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# FEATURE DETECTION FOR REALISTIC IMAGES BASED ON B-SPLINES OF 3rd ORDER RELATED TO INTERPOLAR ON AVERAGE

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***Abstract***

***Purpose:*** *One approach where recognition of objects in a digital image is based on a search for special points of the function of two variables model intensity lighting. As a model proposed to use a two-dimensional polynomial spline-based B-splines of order. For linear operators that determine the characteristics of a digital image is necessary to conduct appropriate studies partial derivatives of first and second order spline said.* ***Methods:*** *Active obtaining explicit views of partial derivatives and study of their standards and quality of approximation, further defining particular cases suitable for implementation in the software.* ***Results:*** *The paper proposes a calculation of differentials and their partial cases for two-dimensional polynomial spline based on B-splines of third order. Based on these, analogs of known operators that determine the local characteristics of digital images were proposed.* ***Discussion****: Further research may be to obtain similar operators based on combinations of two-dimensional B-spline above the second order analysis capabilities and their application to problems of digital imaging and video.*

**Keywords:** asymptotic; B-spline; convolution; digital image; interpolation; kernel; norm of spline operator; operator; polynomial.

## 1. Formulation of the problem

Determining the local features of digital images (DI) are an integral part of procedures for computing and object recognition, photogrammetry, orthorectification data of aerial reconnaissance, tracking goals on digital video, and more.

Talking about the features of DI mean local, low-level features which are not associated with spatial relations - the edges of objects, curvature which is submitted a speed change light intensity towards the edge - that is, those, which are relatively invariant to scaling, rotation and partly to changes regarding the observation point and the intensity of the image.

They are well localized in both spatial and frequency domain, resistant to noise, and individual feature allows you to search for correlations characteristics of objects presented in different scenes images [1].

In general, the mathematical basis of search features is the wide analysis of partial derivatives of first and second order, which are ported-based on model of continuous image. Convolution of discrete counterparts from DI is often used in software like Roberts and Previta operators [2]. Another widely used approach is to analyze the original model image after convolution with Gaussian function [3 - 8]:

$$L(t,q,g)=\int_{(\zeta,\eta)} p(\zeta,\eta)G(t-\zeta,q-\eta,g)d\zeta d\eta \quad (1)$$

where

$$G(t,q,g)=\frac{1}{2\pi g}\exp\left\{-(t^2+g^2)/(2g)\right\}. \quad (2)$$

Functions (1) $L(t,q,g)$ represent the same information, as the output image $p(t,q):R^2\to R$ but at different zoom levels $g=\sigma^2$. To calculate the Gauss derivative function using the formula:

$$L_{t^\alpha q^\beta}(t,q,g)=\frac{\partial^{\alpha+\beta}L(t,q,g)}{\partial t^\alpha \partial q^\beta}=\left(\frac{\partial^{\alpha+\beta}G(t,q,g)}{\partial t^\alpha \partial q^\beta}\right)*p(t,q).$$

An alternative to the smoothed image model based on Gaussian model can be the model based on linear combinations of *B*-splines related to interpolar on average [9].

Having actually similar properties in the frequency domain as a Gauss function, *B*-splines are easier to calculate and allow to build a image model which by analytical form provides the possibility of partial derivatives on which we might to build a high-speed invariant under rotations and zoom of operators which search DI features.

By analogy with [10] we present the researches of partial derivatives of the local polynomial spline based on *B*-splines in the third row, which can be used in the processing of digital images.

## 2. Analysis of the research and publications

Let's consider that the image plane is defined by *T* and *Q* axis. Equal subintervals of length *h>0* are generated along *T* and *Q* axis, so we have a sequence of equally-spaced points $\Delta h, h: t_i = ih, q_j = jh$, $i = \overline{0, H-1}$, $j = \overline{0, W-1}$, where *H* and *W* –linear dimensions of digital image. Let's consider $\phi(t,q)$ to be a function of the impulse response of system, which registers $p(t,q)$ – two-dimensional light intensity function (analogue image). Then, because of technical properties of registration system, the result of convolution of $p(t,q)$ with response function will be:

$$(p*\phi)(ih, jh) = \frac{1}{h^2} \int_{ih}^{ih+h} \int_{jh}^{jh+h} p(t,q) \times$$

$$\times \phi(t-ih, q-jh) dt dq = \overline{p_{i,j}},$$

So we can represent the values of intensity of a digital image in a form:

