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PARAMETRIC ROBUST OPTIMIZATION OF THE DIGITAL FLIGHT CONTROL SYSTEMS

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The paper is devoted to the parametric optimization of the digital flight control systems from the viewpoint of achieving the necessary compromise between their robustness and performance. This compromise could be reached by using modern approach of multi-model H_2/H_∞ *– robust optimization, which was described mainly for continuous systems. In this paper the optimization procedure for discrete control systems is proposed. Its efficiency is illustrated by the example of the digital flight control of the small Unmanned Aerial Vehicle.*

Introduction

Each Flight Control System (FCS) has to suppress exogenous stochastic disturbances produced by turbulent atmosphere and to provide required performance and stability in the presence of parametrical internal disturbances in all flight envelope of unmanned aerial vehicles (UAV).

FCS for small UAV must have low price, weight, power consumption and size. In this case FCS performance is frequently sacrificed in order to satisfy the last demands, which strongly influence not only FCS but also navigation equipment. The limited number of navigation sensors restricts the number of measured flight parameters (state space variables), which are used as the inputs of a controller.

From the other hand aforementioned demands restrict capability of the airborne computer. In this situation it is impossible to apply complicated control laws with high performance and only known simple structures of control systems [1; 2] are relevant.

That is why it is possible to enhance performance and robustness only by parametric optimization. H_2/H_∞ multi-model approach is one of the best ways allowing to reach the compromise between several controversial requirements to performance and robustness for deterministic and stochastic models with nominal and disturbed parameters [3]. This approach was successfully applied for control laws design for UAV [4] and airships [5; 6]. In these papers H_2/H_∞ robust optimization procedure was developed for continuous time, because it was supposed that the airborne computer is powerful enough to process data with high sampling frequency in comparison with the bandwidth of the closed-loop control system. This assumption sometimes is not valuable for the small UAV, which bandwidth is much wider in comparison with the bandwidth of the aircraft or airship; meanwhile the capability of the airborne computer is much more limited and high sampling frequency is unavailable. In this case the condition of Kotelnikov-Shannon theorem could be satisfied with minimal margin. As a result, if the optimization of continuous system would be done, any conversion of the analog controller into the digital one produces closed-loop system, whose dynamics would be very far from optimal (sometimes even unstable). This circumstance requires development of H_2/H_∞ – robust optimization directly for discrete time system, which is the final goal of this paper.

The statement of the optimization problem

The combination of the nominal performance with robust stability (NPRS) could be achieved using mixed H_2/H_∞ control of multi-model plants [3], which can incorporate deterministic as well as stochastic criteria in one performance index, thus permitting the reasonable trade-off between contradictory conditions of deterministic and stochastic performing indices (PI) minimization, meanwhile incorporation of H_2 and H_∞ norms allows to achieve compromise between requirements to suppress external and internal disturbances. The set of models of the plant includes all parametrically disturbed models in the different conditions of flight, which cover all flight envelopes of the given aircraft. So it is necessary to design the composite criterion, which contains all aforementioned partial PI.

Consider the standard form of the MIMO-control system optimization problem, represented in fig. 1, where vector η represents the white noise exogenous disturbances, which along with forming filter creates vector $\mathbf{g}$ of stochastic wind velocities. Spectral densities of components of vector $\mathbf{g}$ in accordance with standard Dryden models [1] describe their dynamics. The matrix of forming filter transfer functions can be produced by Wiener factorization of these spectral densities.

