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FAST TRANSFORMATION ALGORITHMS OF HAAR

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The below is the structure chart of the tree for the direct and inverse discrete transformation of the signals on the finite interval in the basis of the Haar functions that allow minimizing the resource expenditures for the software or hardware implementation of the algorithm.

Introduction and objectives

The problem of the correct choice of the basis for the direct Fourier transformation (DFT) is the paramount in the theory and the practice of the spectrum signal analysis on finite intervals.

The quality of the signals filtration depends on both the choice of the basis estimated by the level of the side lobe gain-frequency characteristics of the DFT channels and the DFT processor speed.

The processor speed is the most crucial characteristics of the processor as it sets the possibility of the signal spectrum calculation in the real time otherwise in the rate of the selective reading entry. In the harmonic spectrum analysis, the classis basis of the discrete exponential functions (DEF) is traditionally used, the components (phase factors) of which are single-step changing irrational number. The indicated feature of DEF basis gradually reduces the computation speed of the spectrum signal, resulting from the necessity of the complex multiplication accomplishments of the irrational operands (the entry data and basis functions).

In connection with the abovementioned, the problem of the transition to the step function basis the main functions of which are the function systems of Walsh and Haar [1] becomes topical. The step function basis in the defined time interval has the constant value. In the numerical evaluation of the expansion coefficient to the basis functions of Walsh, taking on a value +1 or –1, the unsophisticated operation of adding and subtracting should be carried out other then multiplication. Even if the step function basis on the separate intervals take the positive value, differs from the above mentioned, the multiplication of the function by the continual single for the whole interval coefficient (function system of Haar) is carried out. This is much simpler than the multiplication by the cosine and sinus (as in the DEF basis) which are changing from point to point.

In the literary sources (for e. g.[2; 3]), the matrix methods are viewed as the methods for the calculation of the discrete signal analysis of the spectrum in the basis function of Haar. In this work, the objective arises to develop the pictorial enough and easily realized by the software and hardware means fast translation algorithms of Haar.

The Basic Ratios

In analogue, the expansion of the complex discrete signal $\dot{x}(n)$ on the finite interval N of the Fourier series in DEF basis

$$\dot{x}(n) = \sum_{k=0}^{N-1} \dot{c}_k W^{kn},$$

where $\dot{c}_k$ – expansion coefficient, and $W = \exp(j2\pi / N)$ – phase factor, the signal expansion is carried out $x(n)$, $n = \overline{0, N-1}$, which is supposed to be substantial in the Haar series

$$x(n) = \sum_{k=0}^{N-1} c_k h(k,n), \quad n = \overline{0, N-1}, \qquad (1)$$

where $h(k,n)$ – Haar basis function with the n argument playing the role of the normalized discrete time. The aggregate of the basis functions $h(k,n)$ forms the square matrix of the Haar transformation of the $N \times N$ order. Let us designate the Haar matrix as

$H_N = [h(k,n)]$, $\quad k,n = \overline{0, N-1}$.

The expansion coefficients are defined by the following expressions

$$c_k = \frac{1}{N} \sum_{k=0}^{N-1} x(n) h(k,n), \quad k = \overline{0, N-1}. \qquad (2)$$

The ratios (1) and (2) identify as Haar discrete transformation pair (DTH) at that the (2) defines the direct and (1) – inverse transformation correspondingly.

The aggregate of $\{ c_k \}$ forms the discrete signal spectrum $x(n)$ in the basis of $\{ h(k,n) \}$ of Haar function. The main objective of the spectrum analysis is in the calculation of spectrum signal in the given basis. The matrix of the Haar transformation H_N is defined on the binary interval N, or $N = 2^m$ $m = 1,2,\ldots$, and is formed in accordance to the following rules

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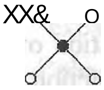

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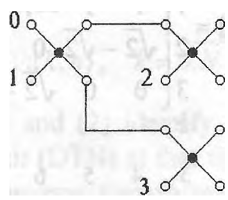

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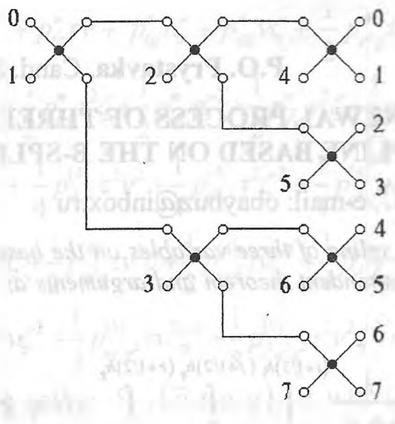

Fig. 5. The tree of the inverse eight-point HFT

Thus, the process of the inverse HFT forming can be presented in the form of the pyramid consisting of m butterfly-like transformations layers. At the i level of the pyramid ($i = \overline{1,m}$, and m is calculated by the ratio $m = \log_2 N$) there are $l_i = 2^{i-1}$ of butterfly operators. The coefficients c_k, which are added to the transformation operators, should be chosen with the weight $\gamma_i = 2^{\frac{i-1}{2}}$, where $i = \overline{1,m}$ is the number of the layer or the transformation phase.

Conclusions

The algorithms for the fast calculations of the Haar transformation were introduced by Andrews [8] and published in Russian [9]. Concerning the structure schemes of HFT [9], the following remarks can be made. First, the column of the inverse Haar transformation has typos Coefficients $y_x(2)$ and $y_x(3)$ should have the multiplier $\sqrt{2}$ which is absent at the picture. Second, the column of the direct HFT does not have any indicators regarding the rules for the data permutations formed by the butterfly-like operators. The underlined remarks are corrected in the offered algorithms of the direct and inverse HFT a can be classified as suitable for technical calculations.

Haar transformation (HT) has local as well as the global sensitivity characteristics. This quality of HT can be demonstrated via turning for instance to the readings $N/4$ coefficients, and correspond to the connection of the four adjacent points and so on. By matrix (6). By analyzing H_8, we can see that $N/2$ of the Haar transformation coefficients $c_k, k = \overline{4,7}$, this, we can define the quality of the local HT sensitivity. The first tow coefficients of HT (c_0, c_1) are the functions of all coordinates in the space of the input orders (the quality of the global sensitivity). For comparison, let's indicate for instance that the discrete transformations of Fourier and Walsh have only global sensitivity qualities.

The indicated conditions leave broad prospective in the use of Haar fast transformation algorithms easily realized by the software and hardware means in solving spectrum analysis problems.

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А.Я. Білецький, О.А. Білецький, А.О. Давлет'янц
Алгоритми швидкого перетворення Хаара
Наведено структурну схему дерева прямого і оберненого дискретного перетворення сигналів на скінченних інтервалах у базисах функцій Хаара, яка забезпечує мінімум ресурсних витрат при програмній або апаратній реалізації алгоритму.

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Алгоритмы быстрого преобразования Хаара
Приведена структурная схема дерева прямого и обратного дискретного преобразования сигналов на конечных интервалах в базисах функций Хаара, обеспечивающая минимум ресурсных затрат при программной или аппаратной реализации алгоритма.