$$p_{i,j} = \overline{p_{i,j}} + \varepsilon_{i,j}, \ i = \overline{0, H-1}, \ j = \overline{0, W-1}, \quad (3)$$

where $\varepsilon_{i,j}$ is an observation error. So when we have a model of an image in a form of (3) we must use approximations which consider both random nature of data and technical properties of registration system such as operators that are related to the interpolar on average. In [11] for approximation of function $p(t,q)$ with values such as (3) in knots $\Delta h, h$ linear combinations of *B*-splines are given. For example, spline operator based on *B*-spline of 3[nd] order is[12]:

$$S_{3,0}(p,q,q) = \sum_{i \in Z} \sum_{j \in Z} p_{i,j} B_{3,h}(t-ih) B_{3,h}(q-jh), \quad (4)$$

where

$$B_{3,h}(t) = \frac{1}{48} \begin{cases} 0, & t \notin [-2h; 2h], \\ (4 + 2t/h)^3, & t \in [-2h; -h], \\ -3(2t/h)^3 - 12(2t/h)^2 + 32, & t \in [-h; 0], \\ 3(2t/h)^3 - 12(2t/h)^2 + 32, & t \in [0; h], \\ (4 - 2t/h)^3, & t \in [h; 2h], \end{cases}$$

Based on the assumption that the basis of *B*-splines are the Rice basis and from the fact that the fundamental splines based on *B*-splines [12] head to zero exponentially fast while removing from the local area of $(i, j)$ -term of approximation we can say that the use of two-dimensional local polynomial splines based on *B*-splines which are related to interpolar on average as a model DI, is acceptable.

For an image $p(t,q)$ we can choose a model in a form of (4). This evaluation can be asymptotically accurate under certain conditions. In particular, if $p(t,q) \in C^{2,2}$, $|\varepsilon_{i,j}| < \varepsilon$, $i, j \in Z$ and $\forall \varepsilon > 0$, we can get the estimation [11]:

$$\left\| p(t,q) - S_{3,0}(p,t,q) \right\|_C \le \frac{5h^2}{24} \left\| p''_{t^2}(t,q) \right\|_C +$$

$$+ \frac{5h^2}{24} \left\| p''_{q^2}(t,q) \right\|_C + \frac{25h^4}{576} \left\| p^{(4)}_{t^2 q^2}(t,q) \right\|_C +$$

$$+ \varepsilon \cdot \left\| p(t,q) \right\|_C + o(h^4). \quad (5)$$

If select the expression (4) or any other combination of *B*-splines of this type as an approximation of the image:

$$S_{r,0}(p,q,q) = \sum_{i \in Z} \sum_{j \in Z} p_{i,j} B_{r,h_t}(t-ih) B_{r,h_q}(q-jh),$$

$$r = 2,3,..., \quad (6)$$

we'll get a model with properties of pulse low non-recursive filter [13]. In particular, in [14], is given the evidence that, as a function of Gauss, any *B*-spline above the first order can be used to determine Fourier transform, so if there is a need to obtain a digital low DI filter, it is sufficient to determine the value of spline nodes in the partition [15] in model (6).

If we use substitutions:

$$x = \frac{2}{h}(t - ih), \ |x| \le 1, \ y = \frac{2}{h}(q - jh), \ |y| \le 1, \quad (7)$$

we can represent (4) in a form:

$$S_{3,0}(p,t,q) = \frac{1}{2304}((1-x)^3 (1-y)^3 p_{i-1,j-1} +$$
$$+ (1-x)^3 (23 - 15y - 3y^2 + 3y^3) p_{i-1,j} +$$
$$+ (1-x)^3 (23 + 15y - 3y^2 - 3y^3) p_{i-1,j+1} +$$
$$+ (1-x)^3 (1+y)^3 p_{i-1,j+2} +$$
$$+ (23 - 15x - 3x^2 + 3x^3)(1-y)^3 p_{i,j-1} +$$
$$+ (23 - 15x - 3x^2 + 3x^3)(23 - 15y - 3y^2 + 3y^2) p_{i,j} +$$
$$+ (23 - 15x - 3x^2 + 3x^3)(23 + 15y - 3y^2 - 3y^2) p_{i,j+1} +$$
$$+ (23 - 15x - 3x^2 + 3x^3)(1+y)^3 p_{i,j+2} +$$
$$+ (23 + 15x - 3x^2 - 3x^3)(1-y)^3 p_{i+1,j-1} +$$