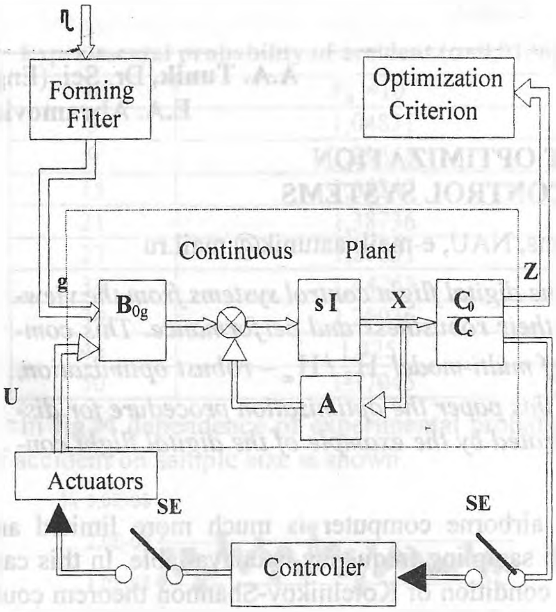


Fig. 1. The standard form of the control system optimization problem

The state-space description of the forming filter associated with Dryden's spectral densities can be easily produced also [1] as the quadruple of matrices $[\mathbf{A_f}, \mathbf{B_f}, \mathbf{C_f}, \mathbf{D_f}]$. Matrix $\mathbf{B_{0g}}$ is designed to incorporate control input **U** and stochastic vector of wind gusts **g** in one input vector; matrix **A** is state-space matrix of UAV. There are 2 observation matrices: $\mathbf{C_0}$, associated with output vector **Z**, that is used for computing the PI of the system, and $\mathbf{C_c}$, associated with other output vector **Y**, which incorporates only really measured outputs, for creating the actual controller feedback. The forming filter is used for computing the stochastic PI, meanwhile for deterministic case it is omitted. Sampling elements (SE) represent the discrete time with sampling period T_s.

The continuous plant is represented by the quadruple of matrices $[\mathbf{A}, \mathbf{B_{0g}}, \mathbf{C_c}, \mathbf{D_c}]$ for creation of the actual closed-loop system and another quadruple $[\mathbf{A}, \mathbf{B_{0g}}, \mathbf{C_0}, \mathbf{D_0}]$ for the computation of optimization criterion. The optimization procedure is based on the composite PI, which includes the following components:

1) H_2-norm for each model (nominal and parametrically disturbed) in deterministic case, which represents the system sensitivity to deterministic disturbances (or command signals):

$$J_d = \sqrt{\sum_{k=0}^{\infty}[\mathbf{X}^T(k)\mathbf{Q}\mathbf{X}(k) + \mathbf{u}^T(k)\mathbf{R}\mathbf{u}(k)]}\,; \qquad (1)$$

2) H_2- norm for each model in stochastic case:

$$J_s = \sqrt{E_M[\mathbf{X}^T(k)\mathbf{Q}\mathbf{X}(k) + \mathbf{u}^T(k)\mathbf{R}\mathbf{u}(k)]}\,; \qquad (2)$$

3) H_∞ - norm for each model:

$$\|\mathbf{G}\|_\infty = \sup_{\omega} \bar{\sigma}(\mathbf{G}(j\omega)) \,,\ 0 \le \omega \le \omega_N\,, \qquad (3)$$

where ω_N-Nyqist frequency: $\omega_N = \dfrac{\pi}{T_s}$.

In expressions (1), (2) **X** stands for vector of state space variables, u is the control vector, E_M stands for expectation operator, and **Q**, **R** are weighting matrices. In expression (3) $\bar{\sigma}$ is the maximal singular value of the transfer function matrix $\mathbf{G}(j\omega)$ of the closed-loop system over the frequency range: $0 \le \omega \le \omega_N$.

It is necessary to note, that the control circuit of the system shown in the fig. 1 is digital, meanwhile the way for the random disturbance **g** propagation in the controlled plant is analog. The obtaining the description of all system in the discrete time requires transformation of this system on the basis of superposition principle in the equivalent system shown in the fig. 2, which permits to obtain discrete time model in the stochastic case.

