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CORRECTED DETERMINATION OF SKIN DEPTH AT DIFFUSION OF THE PULSED MAGNETIC FIELD INTO MOVING CONDUCTING MEDIUM

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The theory of one-dimensional magnetic field penetration into conducting medium (flat plate) has been developed taking into account the motion of plate under the action of magnetic field at the linear growing of field intensity in the time on the external border of conductor. The process has been described by the equation of diffusion with convection term together with the equation of medium motion under the specific boundary conditions. The solution is obtained for the time interval in which the skin depth is less than thickness of plate. The depth of skin layer has increase in comparison with the classical case of immovable medium. This is a summarized effect of the field increase reduction on the medium surface and of the medium motion.

Introduction

Recently in the paper [1] a phenomenological theory of elastic electro-conducting medium reaction to the penetration of external magnetic field has been developed. A connection between the skin depth of a field penetration and velocity of medium motion (both value and direction) has been shown there.

The analysis of this process can be done in the terms of more general equations of electromagnetic field beyond the simple approach used in the paper [1]. In addition, the time dependence of magnetic field on the surface of medium has not been considered in this work. That is why a more detailed consideration is undertaken in this paper to study the real dependence of the medium motion on the field penetration. The change of the applied external field in the time is obviously able to affect the field diffusion into the medium. More strict analysis with respect to movable medium can show the influence of both sides of electromagnetic induction phenomenon (caused by the time variation of applied field and by the medium motion) on the peculiarities of the field diffusion into depth of medium. Thus, a deeper connection between electromagnetic and mechanical processes can be established in this consideration.

Theoretical model for analysis

Analysis of the process described is done using the model of conducting medium that is shown in fig.1. It is like one in the book [2] fig. 1.

The medium surface is designed as the plane YZ, X-axis is directed into the depth of medium. Instead of the condition about immovable external border of medium which has been used in the book [2], we have supposed the possible motion of this border under the action of electromagnetic force caused by the external magnetic field.

Equation of diffusion for a magnetic field intensity H in the moving medium situated at $x' > 0$ can be written as [3]:

$$\frac{\partial \vec{H}}{\partial t} - \text{rot}\left[\vec{u}\vec{H}\right] = D_M \Delta \vec{H}, \tag{1}$$

where $\vec{u} = \vec{u}(x',t)$ is the field of velocity for the medium, x' is an abscissa in the laboratory co-ordinate system connected with the source of external magnetic field of intensity H: D_M is the coefficient of magnetic field diffusion: $D_M = (\sigma\mu)^{-1}$; σ is the coefficient of electrical conductivity of medium; μ is the magnetic permeability of medium.

It is natural to suppose that magnetic field intensity has only z component:

$$\vec{H} = (0, 0, H_z). \tag{2}$$

Then the electrical field and current density have only y components:

$$\vec{E} = (0, E_y, 0); \tag{3}$$

$$\vec{j} = (0, j_y, 0). \tag{4}$$

Commonly usable physical approach allows to restrict our consideration only by x component of motion velocity for medium u_x, due to assumption about non-deformable medium.

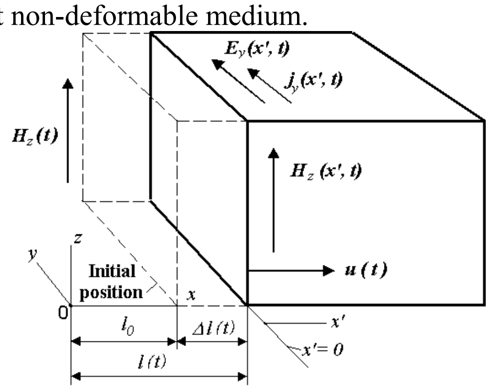

Fig. 1. Schematic view of calculation model (the element of conductor volume under the action of external magnetic field on the surface $x' = 0$)

But in general case of plastic medium it would be necessary to take into consideration the other components of velocity.

The one-dimensional problem is under consideration here with the next simplifications:

$u_x \neq 0$, $u_y = u_z = 0$.