$$+(23+15x-3x^2-3x^3)(23-15y-3y^2+3y^3)p_{i+1,j}+$$
$$+(23+15x-3x^2-3x^3)(23+15y-3y^2-3y^3)p_{i+1,j+1}+$$
$$+(23+15x-3x^2-3x^3)(1+y)^3 p_{i+1,j+2}+$$
$$+(1+x)^3(1-y)^3 p_{i+2,j-1}+$$
$$+(1+x)^3(23-15y-3y^2+3y^3)p_{i+2,j}+$$
$$+(1+x)^3(23+15y-3y^2-3y^3)p_{i+2,j+1}+$$
$$+(1+x)^3(1+y)^3 p_{i+2,j+2}). \qquad (8)$$

Then, if necessary for DI processing, substituting $x=1, y=1$ we can get a linear operator of low-frequency filter:

$$L(p^{i,j}) = S_{3,0}(p,ih,jh) = (p_{i-1,j-1}+4p_{i-1,j}+$$
$$+p_{i-1,j+1}+4p_{i,j-1}+16p_{i,j}+4p_{i,j+1}+p_{i+1,j-1}+$$
$$+4p_{i+1,j}+p_{i+1,j+1})/36, \quad i,j\in Z,$$

or in form of discrete convolution of sequence $p_{i,j}$, $i,j\in Z$ with mask of filter $\gamma$, we can write:

$$L(p^{i,j}) = \sum_{ii=i-1}^{i+1}\sum_{jj=j-1}^{j+1}\gamma_{ii-i,jj-j}\,p_{ii,jj}, \; i,j\in Z,$$

where

$$\gamma = \frac{1}{36}\begin{pmatrix}1 & 4 & 1\\ 4 & 16 & 4\\ 1 & 4 & 1\end{pmatrix}.$$

The main idea of this article is to get partial derivatives of an image model in a form of (7) and operators for extreme points detection.

### 3. Methodology

Using substitutes (6) and expression (7), partial differentials of spline (4) are:

$$S'_{3,0}(p,t,q)_t = \frac{1}{384h}(-(1-x)^2(1-y)^3 p_{i-1,j-1}-$$
$$-(1-x)^2(23-15y-3y^2+3y^3)p_{i-1,j}-$$
$$-(1-x)^2(23+15y-3y^2-3y^3)p_{i-1,j+1}-$$
$$-(1-x)^2(1+y)^3 p_{i-1,j+2}+$$
$$+(-5-2x+3x^2)(1-y)^3 p_{i,j-1}+$$
$$+(-5-2x+3x^2)(23-15y-3y^2+3y^3)p_{i,j}+$$
$$+(-5-2x+3x^2)(23+15y-3y^2-3y^3)p_{i,j+1}+$$
$$+(-5-2x+3x^2)(1+y)^3 p_{i,j+2}+$$
$$+(5-2x-3x^2)(1-y)^3 p_{i+1,j-1}+$$
$$+(5-2x-3x^2)(23-15y-3y^2+3y^3)p_{i+1,j}+$$
$$+(5-2x-3x^2)(23+15y-3y^2-3y^3)p_{i+1,j+1}+$$
$$+(5-2x-3x^2)(1+y)^3 p_{i+1,j+2}+$$
$$+(1+x)^2(1-y)^3 p_{i+2,j-1}+$$
$$+(1+x)^2(23-15y-3y^2+3y^3)p_{i+2,j}+$$
$$+(1+x)^2(23+15y-3y^2-3y^3)p_{i+2,j+1}+$$
$$+(1+x)^2(1+y)^3 p_{i+2,j+2}).$$