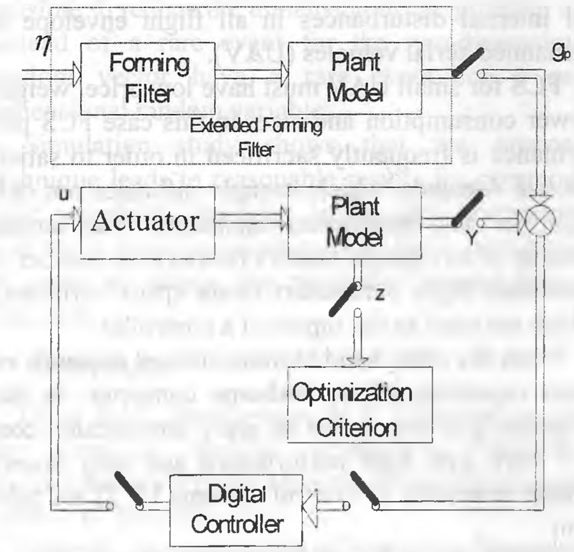


Fig. 2. The block-diagram of the equivalent digital control system

Here the extended forming filter includes the plant model along with primary analog forming filter derived from Dryden's model.

For this system it is possible to determine the quadruple of matrices $[\mathbf{A_{cl}}, \mathbf{B_{cl}}, \mathbf{C_{cl}}, \mathbf{D_{cl}}]$ for the state-space description of the deterministic model of the digital closed-loop system for nominal and parametrically disturbed cases using standard analog-to-digital conversion of "actuator + plant" series connection. The series connection of this closed-loop system model with discrete model of the extended forming filter (fig. 2) produces the model for investigation of the stochastic case. So the following quadruples of matrices are used for the set of models in this case of multi-model approach:

– for description of the real closed-loop system (with vector **Y** as an output) in the nominal and parametrically perturbed cases respectively:

$$\left[\begin{array}{c|c} \mathbf{A}_{cl} & \mathbf{B}_{cl} \\ \hline \mathbf{C}_{cl} & \mathbf{D}_{cl} \end{array}\right] \text{ and } \left[\begin{array}{c|c} \mathbf{A}_{cl}^{p} & \mathbf{B}_{cl}^{p} \\ \hline \mathbf{C}_{cl}^{p} & \mathbf{D}_{cl}^{p} \end{array}\right]; \quad (4)$$

– for description of this system, calculation of optimization criterion (with vector **Z** as an output) in the deterministic case, for nominal and parametrically perturbed plants respectively:

$$\left[\begin{array}{c|c} \mathbf{A}_{cl} & \mathbf{B}_{cl} \\ \hline \mathbf{C}_{o} & \mathbf{D}_{o} \end{array}\right] \text{ and } \left[\begin{array}{c|c} \mathbf{A}_{cl}^{p} & \mathbf{B}_{cl}^{p} \\ \hline \mathbf{C}_{o}^{p} & \mathbf{D}_{o}^{p} \end{array}\right]; \quad (5)$$

– for description of this system, calculation of optimization criterion (with vector **Z** as an output) in the stochastic case, for nominal and parametrically perturbed plants respectively:

$$\left[\begin{array}{c|c} \mathbf{A}_{cls} & \mathbf{B}_{cls} \\ \hline \mathbf{C}_{os} & \mathbf{D}_{os} \end{array}\right] \text{ and } \left[\begin{array}{c|c} \mathbf{A}_{cls}^{p} & \mathbf{B}_{cls}^{p} \\ \hline \mathbf{C}_{os}^{p} & \mathbf{D}_{os}^{p} \end{array}\right]. \quad (6)$$

These models permit to calculate the H_2 – norms (1) and (2) for nominal and perturbed models using the controllability Gramians of closed loop systems for deterministic $\mathbf{G}_d$ and stochastic $\mathbf{G}_s$ cases. These Gramians can be found as the solutions of Lyapunov equations for deterministic and stochastic discrete systems [7; 8] used in multi-model approach:

$$\mathbf{A}_{cl}\mathbf{G}_d\mathbf{A}_{cl}^{T} - \mathbf{G}_d + \mathbf{B}_{cl}\mathbf{B}_{cl}^{T} = 0;$$

$$\mathbf{A}_{cls}\mathbf{G}_s\mathbf{A}_{cls}^{T} - \mathbf{G}_s + \mathbf{B}_{cls}\mathbf{B}_{cls}^{T} = 0.$$

