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MODELING OF SOUND RADIATION BY A BEAM

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An analytical formulation and numerical analysis are presented for the vibration and following noise radiation of a simply supported beam. The analytical solution was found in a form that is appropriate to describe the control of the beam vibration using active structures. The modeling approach can be used for forming of feedback control of the system of vibration loads by means of distribution of the actuators along the beam length.

Introduction

Extensive literature is devoted to investigations of optimum control of the oscillations of elastic beams and plates [1; 2]. Active damping for the flexible distributed systems was introduced more then 40 years ago [3; 4]. These researches are included now into the general theory of optimum control. In recent years a number of researches have been carried out for active structural control with usage of sensors and actuators as elements of these control systems. In given article a one-dimensional problem of the oscillations of a beam for criteria based on the minimal acoustic radiation is considered. The solution of this problem provides an ideal control system with maximum active damping of the beam vibration and minimum of correspondent sound emission.

Theoretical grounds of the problem

A simply supported beam has been exemplified to demonstrate a new method of multi-criteria optimization. We consider the sound radiation of a beam of finite length L which is simply supported at its ends, so we may consider the displacement and acceleration of the oscillation at these points like absent:

$$u(0)=\frac{\partial^2 u(0)}{\partial x^2}=0;\qquad u(L)=\frac{\partial^2 u(L)}{\partial x^2}=0. \tag{1}$$

The solution of the problem is defined by solving of Helmholtz equation and equation of transverse motion of the beam (for harmonic waves):

$$\Delta p+k^2p=0;$$

$$\frac{d^4u}{dx^4}-k_f^4u=\frac{F+M}{EI};$$

$$\frac{\partial p}{\partial z}=\rho\omega^2 u(x)\quad \text{for } z=0, \tag{2}$$

$$k_f^4=\frac{\omega^2\rho_s S}{EI};$$

where F is a load per unit of beam length;

$$F=\overline{F}_F\exp(i\varphi_F)\delta(x-x_F)+\sum_{j=1}^{M}F_j\exp(i\varphi_j)\delta(x-x_j);$$

φ_j is a phase of j-th mode; M – is the sum of force's moments:

$$M=\sum_{k=1}^{K}M_k\exp(i\psi_k)\delta'(x-x_k);$$

ψ_k is a phase of k-th force's moment; $\delta'(x)$ is a derivative of Dirac function; ω is an angular frequency; E is a Young's modulus; I is a moment of inertia of cross section S of the beam.

For second equation in (2) the finite Fourier transformation may be used:

$$u(n)=\int_0^L u(u)\sin\left(\frac{n\pi x}{L}\right)dx;$$

$$F(n)=\int_0^L F(x)\sin\left(\frac{n\pi x}{L}\right)dx.$$

A general solution of the equation for with boundary conditions (1) can be obtained $u(x)$ in form of the following expansion:

$$u(x)=-\frac{2}{\rho_s SL}\sum_{n=1}^{\infty}\frac{\sin\left(\frac{n\pi x}{L}\right)}{\left(\omega^2-\omega_n^2\right)}\left[\overline{F}_F\exp(i\varphi_F)\sin\left(\frac{n\pi x_F}{L}\right)+\sum_{j=1}^{M}F_j\exp(i\varphi_j)\sin\left(\frac{n\pi x_j}{L}\right)-\sum_{k=1}^{K}M_k\frac{n\pi}{L}\exp(i\psi_k)\cos\left(\frac{n\pi x_k}{L}\right)\right], \tag{3}$$

where modal frequencies of the beam oscillations are defined as:

$$\omega_n = \sqrt{\frac{IE}{\rho_s S}\left(\frac{n\pi}{L}\right)^2}.$$

For calculation of the acoustic field around the beam a model of plane piston, which is set in an infinite rigid baffle may be proposed.

For the vibrating plane piston following conditions on the beam

$$\frac{\partial p}{\partial z} = \rho\omega^2 u(x)$$

and on the baffle

$$(z=0)\ \frac{\partial p}{\partial z} = 0$$

must be fulfilled. In this case exact solution for acoustic pressure can be presented in a form [5]

$$p(x, y, 0) = -\frac{\rho\omega^2}{2\pi}\int_0^L\int_0^b \frac{\exp(ikr)}{r} u(x_0)dx_0dy_0\,, \qquad (4)$$

where

$$r = \sqrt{(x-x_0)^2 + (y-y_0)^2}\ .$$

Equation (4) gives an exact solution of the task under consideration in terms of the prescribed displacement distribution of the beam. Acoustical power of the beam in this case is specified by

$$W = \frac{1}{2}\mathrm{Re}\int_0^L\int_0^b p(x, y, 0)w^*(x)dxdy\,, \qquad (5)$$

where

$$w(x) = -i\omega u(x)\,.$$

Sound power level can be written

$$L_W = 10\lg\frac{W}{W_0}\,,$$

where

$$W_0 = 10^{-12}\ \mathrm{Wt}.$$

Equation (4) may be used as a functional to minimize of an acoustical power of the beam.