$$S'_{3,0}(p,t,q)_q = \frac{1}{384h}(-(1-x)^3(1-y)^2 p_{i-1,j-1}+$$
$$+(1+x)^3(-15-2y+3y^2)p_{i-1,j}+$$
$$+(1+x)^3(15-2y-3y^2)p_{i-1,j+1}+$$
$$+(1+x)^3(1+y)^2 p_{i-1,j+2}-$$
$$-(23-15x-3x^2+3x^3)(1-y)^2 p_{i,j-1}+$$
$$+(23-15x-3x^2+3x^3)(-15-2y+3y^2)p_{i,j}+$$
$$+(23-15x-3x^2+3x^3)(15-2y-3y^2)p_{i,j+1}+$$
$$+(23-15x-3x^2+3x^3)(1+y)^2 p_{i,j+2}-$$
$$-(23+15x-3x^2-3x^3)(1-y)^2 p_{i+1,j-1}+$$
$$+(23+15x-3x^2-3x^3)(-15-2y+3y^2)p_{i+1,j}+$$
$$+(23+15x-3x^2-3x^3)(15-2y-3y^2)p_{i+1,j+1}+$$
$$+(23+15x-3x^2-3x^3)(1+y)^2 p_{i+1,j+2}-$$
$$-(1+x)^3(1-y)^2 p_{i+2,j-1}+$$
$$+(1+x)^3(-15-2y+3y^2)p_{i+2,j}+$$
$$+(1+x)^3(15-2y-3y^2)p_{i+2,j+1}+$$
$$+(1+x)^3(1+y)^2 p_{i+2,j+2}).$$

The following assessment standards and quality of approximation occur to received partial differentials.

Statements 1. For operators $S'_{3,0}(p,t,q)_t$, $S'_{3,0}(p,t,q)_q$ is true

$$\left\|S'_{3,0}(p,t,q)_t\right\|_C = \frac{3}{4}\left\|p(t,q)\right\|_{C,}$$

$$\left\|S'_{3,0}(p,t,q)_q\right\|_C = \frac{3}{4}\left\|p(t,q)\right\|_{C.}$$

Statements 2. If $p(t,q)\in C^{3,3}$, then there is asymptotic equality

$$p'(t,q)_t - S'_{3,0}(p,t,q)_t = \frac{-5h^2}{24} p'''_{t^3}(t,q) - $$
$$-\frac{5h^2}{24} p'''_{tq^2}(t,q) - \frac{25h^4}{576} p^{(5)}_{t^3q^2}(t,q) + o(h^5),$$
$$p'(t,q)_q - S'_{3,0}(p,t,q)_q = \frac{-5h^2}{24} p'''_{t^2q}(t,q) - $$
$$-\frac{5h^2}{24} p'''_{q^3}(t,q) - \frac{25h^4}{576} p^{(5)}_{t^2q^3}(t,q) + o(h^5).$$

Corollary 1. For $\forall p(t,q) \in C^{3,3}$ is true

$$\left\| p'(t,q)_t - S'_{3,0}(p,t,q)_t \right\|_C \le \frac{5h^2}{24} \left\| p'''_{t^3}(t,q) \right\|_C + $$
$$+\frac{5h^2}{24} \left\| p'''_{tq^2}(t,q) \right\|_C + \frac{25h^4}{576} \left\| p^{(5)}_{t^3q^2}(t,q) \right\|_C + $$
$$+\frac{3}{4}\varepsilon \left\| p(t,q) \right\|_C + o(h^5),$$
$$\left\| p'(t,q)_q - S'_{3,0}(p,t,q)_q \right\|_C \le \frac{5h^2}{24} \left\| p'''_{t^2q}(t,q) \right\|_C + $$
$$+\frac{5h^2}{24} \left\| p'''_{q^3}(t,q) \right\|_C + \frac{25h^4}{576} \left\| p^{(5)}_{t^2q^3}(t,q) \right\|_C + $$
$$+\frac{3}{4}\varepsilon \left\| p(t,q) \right\|_C + o(h^5).$$

Discrete analogues $S'_{3,0}(p,t,q)_t$, $S'_{3,0}(p,t,q)_q$, which can be used for image processing, when $x$=1, $y$=1 and $h_t = h_q = 1$, can be represented as:

$$S'_{3,0}(p,t,q)_l = \sum_{ii=i-1}^{i+1} \sum_{jj=j-1}^{j+1} \gamma'_{l,ii-i,jj-j} p_{ii,jj}\ , \qquad (9)$$

where $l = \{t, q\}$;

$$\gamma'_t = \frac{1}{12}\begin{pmatrix} -1 & -4 & -1 \\ 0 & 0 & 0 \\ 1 & 4 & 1 \end{pmatrix};\ \gamma'_q = \frac{1}{12}\begin{pmatrix} -1 & 0 & 1 \\ -4 & 0 & 4 \\ -1 & 0 & 1 \end{pmatrix}.$$