They are for nominal and parametrically perturbed models. Note, that observation matrices of closed loop systems $\mathbf{C}_0$ and $\mathbf{C}_{0s}$ in (4)–(6) include the control inputs also. According to [7; 8] the squares of H_2-norms (1) and (2) can be calculated as follows:

$$J_d^2 = \text{trace } (\mathbf{C}_o^{w}\mathbf{G}_d(\mathbf{C}_o^{w})^{T}); \quad (7)$$

$$J_s^2 = \text{trace } (\mathbf{C}_{os}^{w}\mathbf{G}_s(\mathbf{C}_{os}^{w})^{T}), \quad (8)$$

where $\mathbf{C}_0^{w}, \mathbf{C}_{0s}^{w}$ – weighted observation matrices:

$$\mathbf{C}_o^{w} = \mathbf{C}_o\mathbf{Q}; \ \mathbf{C}_{os}^{w} = \mathbf{C}_{os}\mathbf{Q};$$

$$\mathbf{Q} = \text{diag}(q_1, \ldots, q_n), \quad (9)$$

$q_1, \ldots, q_n$ – weights of corresponding state space variables in H_2-norms (7); (8).

The usage of controllability Gramians for calculations of PI is very useful especially in the aerospace control problems, because it can determine the contribution of each component of the state space vector in PI calculations. So the optimization criteria could be closely matched with the airworthiness requirements [9]. Linear combination of the partial PI corresponding to different models (5), (6) defines the composite PI of all set of models:

$$J_c = \lambda_d J_d^2 + \lambda_d^p (J_d^p)^2 + \lambda_s J_s^2 + \lambda_s^p (J_s^p)^2 \quad (10)$$

where $\lambda_d, \lambda_d^p, \lambda_s, \lambda_s^p$ are weighting coefficients of partial PI of deterministic and stochastic nominal and parametrically perturbed models respectively.

The robustness of the system is determined by H_∞-norm (3) of the complementary sensitivity matrix [10]:

$$\|T\|_\infty = \operatorname*{Sup}_{\omega} \bar{\sigma}(\mathbf{T}(j\omega)); \ 0 \le \omega \le \omega_N,$$

where $\mathbf{T(z)}=\mathbf{C(z)G(z)}[\mathbf{E}+\mathbf{C(z)G(z)}]^{-1}$, $z=e^{j\omega Ts}$ $\mathbf{G(z)}$ and $\mathbf{C(z)}$ are matrices of transfer functions of the plant with the actuator and controller respectively. Adding these H_∞-norms forthe nominal and parametrically perturbed models with corresponding weighting coefficients λ_∞, λ_∞^p to the composite PI, we can obtain aggregated performance-robustness index (PRI):

$$P_{p-r} = J_c + \lambda_\infty \|T\|_\infty + \lambda_\infty^p \|T^p\|_\infty. \quad (11)$$

Increasing or decreasing weights $\lambda_\infty, \lambda_\infty^p$ relatively to the weights for performance components λ_d, λ_d^p, λ_s, λ_s^p it is possible to reach the trade-off between robustness and performance of the system. PRI (11) is a function of the vector of variable parameters of controller $\overrightarrow{\mathbf{C}_n}$, including gains of all its input signals. Components $\overrightarrow{\mathbf{C}_n}$ appear in all quadruples of matrices (4)-(6) from controller's description. The optimization procedure must find such value of vector $\overrightarrow{\mathbf{C}_n}$, which provides the minimum of PRI. So far as controllability Gramians could be defined only for the stable and fully controllable system, it is possible to find the minimal value of composite PRI (11) over the space of variable parameters $\overrightarrow{\mathbf{C}_n}$, if and only if, in the process of performing optimization procedure the closed loop system would be stable and the search of optimal value $\overrightarrow{\mathbf{C}_n}^*$ of vector $\overrightarrow{\mathbf{C}_n}$ would be made within stability domain of variable parameter space. Therefore total cost function for running optimization procedure has to include some penalty function (PF) for violation of the location's area of the closed loop system poles in the complex plane. This area D is represented in the fig. 3, *a*, where it is restricted with two bold circles. The 1st circle with larger radius determines the stability margin(distance to the unit circle d_0), while the 2nd one with smaller radius determines the maximal bandwidth of the closed loop system. So it is necessary to find the minimal value d_m of all distances