Influence of the load location

The load value and its position on a beam are the main parameters, influencing on noise radiation by a beam without actuators, which may used for decreasing of this noise.

For a simply supported steel beam with length 0,61 m, width 0,051 m, and thickness 0,00635 m, the noise spectra in dependence with load (equal to F_F = 1 in every case) location are shown in fig. 1. One may see that not only the level of noise, but significance of the particular frequency modes may change with load location. For example, for the load position in center of the beam, X_F = 0,5, first, third and fifth modes are significant only.

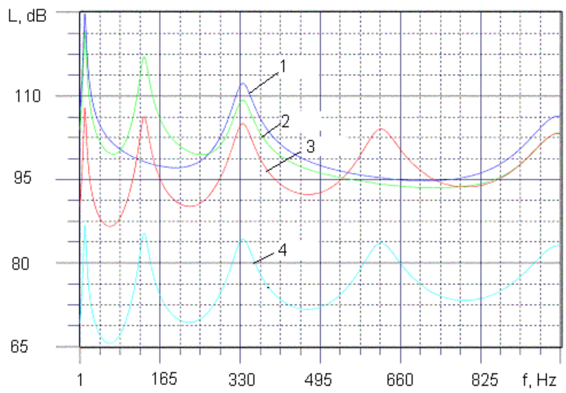

Fig. 1. Noise spectra of the beam radiation in dependence with load location:
1 – X_F = 0,5; *2* – X_F = 0,75; *3* – X_F = 0,95; *4* – X_F = 0,995

For the load position in X_F = 0,75, besides them a second mode is quite dominant, and so on.

Spectra of displacement of the bean in oscillation (fig. 2) show that the first mode is a dominant always, for it the calculated values are following: 0,000204, 0,000102, 0,000005 and less then 0,000001 m for the locations X_F = 0,5, X_F = 0,75, X_F = 0,95, and X_F = 0,995 accordingly.

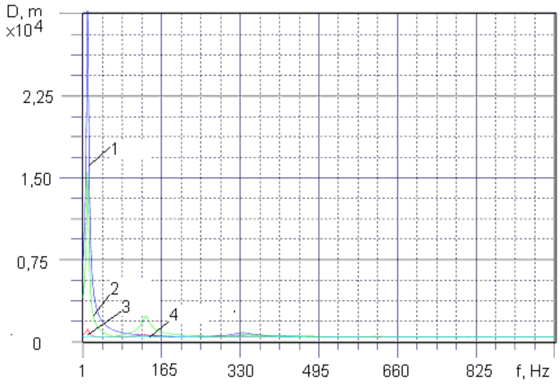

Fig. 2. Displacement spectra of the beam oscillation in dependence with load location:
1 – X_F = 0,5; *2* – X_F = 0,75; *3* – X_F = 0,95; *4* – X_F = 0,995

For most of the cases the results show that first four modes of the beam oscillations are enough to be considered for the noise radiation investigation. For example, the set of sound pressure levels the first seven modes for beam with load in the center is shown (Total SPL = 124,28 dB) in table

First seven modes of the beam oscillation

Mode frequen-cy, Hz	Angular frequency of the mode	Displace-ment, m× 10^4	Sound pres-sure, Pa× 10^{-4}	SPL, dB
40,15	252,2	2,04	0,58897	123,9
160,61	1009,1	0,01	0,05445	98,58
361,37	2270,5	0,03	0,36950	111,9
642,43	4036,5	< 0,01	0,09447	95.80
1003,8	6307,0	< 0,01	0,31765	106,3
1445,5	9082,1	< 0,01	0,12084	93,7
1967,4	12361,8	< 0,01	0,25249	102,5

Influence of number of loads and their location

Load on the beam may be presented in distributed form along the beam.

Influence of the load distribution was investigated using the same model (3)–(5) in comparison with the case of the single load located in a center of the beam.