Continuous partial derivatives of second order are defined by expressions:

$$S''_{3,0}(p,t,q)_{t^2} = \frac{1}{96h^2}((1-x)(1-y)^3 p_{i-1,j-1} + $$
$$+(1-x)(23-15y-3y^2+3y^3) p_{i-1,j} + $$
$$+(1-x)(23+15y-3y^2-3y^3) p_{i-1,j+1} + $$
$$+(1-x)(1+y)^3 p_{i-1,j+2} + $$
$$+(-1+3x)(1-y)^3 p_{i,j-1} + $$
$$+(-1+3x)(23-15y-3y^2+3y^3) p_{i,j} + $$
$$+(-1+3x)(23+15y-3y^2-3y^3) p_{i,j+1} + $$
$$+(-1+3x)(1+y)^3 p_{i,j+2} + $$
$$+(-1-3x)(1-y)^3 p_{i+1,j-1} + $$
$$+(-1-3x)(23-15y-3y^2+3y^3) p_{i+1,j} + $$
$$+(-1-3x)(23+15y-3y^2-3y^3) p_{i+1,j+1} + $$
$$+(-1-3x)(1+y)^3 p_{i+1,j+2} + $$
$$+(1+x)(1-y)^3 p_{i+2,j-1} + $$
$$+(1+x)(23-15y-3y^2+3y^3) p_{i+2,j} + $$
$$+(1+x)(23+15y-3y^2-3y^3) p_{i+2,j+1} + $$
$$+(1+x)(1+y)^3 p_{i+2,j+2}).$$

$$S''_{3,0}(p,t,q)_{q^2} = \frac{1}{96h^2}((1-x)^3(1-y) p_{i-1,j-1} + $$
$$+(1-x)^3(-1+3y) p_{i-1,j} + $$
$$+(1-x)^3(-1-3y) p_{i-1,j+1} + $$
$$+(1-x)^3(1+y) p_{i-1,j+2} + $$
$$+(23-15x-3x^2+3x^3)(1-y) p_{i,j-1} + $$
$$+(23-15x-3x^2+3x^3)(-1+3y) p_{i,j} + $$
$$+(23-15x-3x^2+3x^3)(-1-3y) p_{i,j+1} + $$
$$+(23-15x-3x^2+3x^3)(1+y) p_{i,j+2} + $$
$$+(23+15x-3x^2-3x^3)(1-y) p_{i+1,j-1} + $$
$$+(23+15x-3x^2-3x^3)(-1+3y) p_{i+1,j} + $$
$$+(23+15x-3x^2-3x^3)(-1-3y) p_{i+1,j+1} + $$
$$+(23+15x-3x^2-3x^3)(1+y) p_{i+1,j+2} + $$
$$+(1+x)^3(1-y) p_{i+2,j-1} + $$
$$+(1+x)^3(-1+3y) p_{i+2,j} + $$
$$+(1+x)^3(-1-3y) p_{i+2,j+1} + $$
$$+(1+x)^3(1+y) p_{i+2,j+2}).$$

$$S''_{3,0}(p,t,q)_{tq} = \frac{1}{64h^2}((1-x)^2(1-y)^2 p_{i-1,j-1} - $$
$$-(1-x)^2(-5-2y+3y^2) p_{i-1,j} - $$
$$-(1-x)^2(5-2y-3y^2) p_{i-1,j+1} - $$
$$-(1-x)^2(1+y)^2 p_{i-1,j+2} - $$

$$-(-5-2x+3x^2)(1+y)^2 p_{i,j-1}+$$
$$+(-5-2x+3x^2)(-5-2y+3y^2)p_{i,j}+$$
$$+(-5-2x+3x^2)(5-2y-3y^2)p_{i,j+1}+$$
$$+(-5-2x+3x^2)(1+y)^2 p_{i,j+2}-$$
$$-(5-2x-3x^2)(1-y)^2 p_{i+1,j-1}+$$
$$+(5-2x-3x^2)(-5-2y+3y^3)p_{i+1,j}+$$
$$+(5-2x-3x^2)(5-2y-3y^3)p_{i+1,j+1}+$$
$$+(5-2x-3x^2)(1+y)^2 p_{i+1,j+2}-$$
$$-(1+x)^2(1-y)^2 p_{i+2,j-1}+$$
$$+(1+x)^2(-5-2y+3y^2)p_{i+2,j}+$$
$$+(1+x)^2(5-2y-3y^2)p_{i+2,j+1}+$$
$$+(1+x)^2(1+y)^2 p_{i+2,j+2}).$$