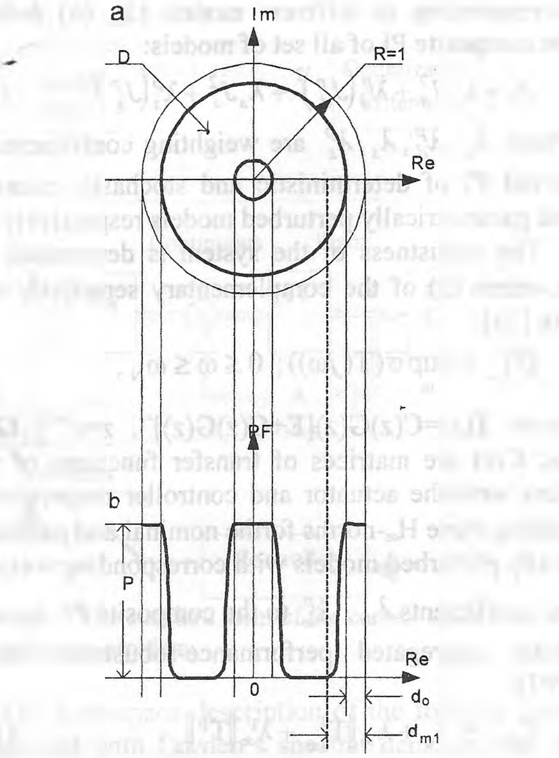


Fig. 3. Penalty function in the complex z-plane:
a – requested location of the closed-loop system poles;
b – graph of PF(dm)

from all poles of nominal and perturbed models to the 1st and the 2nd borders of area D in complex plane z. The penalty $PF_1(d_m)$ as a function of minimal distance could be graphically shown in fig. 3, *b* and defined over area D for its 1st border as follows:

$$PF_i(d_m)=\begin{cases} 0, \; if \; d_m \geq d_{m1} \\ \dfrac{P}{2}\left[1+\cos\left(\dfrac{\pi(d_m-d_0)}{d_{m1}-d_0}\right)\right], \\ P, \; if \; d_m \leq d_0 \end{cases} \quad (12)$$

it $d_0 < d_m < d_{m1}$,

where P – of large value (for example $P=10^4 \div 10^6$). This function is smoothed and differentiable inside the unit circle.

The penalty $PF_2(d_m)$ for the 2nd border is defined in the same way. Eventually the total cost function for the optimization procedure has the following form:

$$J_\Sigma = J_c + \sum_{i=1}^{2} Pf_i \quad (13)$$

and the optimization procedure will have to find optimal values for the components of vector of variable parameters under the following condition:

$$\overrightarrow{\mathbf{C}_n^*} = \arg\min J_\Sigma\left(\overrightarrow{\mathbf{C}_n}\right), \; \mathbf{C}_n \in D_c, \quad (14)$$

where D_c is the stability domain within the parameter's space, which is defined by D.

Penalty function like (12) is of great importance in the design procedure because of following reasons:

1) if the optimized system is stable during all optimization process, the PI is convex function [11], thus ensuring the unique solution of optimization process;

2) pole placement in the aforementioned area is closely related to the robust properties of closed-loop system [12].

Finally it is necessary to add, that in the optimization procedure some parameters of controller could sometimes be chosen not reasonable large. In this case it is useful to add to PF well known restrictive term:

$$PF_r = \sum_{r=1}^{l} \lambda_r p_r^2,$$

where l – a number of parameters p_r, which is to be restricted; λ_r – the weight factor.

