!}y_0^{c-1}\int\frac{ydy}{\left(y^2-1\right)}+$$

$$+\frac{c(c-1)}{2!}y_0^{c-2}\int\frac{y^2dy}{\left(y^2-1\right)}+\frac{c(c-1)(c-2)}{2!}\times$$

$$\times y_0^{c-3}\int\frac{y^3dy}{\left(y^2-1\right)}+K+\frac{1}{n!}c(c-1)(c-2)\times K\times$$

$$\times[c-(n-1)]y_0^{c-n}\int\frac{y^ndy}{\left(y^2-1\right)}+P_n(y).$$

The integration is true because it does not alter series convergence radius. Integrals of I_{41} with the even powers of y have the solution

$$I_{41}^{2m}=\frac{y_0^{c-2m}}{(2m)!}\prod_{n=0}^{2m-1}(c-n)\left\{\frac{1}{2}(-1)^m\ln\left|\frac{y-1}{y+1}\right|+\right.$$

$$\left.+\sum_{k=0}^{m-1}(-1)^k\frac{y^{2m-2k-1}}{2m-2k-1}\right\},$$

but for odd powers of y the solution is

$$I_{41}^{2m+1}=\frac{y_0^{c-(2m+1)}}{2m+1}\prod_{n=0}^{2m}(c-n)\left\{\frac{1}{2}(-1)^m\ln\left|y^2+a^2\right|+\right.$$

$$\left.+\frac{1}{2}\sum_{k=0}^{m-1}(-1)^k\frac{y^{2m-2k}}{m-k}\frac{1}{2}\right\},$$

then integral I_4 at $a^2>0$ takes the form

$$I_{4,\left(a^2>0\right)}=\frac{\dfrac{\left(w_x-f_x\right)}{2\lambda}}{\dfrac{c_p}{\omega D_h}}a\left[y_0^c\frac{1}{2}\ln\left|\frac{x-1}{x+1}\right|+\right.$$

$$\left.+cy_0^{c-1}\frac{1}{2}\ln\left|y^2+1\right|+\left(I_{41}^{2m}+I_{41}^{2m+1}\right)\right].$$

In a similar manner, for integral I_{31} we have the following expressions for even and odd powers, respectively

$$I_{31}^{2m}=\frac{x_0^{c-2m}}{(2m)!}\prod_{n=0}^{2m-1}(c-n)\times\left\{\frac{y^{2m+1}}{2\left(y^2+1\right)}+\frac{1}{2}(1-2m)\times\right.$$

$$\left.\times\left[\sum_{k=1}^{m}\frac{(-1)^{m+k}}{2k-1}(y)^{2k-1}+\frac{1}{2}(-1)^m\ln\left|\frac{y+1}{y-1}\right|\right]\right\},$$

$$I_{31}^{2m+1}=-\frac{y_0^{c-(2m+1)}}{2m+1}\prod_{n=0}^{2m}(c-n)\times$$

$$\times\sum_{k=0}^{m}(-1)^k\binom{m}{k}\frac{1}{2(2-m+k-1)\left(y^2+1\right)^{2-m+k-1}},$$

and integral I_3 at $a^2>0$ is

$$I_{3,\left(a^2>0\right)}=\frac{\left(\dfrac{gRG}{F}\right)^2}{c_p}a^2\left\{y_0^c\left[\frac{y}{2\left(y^2+1\right)}+\frac{1}{4}\ln\left|\frac{y-1}{y+1}\right|\right]-\right.$$

$$\left.-cy_0^{c-1}\frac{1}{2\left(y^2+1\right)}+\left(I_{31}^{2m}+I_{31}^{2m+1}\right)\right\}.$$

Finally, taking into account (7) the specific solution of the energy equation (6) at $a^2>0$ has the form

$$P_{\left(a^2>0\right)}=\frac{\left[I_{3,\left(a^2>0\right)}+I_{4,\left(a^2>0\right)}+\text{const}\right]}{z^{2b}\left(z^2+a^2\right)^{\frac{1-2b}{2}}}. \quad (10)$$

For the case when $a^2<0$ we expand into a series the function

$$\frac{1}{y^c}=y_0^{-c}-\frac{c}{1!}y_0^{-(c+1)}\left(y-y_0\right)+\frac{c(c+1)}{2!}y_0^{-(c+2)}\left(y-y_0\right)^2-$$

$$-\frac{c(c+1)(c+2)}{3!}y_0^{-(c+3)}\left(y-y_0\right)^3+K+$$

$$+(-1)^n\frac{c(c+1)(c+2)K[c+(n-1)]}{n!}y_0^{-(c+n)}\left(y-y_0\right)^n+$$

$$+R_n(y),$$

which convergence radius is the similar to the one in case of $a^2>0$

$$r=\lim_{n\to\infty}\left|\frac{(-1)^{n+1}\dfrac{c(c+1)(c+2)K[c+n]}{(n+1)!}y_0^{-(c+n+1)}\left(y-y_0\right)^{n+1}}{(-1)^n\dfrac{c(c+1)(c+2)K[c+(n-1)]}{n!}y_0^{-(c+n)}\left(y-y_0\right)^n}\right|=$$

$$=\lim_{n\to\infty}\left|\frac{(-1)(c+n)\left(y-y_0\right)}{(n+1)y_0}\right|=\frac{\left|y-y_0\right|}{y_0}<1.$$