Statements 3. For operators $S''_{3,0}(p,t,q)_{t^2}$, $S''_{3,0}(p,t,q)_{tq}$, $S''_{3,0}(p,t,q)_{q^2}$ is true

$$\left\|S''_{3,0}(p,t,q)_{t^2}\right\|_C=\|p(t,q)\|_C,$$

$$\left\|S''_{3,0}(p,t,q)_{tq}\right\|_C=\frac{9}{16}\|p(t,q)\|_C,$$

$$\left\|S''_{3,0}(p,t,q)_{q^2}\right\|_C=\|p(t,q)\|_C.$$

Statements 4. If $p(t,q)\in C^{3,3}$, then there is asymptotic equality

$$p''(t,q)_{t^2}-S''_{3,0}(p,t,q)_{t^2}=\frac{-5h^2}{24}p^{(4)}_{t^2q^2}(t,q)+o\left(h^4\right),$$

$$p''(t,q)_{tq}-S''_{3,0}(p,t,q)_{tq}=\frac{-5h^2}{24}p^{(4)}_{t^3q}(t,q)-$$

$$\frac{-5h^2}{24}p^{(4)}_{tq^3}(t,q)-\frac{25h^4}{576}p^{(6)}_{t^3q^3}(t,q)+o(h^4),$$

$$p''(t,q)_{q^2}-S''_{3,0}(p,t,q)_{q^2}=\frac{-5h^2}{24}p^{(4)}_{t^2q^2}(t,q)+o\left(h^4\right).$$

Corollary 2. For $\forall p(t,q)\in C^{3,3}$ is true

$$\left\|p''(t,q)_{t^2}-S''_{3,0}(p,t,q)_{t^2}\right\|_C\le\frac{5h^2}{24}\left\|p^{(4)}_{t^2q^2}(t,q)\right\|_C+$$
$$+\varepsilon\|p(t,q)\|_C+o\left(h^4\right),$$

$$\left\|p''(t,q)_{tq}-S''_{3,0}(p,t,q)_{tq}\right\|_C\le\frac{5h^2}{24}\left\|p^{(4)}_{t^3q}(t,q)\right\|_C+$$
$$+\frac{5h^2}{24}\left\|p^{(4)}_{tq^3}(t,q)\right\|_C+\frac{25h^4}{576}\left\|p^{(6)}_{t^3q^3}(t,q)\right\|_C+$$
$$+\frac{9}{16}\varepsilon\|p(t,q)\|_C+o\left(h^4\right),$$

$$\left\|p''(t,q)_{q^2}-S''_{3,0}(p,t,q)_{q^2}\right\|_C\le\frac{5h^2}{24}\left\|p^{(4)}_{t^2q^2}(t,q)\right\|_C+$$
$$+\varepsilon\|p(t,q)\|_C+o\left(h^4\right),$$

Respectively, discrete convolutions of the 3nd order differential operators based on model (4) are:

$$S''_{3,0}(p,t,q)_l=\sum_{ii=i-1}^{i+1}\sum_{jj=j-1}^{j+1}\gamma''_{l,ii-i,jj-j}\cdot p_{ii,jj}\,,\qquad(10)$$

where

$$l=\left\{t^2,q^2,tq\right\};\ \gamma''_{t^2}=\frac{1}{6}\begin{pmatrix}1&4&1\\-2&-8&-2\\1&4&1\end{pmatrix};$$

$$\gamma''_{q^2}=\frac{1}{6}\begin{pmatrix}1&-2&1\\4&-8&4\\1&-2&1\end{pmatrix};\ \gamma''_{tq}=\frac{1}{4}\begin{pmatrix}1&0&-1\\0&0&0\\-1&0&1\end{pmatrix}.$$

Finding extreme points of objects of digital images using operators (8) is based on the assumption that differentiation identifies abrupt changes in intensity on angles and edges of objects. Condition for detection of points, where intensity change is at its maximum, requires to determine local maximums of first partial derivatives. So we can analyze gradient $\Delta S_{3,0}$ of an image using model (4):

$$\Delta S_{3,0}=S'_{3,0,t}+S'_{3,0,q}=$$
$$=\sum_{ii=i-1}^{i+1}\sum_{jj=j-1}^{j+1}gr(3,0)_{ii-i,jj-j}\cdot p_{ii,jj}\,,$$