Optimization and design procedure

The 1st stage of a design procedure consists of determination of a structure and the initial values of its parameters from the viewpoint of closed loop pole placement in the prescribed area.

As the 1st help to solve this task the standard LQR-procedure could be used for determination of signs and values of all CL components, when full state vector is measured.

Then at the 2nd stage actual sensors are taken into account adding PD-controllers or any other elements of dynamic feedback, using some known structures [1; 2].

Standard MatLab procedures for determination of state space description of series and feedback connections permit to find such values of controllers parameters, which place closed loop system poles to left half-plane inside prescribed area or with some its minor violations.

Known pole-placement methods [1; 2] could be used as well.

At the 3rd stage aforementioned optimization procedure could be applied to find the optimal parameters, which give minimum to the total cost function (13).

In our case simple but reliable Nelder-Mead optimization procedure [13] was used.

It requires more steps to be done to finde minimal value of cost function (13), but on the other hand it doesn't require determination of cost function gradients, which is not trivial task as itself.

Eventually at the 4th stage it is necessary to evaluate the actual performance of designed system.

It could be done analytically with the help special program, by evaluation of all components of PRI (11) without the weighting coefficients, and experimentally on the basis of simulation.

In the simulation procedure it is possible to use some inevitable non-linear functions (such as saturation of actuator, dead zones etc.) in combination with linear or full nonlinear model of UAV.

If some state space variables don't satisfy the required specifications and are unreasonably large, it is necessary to increase corresponding coefficients in the weighting matrix **Q** in expression (10) and to execute the optimization procedure again.

The same situation occurs, if it is necessary to diminish H_2- or H_∞-norms of nominal or parametrically disturbed model; in this case it is necessary to change corresponding weighting coefficients $\lambda_\infty, \lambda_\infty^p, \lambda_d, \lambda_d^p$ etc. and to execute optimization procedure with new coefficients again.

Therefore this optimization procedure has to be repeated several times until appropriate values of separate components of PRI (11) are reached.

Case study

Consider the altitude-hold mode for small UAV with the following parameters [4]:

- cruise speed U_0=250 km/hr,
- altitude H_0 = 2 km;
- maximum take-off weight MTO W=146 kg;
- moment of inertia J_{zz}= 124 kg·m^2;
- wings area S = 1,84 m^2;
- mean aerodynamic hord $\bar{c}$ = 0,51 m.

Control surfaces are: elevator and ailerons only. Navigation system provides the longitudinal channel of autopilot with three sensors: altitude, pitch angle and rate.

The control law has the following form:

$$\delta e(z) = W_a(z)\left[W_h(z), K_\vartheta, K_q\right]\left[h, \vartheta, q\right]^T;$$

$$W_a(z) = 1 + \frac{T_{d\vartheta q}}{T_s z}(z-1);$$

$$W_h(z) = K_h + \frac{T_{dh}}{T_s z}(z-1),$$

where δe – the deflection of elevator; K_ϑ, K_q, K_h – pitch angle, rate and altitude gained respectively; $T_{d\theta q}$, T_{dh} – time constants of the 1st difference elements in the angular and altitude circuits.

Vector of adjustable parameters of autopilot $\vec{\mathbf{C}}_\mathbf{n}$, which has to be determined from optimization procedure (14), has the following components:

$$\vec{C}_n = [K_\vartheta,\ K_q, K_h, T_{d\theta q}, T_{dh}]. \qquad (15)$$

Parametrically perturbed model occurs, when the true air speed decreases to value 200 km/hr. UAV longitudinal dynamics state space models corresponding to the nominal and perturbed cases have the following form:

$$\mathbf{A} = \begin{bmatrix} -0,0345 & 6 & -9,78 & 0 & 0 \\ -0,0041 & -1,76 & 0 & 0,99 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0,0033 & -25,7 & 0 & -2,19 & 0 \\ 0 & -69,4 & 69,4 & 0 & 0 \end{bmatrix};$$