The sum of distributed loads always was equal to 1, so to the value of point-located load. In the case of the distributed loads the displacement of the beam oscillation becomes less and noise radiation – less efficient too (fig. 3).

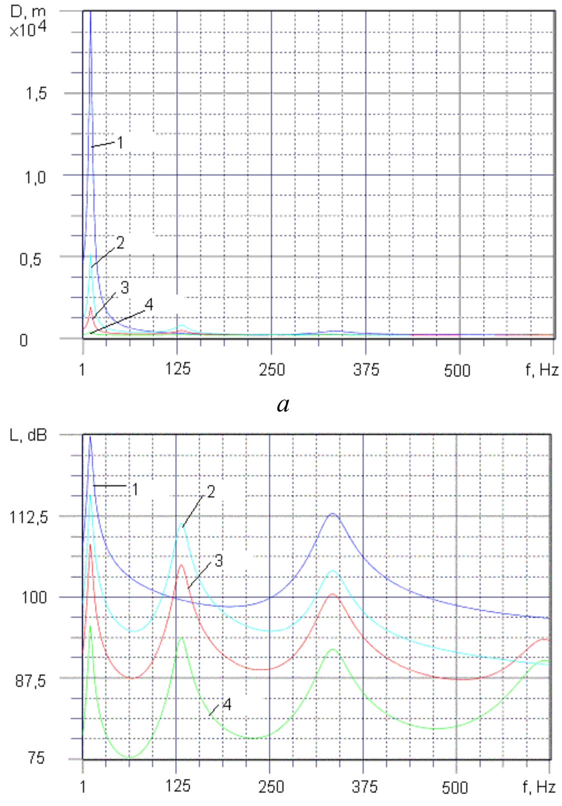

Fig. 3. Number of loads influence on displacement (*a*) and noise (*b*) spectra of oscillated beam:
1–1 load; *2*–2 loads; *3*–4 loads; *4*–9 loads

In these cases the distances between the loads were equalized, for example for 9 loads $\Delta X_F = 0{,}1111$, for 4 loads $\Delta X_F = 0{,}2$.

Efficiency of the noise radiation may rise if to minimize the total distance of the range of distributed loads' location. For example in fig. 4 the cases for 4 distributed loads are analyzed. They are located in ranges $X_1 \ldots X_4$ in following way: 0,2…0,8; 0,4…0,6; 0,47…0,53; 0,485…0,515; 0,496…0,504 (it means that in first case $\Delta X_F = 0{,}2$ m and in last case $\Delta X_F = 0{,}02$). Last a spectrum for $\Delta X_F = 0{,}02$ in range 0,496…0,504 is very similar in form to the spectrum with single load (fig. 1 or fig. 3, *b*, but less noisy).

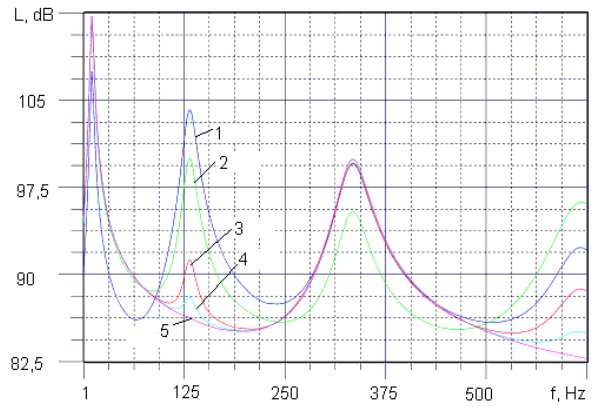

Fig. 4. Influence of the distance range of loads location for 4 distributed loads:
1 – 0,2…0,8; *2* – 0,4…0,6; *3* – 0,47…0,53; *4* – 0,485…0,515; *5* – 0,496…0,504

This effect o a number of loads on noise efficiency is shown in fig. 5 – forms of the spectra are the same, but if the number of the loads is more, the SPL of their noise is less.

All the distributed loads located inside the length range 0,496 – 0,504 m.