where

$$gr(3,0)=\frac{1}{6}\begin{pmatrix}-1&-2&0\\-2&0&2\\0&2&1\end{pmatrix}.$$

An alternative choice to 1st order differentiation and finding maximums of 1st derivative is differentiation of 2nd order followed by determining zero-value points of 2nd derivative. In digital image processing Laplace operator or Laplacian with kernel, which is approximated by 2nd order differentials, is commonly used. According to model (4) and operators (9), Laplacian $\Delta^2 S_{3,0}$ based on local polynomial spline $S_{3,0}(p,t,q)$ can be represented as:

$$\Delta^2 S_{3,0} = S''_{3,0,t^2} + S''_{3,0,q^2} =$$
$$= \sum_{ii=i-1}^{i+1} \sum_{jj=j-1}^{j+1} lp(3,0)_{ii-i,jj-j} \cdot p_{ii,jj} \ , \quad (11)$$

where

$$lp(3,0) = \frac{1}{3}\begin{pmatrix} 1 & 1 & 1 \\ 1 & -8 & 1 \\ 1 & 1 & 1 \end{pmatrix}.$$

Except Laplacian $\Delta^2 S_{3,0}$, using model (4) we can define another three differential invariants: gradient magnitude $|\Delta S_{3,0}|$, determinant of the Hessian $\det H_{S_{3,0}}$ and rescaled level curve curvature $k(S_{3,0})$.

In particular, affine covariant blob detector can be represented as minimum and maximum of Hessian's determinant:

$$\det H_{S_{3,0}} = S''_{3,0,t^2} \cdot S''_{3,0,q^2} - (S''_{3,0,tq})^2 , \quad (12)$$

and straight-forward and affine covariant corner detector as minimum and maximum of curve curvature $k(S_{3,0})$:

$$k(S_{3,0}) = (S'_{3,0,t})^2 \cdot S''_{3,0,q^2} + (S'_{3,0,q})^2 \cdot S''_{3,0,t^2} -$$
$$- 2S'_{3,0,t} \cdot S'_{3,0,q} \cdot S'_{3,0,tq} , \quad (13)$$

where operators $S'_{3,0,t}$, $S'_{3,0q}$, $S''_{3,0,t^2}$, $S''_{3,0,q^2}$ and $S''_{3,0,tq}$ are defined in (9) and (10).

To find descriptor of extreme point of digital image for SIFT method [16], using model (4) and operators (9) we can suggest to use following estimations of gradient magnitude $|\Delta S_{3,0}|$

$$|\Delta S_{3,0}| = \sqrt{(S'_{3,0,t})^2 + (S'_{3,0,q})^2} \ , \quad (14)$$

and orientation value $\Theta_{S_{3,0}}$

$$\Theta_{S3,0} = arctg\left(\frac{S'_{3,0,q}}{S'_{3,0,t}}\right). \quad (15)$$

**4. Analysis of computational complexity descriptors**

Application of received descriptors based on spline model (9-10) is appropriate, because as seen from (9-10) all operations are linear, so the computational complexity of the algorithm specific points using detectors (11-14) will be $O(h * w)$, where $h$ i $w$ image size, compared with the use of model (1) containing the exponential complexity in the core (2).

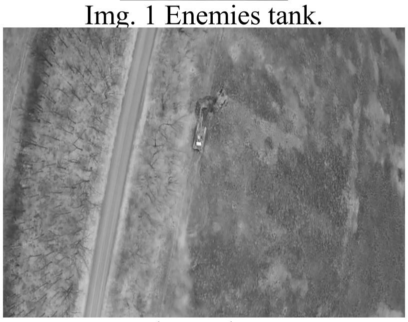
**5. Practical implementation**

All the proposed operators are implemented in software processing DI camera drone. Set an example of practical use imposed by operators. When processing data aerial photography camera drones, which was held in the area ATO [17].

On images are reference image of the enemy tank (Img 1) and a test image (Img.2-3). Note that the use of detectors based on the curvature of the level gradient magnitude, determinants and Hessian Laplacian object target was clearly detected on test frames. The following table (Table 1) shows the performance of the detector by the number of isolated singular points.

Analyzing the results given in the table can be noted that all descriptors for almost the same number of points, which coincided with a slight advantage based on curvature detection level.

Img. 1 Enemies tank.



Img. 2 Enemies tank near to stone.

Img. 3 Enemies tank on the road.