$$\mathbf{B} = \left[0,36\ \ -0,16\ \ 0\ \ -31,1\ \ 0\right]^T;$$

$$\mathbf{A_p} = \begin{bmatrix} -0,0273 & 6 & -9,78 & 0 & 0 \\ -0,0064 & -1,76 & 0 & 1 & 0 \\ 39\ \ 0 & 0 & 0 & 1 & 0 \\ 0,0036 & -16,1 & 0 & -1,73 & 0 \\ 0 & -55,6 & 55,6 & 0 & 0 \end{bmatrix};$$

$$\mathbf{B_p} = [0.36\ \ -0.13\ \ 0\ \ -19.9\ \ 0]^T,$$

where matrices for perturbed case are provided with subscript "p".

The performance-robustness index would have the following form:

$$J = \lambda_{0d} J_{0d} + \lambda_{0s} J_{0s} + \lambda_{pd} J_{pd} + \lambda_{ps} J_{ps} +$$

$$+ \lambda_\infty T_\infty + \lambda_{p\infty} T_{p\infty}, \qquad (16)$$

where J_{0d}, J_{0s} stand respectively for deterministic and stochastic partial PI of nominal system, $\lambda_{0d}, \lambda_{0s}$ stand for corresponding weight factors, while the same symbols with subscripts "p" stand for the same values of perturbed system. Symbols $T_\infty (T_{p\infty})$ and $\lambda_\infty (\lambda_{p\infty})$ denote H_∞-norms for complementary sensitivity functions and weight factors for nominal and perturbed systems. After few tentative executions of optimization procedure these parameters were chosen as

$$\lambda_{0d} = \lambda_{pd} = 1,2;$$

$$\lambda_{0s} = \lambda_{ps} = 10;$$

$$\lambda_\infty = \lambda_{p\infty} = 0,4.$$

Parameters of PF were chosen as

$$R_1 = 0,9999;$$

$$R_2 = 0,0005.$$

Using PRI (16) and PF (12) with aforementioned parameters, optimization procedure determined the following vector (15) of control law's parameters:

$$\vec{\mathbf{C}}_\mathbf{n} = [-9,2\ -1 - 0,05\ 0,14\ 0,008].$$

Numerical characteristics of nominal and perturbed systems are represented in the tab. 1, 2.

Table 1

Comparison of characteristics for nominal and perturbed plant of the stochastic model

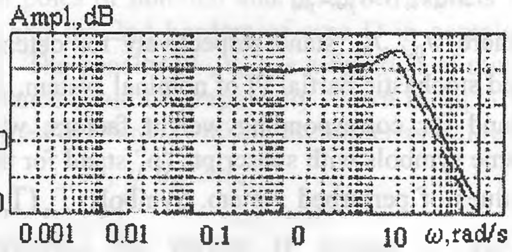
Plant	R.m.s. of state space variables				
	α, rad	ϑ, rad	Q, rad·s^{-1}	h, m	δe, rad
N	0,0007	0,0034	0,003	1,07	0,0017
P	0,0009	0,0063	0,0035	1,39	0,0019

Table 2

Comparison of characteristics for nominal and perturbed plant of the deterministic model

Plant	Stability margin		H_2	H_∞
	Phase (deg)	Ampl. (dB)		
N	145	20	1,59	0,43
P	152	23	1,15	0,29

As it follows from this table r.m.s. of state variables in stochastic case are varying within reasonable limits, whereas such integral characteristics as H_2-and H_∞-norms, are defined by expressions (2), (3) and phase stability margins are varying in a vary small limits. Amplitude Bode plots of closed loop systems (fig. 4) are very flat, thus proving robustness of system. The fig. 4 is the nominal(solid line) and the perturbed (dotted line) closed-loopsystem.