In a similar way as for $a^2>0$ and taking into account (7), the specific solution of energy equation (6) has the following form for $a^2<0$

$$P_{\left(a^2<0\right)}=\frac{\left[I_{3,\left(a^2<0\right)}+I_{4,\left(a^2<0\right)}+\text{const}\right]}{z^{2b}\left(z^2+a^2\right)^{\frac{1-2b}{2}}}, \quad (11)$$

where

$$I_{3,\left(a^2<0\right)}=\frac{\left(\dfrac{gRG}{F}\right)^2}{c_p}a^2\left\{y_0^{-c}\left[\frac{y}{2\left(y^2+1\right)}+\frac{1}{4}\ln\left|\frac{y-1}{y+1}\right|\right]+\right.$$

$$\left.+cy_0^{-c-1}\frac{1}{2\left(y^2+1\right)}+\left(I_{32}^{2m}+I_{32}^{2m+1}\right)\right\};$$

$$I_{32}^{2m}=\frac{y_0^{-(c+2m)}}{(2m)!}\prod_{n=0}^{2m-1}(c+n)\left\{\frac{y^{2m+1}}{2\left(y^2+1\right)}+\frac{1}{2}(1-2m)\times\right.$$

$$\times\left[\sum_{k=1}^{m}\frac{(-1)^{m+k}(y)^{2k-1}}{2k-1}+\frac{1}{2}(-1)^m\ln\left|\frac{y+1}{y-1}\right|\right]\Bigg\};$$

$$I_{32}^{2m+1}=-\frac{y_0^{-(c+2m+1)}}{2m+1}\prod_{n=0}^{2m}(c-n)\times\sum\left[(-1)^k\binom{m}{k}\times\right.$$

$$\left.\times\frac{1}{2(2-m+k-1)(y^2+1)^{2-m+k-1}}\right].$$

$$I_{4,(a^2<0)}=\frac{(w_x-f_x)}{c_p\dfrac{2\lambda}{\omega D_h}}a\left[y_0^{-c}\frac{1}{2}\ln\left|\frac{y-1}{y+1}\right|-\right.$$

$$\left.-cy_0^{-c-1}\frac{1}{2}\ln\left|y^2+1\right|-\left(I_{42}^{2m}+I_{42}^{2m+1}\right)\right];$$

$$I_{42}^{2m}=\frac{y_0^{-(c+2m)}}{(2m)!}\prod_{n=0}^{2m-1}(c+n)\left\{\frac{1}{2}(-1)^m\ln\left|\frac{y-1}{y+1}\right|+\right.$$

$$\left.+\sum_{k=0}^{m-1}(-1)^k\frac{y^{2m-2k-1}}{2m-2k-1}\right\};$$

$$I_{42}^{2m+1}=\frac{y_0^{-(c+2m+1)}}{2m+1}\prod_{n=0}^{2m}(c+n)\left\{\frac{1}{2}(-1)^m\ln\left|y^2+a^2\right|+\right.$$

$$\left.+\frac{1}{2}\sum_{k=0}^{m-1}(-1)^k\frac{y^{2m-2k}}{m-k}\right\}.$$

Introducing new unknown *z* and substituting into (4) solutions (10), (11) the mathematical model of one-dimensional stationary flow of compressible gas with the given constant heat flux, friction and mass forces in the constant-area channel may be reduced to the single ODE.

But for this common flow case it is not warranted because it will have to differentiate series and the resulting ODE will be very complex.

Hence, rather pure numeric methods should be applied to solving this common flow model.

For one very important case of flow model for ideal gases such ODE reduction gives good results, because the energy equation is solved in elementary functions

$$P=-\frac{1}{gR}\left(\frac{gRG}{F}\right)^2\frac{1}{\sqrt{c}}\left\{\frac{a}{\sqrt{a+cz^2}}\left(1-\frac{b}{2c}\right)\times\right.$$

$$\left.\times\ln\left|z+\frac{\sqrt{a+cz^2}}{\sqrt{c}}\right|+\frac{b}{\sqrt{c}}\frac{z}{2}\right\}+\text{const}\frac{\sqrt{c}}{\sqrt{a+cz^2}};$$

$$a=q_x;$$

$$b=\left(\frac{gRG}{F}\right)^2\frac{\lambda}{2D_h};$$

$$c=c_P\frac{1}{gR}\left(\frac{gRG}{F}\right)^2\frac{\lambda}{2D_h}.$$

That is resulted in the explicit nonlinear ODE which can be solved with any explicit integration method.

Such a model permits modeling both subsonic flow acceleration and supersonic flow deceleration.

Conlusions

Thus, it is shown that general solution exists for the mathematical model of one-dimensional stationary flow of compressible gas with the given constant heat flux, friction and mass forces that can perform work in the constant cross area channel.

The analytical solution of the energy equation of such flow is obtained, and this kind of a flow model limits application semi-analytical methods for their formalisation.

Beginning with this problem statement the other integration methods for its solving should be developed.

References

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М.Ю. Федоров

Математичний аналіз моделювання стисливого потоку ідеального газу

Розглянуто можливість формалізації математичної моделі одновимірного стаціонарного потоку стисливого газу з заданими постійними значеннями теплового потоку, тертя, масовими силами та їх роботою в каналі постійного перерізу. Отримано аналітичний розв'язок рівняння енергії для даного виду течії.

М.Ю. Федоров

Математический анализ моделирования сжимаемого потока идеального газа

Рассмотрена возможность формализации математической модели, описывающей одномерный стационарный поток сжимаемого газа с заданными постоянными значениями теплового потока, трения, массовыми силами и работой массовых сил в канале постоянного сечения. Получено аналитическое решение уравнения энергии для данного вида течения.