*Table.1*

**Comparing the number of matches singular points for DI descriptors**

| Operator | I | II | III | IV | V |
|---|---|---|---|---|---|
| $k(S_{3,0})$ | 86 | 8044 | 25 | 812 | 10 |
| $\|\Delta S_{3,0}\|$ | 54 | 6059 | 19 | 1787 | 7 |
| $\Delta^2 S_{3,0}$ | 66 | 6695 | 17 | 1716 | 8 |
| $\det H_{S_{3,0}}$ | 77 | 15347 | 21 | 6461 | 7 |

where

I - The number of singular points for Img. 1.

II - The number of singular points for Img. 2.

III - The number of singular points that coincided for Img. 1 and Img. 2.

IV - The number of singular points for Img. 3.

V - The number of singular points that coincided for Img. 1 and Img. 3.

Conducted practical research on real data confirm the adequacy of the proposed linear operators detection of search and allow them to recommend the implementation of the software processing of video cameras purpose unmanned aircraft.

## 6. Conclusion

Clear views were got and the quality of approximation of first and second differentials was explored and identified the cases of partial local two-dimensional polynomial spline based on *B*-splines of third order which is related to interpolar on average, also analogues of known operators for determining the characteristics of the local DI were given.

Unlike the similar operators based on *B*-splines of second order, received operators have an advantage in determining the specific pixel due to the greater degree of smoothing at the same computational complexity, which gives the right to recommend them for implementation in software processing the DI and digital video, including in real time.

Further research may be dedicated towards defining similar operators based on two-dimensional combinations of B-splines of higher orders and analyzing their application in digital image and video processing.

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**П.О. Приставка[1], О.В. Тиводар[2], Б.І. Мартюк[3]**

**Визначення особливостей зображень на основі комбінацій В-сплайнів третього порядку, близьких до інтерполяційних у середньому.**

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**Мета:** Одним із підходів для розпізнавання об'єктів на цифровому зображенні є такий, що базується на пошуку особливих точок функції двох змінних моделі інтенсивності освітлення. У якості моделі пропонується використовувати двовимірний поліноміальний сплайн на основі В-сплайнів третього порядку. Для отримання лінійних операторів, які визначають особливості цифрового зображення необхідно провести відповідні дослідження часткових похідних першого та другого порядків зазначеного сплайну. **Методи:** Здійснюється одержання явних виглядів часткових похідних та дослідження їх норм та якості апроксимації, у подальшому визначенні часткових випадків придатних для реалізації у програмному забезпеченні. **Результати:** Отримано диференціали та їх часткові випадки для двовимірного поліноміального сплайну на основі B-сплайнів третього порядку. На їх основі запропоновано аналоги відомих операторів для визначення локальних особливостей цифрових зображень. **Обговорення**: Подальші дослідження можуть полягати в отриманні аналогічних операторів на основі двовимірних комбінацій B-сплайнів порядку вище другого та аналізу можливостей їх застосування в задачах обробки цифрових зображень та відео.

**Ключові слова:** асимптотика; B-сплайн; згортка; інтерполяція; норма оператору; оператор; поліном; цифрове зображення; ядро.

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**Определение особенностей изображений на основе комбинаций B-сплайнов третьего порядка, близких к интерполяционным в среднем.**

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**Цель:** Одним из подходов для распознавания объектов на цифровом изображении основан на поиске особых точек функции двух переменных модели интенсивности освещения. В качестве модели предлагается использовать двумерный полиномиальный сплайн на основе B-сплайнов третьего порядка. Для получения линейных операторов которые определяют особенности цифрового изображения необходимо провести соответствующие исследования частных производных первого и второго порядков указанного сплайна. **Методы:** Осуществляется получения явных видов частных производных и исследования их норм и качества аппроксимации, в дальнейшем определении частичных случаев пригодных для реализации в программном обеспечении. **Результаты:** Получены дифференциалы и их частные случаи для двумерного полиномиального сплайна на основе B-сплайнов третьего порядка. На их основе предложены аналоги известных операторов для определения локальных особенностей цифровых изображений. **Обсуждение:** Дальнейшие исследования могут заключаться в получении аналогичных операторов на основе двумерных комбинаций B-сплайнов порядка выше второго и анализа возможностей их применения в задачах обработки цифровых изображений и видео.

**Ключевые слова:** асимптотика; B-сплайн; интерполяция; норма оператора; оператор; полином; свертка; цифровое изображение; ядро.

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