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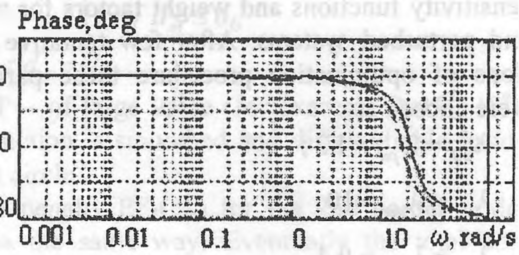


b

Fig. 4. Bode plots of closed loop systems:
a – Magnitude Bode plot; *b* – Phase Bode plot

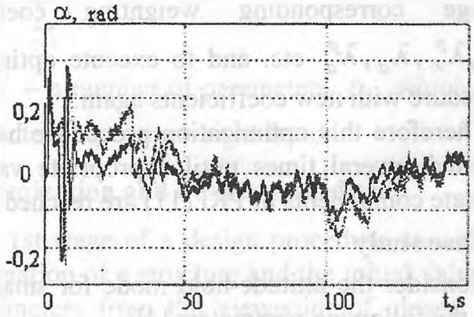
Transient processes in nominal and perturbed systems, which were simulated taking into account all nonlinear functions inherent to the live autopilot as well as the influence of the random wind, simulated according to the standard Dryden model of turbulence [1]. Results of the simulation are shown at fig. 5.

These figures along with numerical results, represented in tab. 1, 2 show that desired robustness-performance trade-off is achieved.

Conclusions

1. H2/H_∞-optimization procedure for digital control systems permits to obtaine the flight control law, which could be directly implemented in the airborne computer without any additional firstly the continuous systems has to be designed and adjustments. These adjustments are inevitable when its conversion to the digital form must be done at the 2nd step.

2. Proposed optimization procedure permits to achieve desirable compromise between performance and robustness of the FCS, when essential variations of the aircraft's dynamic parameters due to the variations of the state of rest true airspeed doesn't affect essentially the performance of system.



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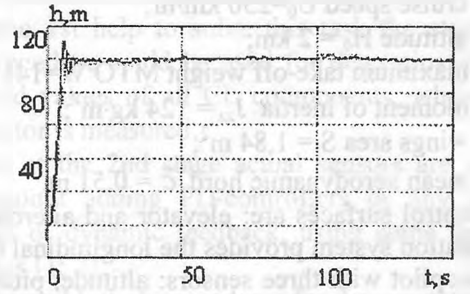

b

Fig. 5. Transient processes in the digital control systems in the altitude-hold mode:
a – angle of attack; *b* – altitude

3. This procedure is used to be performed several times. Each running of this procedure is made with certain values of all weight coefficients of composite PRI (10). The values of control law's parameters then are used for the evaluation of all characteristics of the closed loop control system under deterministic and stochastic disturbances as well as for the simulation of this system under the same disturbances. If the deflections of the state-space variables under deterministic disturbances (or their r.m.s. under stochastic disturbances) of the controlled plant are tolerable from the viewpoint of the system's performance, these adjustable parameters must be used in the actual controller. Otherwise, it is necessary to increase in PRI (16) certain weight coefficients corresponding to the state-space variables, which have

intolerably large values, and to repeat this procedure again until these values are restricted inside the desirable tolerances.

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Параметрична робастна оптимізація цифрових систем керування польотом

Запропоновано метод досягнення компромісу між робастністю і якістю систем керування при номінальних і параметрично збурених моделях об'єкта в детермінованому і стохастичному випадках. Для вирішення цієї задачі використано багатомодельний H_2/H_∞ підхід робастної оптимізації. У вітчизняній та зарубіжній літературі можна знайти застосування цього багатомодельного H_2/H_∞ підходу робастної оптимізації для неперервних систем. Розглянуто приклад використання для дискретної моделі робастної оптимізації повздовжнього каналу малого безпілотного літального апарата.

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Параметрическая робастная оптимизация цифровых систем управления полетом

Предложен метод достижения компромисса между робастностью и качеством систем управления при номинальных и параметрически возмущенных моделях объекта в детерминированном и стохастическом случаях. Для решения этой задачи используется многомодельный H_2/H_∞ подход робастной оптимизации. В качестве примера рассмотрено применение робастной оптимизации продольного канала малого беспилотного летательного аппарата для дискретной модели.