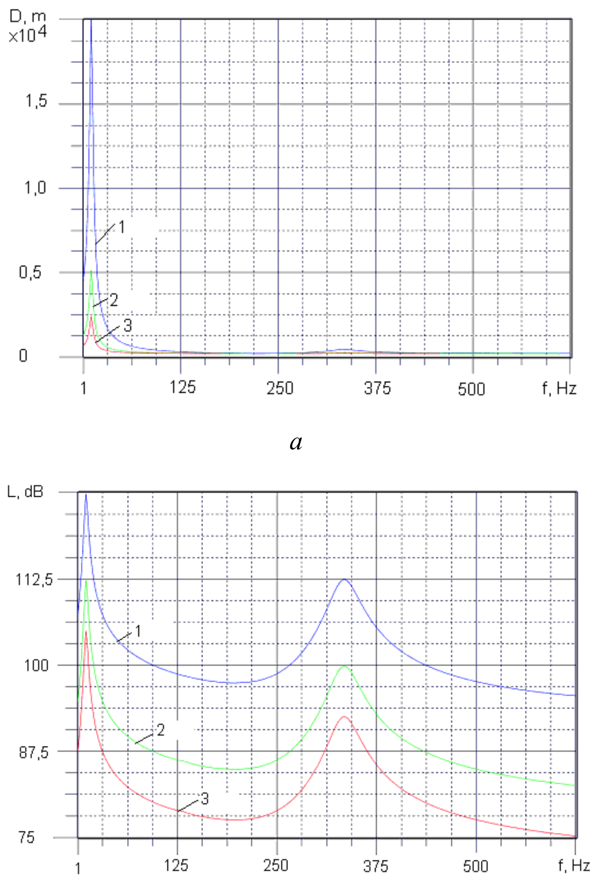

Fig. 5. Number of loads influence on displacement (*a*) and spectra of noise (*b*) radiated by a beam:
1–1 load; *2*–9 loads; *3*–4 – loads

Influence of number of actuators and their location for 1 load

The model (3)–(5) was used for analysis of the actuator influence on radiated noise by a beam. In all cases the single load equal to 1 in a center of the beam was investigated. A simple analysis of the expression for displacement (3) show that if an actuator to locate in same place as a load, but with delayed phase on a π, the noise and oscillation of the beam must be absent, please see in fig. 6 a green spectra. All the distributed actuators located inside the length range 0,496 – 0,504 m.

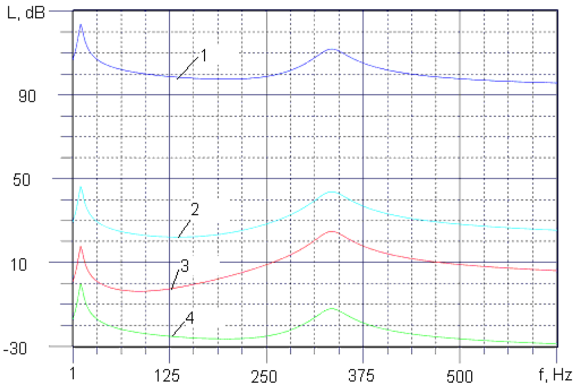

Fig. 6. Number of influence on SPL spectra of noise radiated by a beam with 1 load:
1 – without actuators (total SPL = 124 dB); *2*–1 actuator (total SPL = 0.0 dB); *3*–2 actuators (total SPL = 25,6 dB); *4*–9 actuators (total SPL = 48,3 dB)

But if an actuator will be distributed its efficiency becomes less in dependence of the number of actuator forces (fig. 6). Efficiency becomes less if the location of the actuator or its phase are shifted from the optimum values.

Conclusion

New model for oscillation and noise radiation of a simply supported beam was defined. Results of its analysis show that it is appropriate for optimization of the actuator location depending of character of the load and actuator

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Моделювання випромінювання шуму балкою

Наведено аналітичний розв’язок та його числовий аналіз для визначення вібрації та випромінювання створюваного нею шуму простою закріпленою з обох боків балкою. Розв’язок визначено у формі, сприятливій для обґрунтування управління вібрацією балки з використанням активних методів гасіння. Отриманий розв’язок може бути застосований для обґрунтування управління системою навантаження на балку, яка створює вібрацію, розподіленими вздовж балки актуаторами у вигляді сил і моментів.

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Моделирование излучения шума балкой

Приведено аналитическое решение и его численный анализ для определения вибрации и излучения создаваемого ею шума простой закрепленной с обеих сторон балкой. Решение определено в форме, благоприятной для обоснования управления вибрацией балки с использованием активных методов гашения. Полученное решение может быть применен для обоснования управления системой нагрузки на балку, создающей вибрацию, распределенными вдоль балки актуаторами в виде сил и моментов.