Thus, at such conditions the equation (1) has the view

$$\frac{\partial H_z}{\partial t} + \frac{\partial (u_x H_z)}{\partial x} = D_M \, \Delta H_z$$

or

$$\frac{\partial H_z}{\partial t} = D_M \frac{\partial^2 H_z}{\partial x^2} - u_x \frac{\partial H_z}{\partial x} - \frac{\partial u_x}{\partial x} H_z \qquad (5)$$

i.e. this equation contains, besides time derivative of field intensity, the first and second partial derivatives of the field intensity on the co-ordinate as well as the intensity in zero order.

Now the initial condition and boundary conditions must be written. The presence of medium velocity indicates a taking into account of convective transfer of magnetic field by the moving medium. Let suppose that the magnetic field in the medium is absent at the initial time instant $t = 0$:

$$H_z(x', 0) = 0 . \qquad (6)$$

It is most simple to suppose that the external magnetic field on the surface of medium is growing in the time according to linear correlation

$$H_1 = H_0 \frac{t}{t_0} = v_H \, t , \qquad (7)$$

while the medium is immovable.

In the correlation (7) the magnitude

$v_H = H_0 / t_0$

is the speed of magnetic field intensity increase on the surface of immovable medium measured in A/(m·s). A displacement of the border is considered going in the positive direction of *X*-axis and can be taken into account in the expression of the field intensity on the surface of medium.

Given the interval of length occupied by the magnetic field area outside of conducting medium along the *X*-axis is equal to l_0 at the initial time instant. Taking into account a motion of medium with variable velocity $u(t)$, this interval of length will increase in the time as

$$l(t) = l_0 + \int_0^t u(t') \, dt' . \qquad (8)$$

So, at the time variation of magnetic field for an immovable medium given by (7), magnetic field intensity at the surface of movable medium will have reduced resulting value through an influence of the linear interval increase (8) between this surface and the source of the field:

$$H_z(0,t) = H_0 \frac{t}{t_0} \frac{l_0}{l(t)} .$$

Now the boundary condition can be written as

$$H_z(0,t) = \begin{cases} 0, & t < 0; \\ \dfrac{H_0 \, l_0 \, t}{(l_0 + \int_0^t u(t') \, dt') \, t_0}, & t \geq 0. \end{cases} \qquad (9)$$

So, magnetic content of problem is expressed by equation (5), initial condition (6) and boundary condition (9). As seen in (9) it is necessary to define additionally in the boundary condition the velocity of displacement $u(t)$ for the border of conducting medium. It can be done if we shall consider that a conducting medium has the view of a flat plate of thickness L along the motion direction, with density of matter ρ. The magnetic pressure on the external side of this plate facing the source of magnetic field is equal to

$p_H = \mu H_z^2(0,t)/2$

and can be included into the equation of plate motion in the form [2]:

$$\rho L \frac{\partial u}{\partial t} = \frac{\mu H_z^2(0,t)}{2} . \qquad (10)$$

The next expression can be used as the initial condition for the equation of motion (10):

$u(0) = 0$. (11)

Resulting, the boundary conditions for the problem under consideration include two equations:

– integral expression (9), that gives a dependence of the border field intensity on the geometric parameter, namely, on the velocity of border (and whole medium) motion;

– differential equation (10), that gives an interconnection between the velocity of motion for conducting medium and magnetic pressure on the surface of medium.

After obtaining the magnetic field intensity in the medium $H_z(x',t)$ the induced current density can be found using the Ampere's law:

$$\vec{j} = \mathrm{rot}\,\vec{H} . \qquad (12)$$

For our problem that is defined now by equations (2)–(4) the equation (12) has the form

$$j = -\frac{\partial H_z}{\partial x'} . \qquad (13)$$

As we have accepted that the medium under consideration presents the plate of limited thickness, we can suppose it moving as solid, i.e.

$u_x = u$.

Algorithm of solution

At the relatively small time of process the depth of the field penetration will be significantly less than the plate thickness. In this situation in correspondence with general classification of diffusive phenomena our equation (5) belongs to the class of diffusion problems with time-dependent value of diffusive flux on the surface of semi-infinite medium.

The expression of the field intensity can be obtained by analytical way for the initial interval of time. On condition of relatively small displacement of border for medium

$$\int_0^t u(t_1)\,dt_1 << l_0$$

the magnetic field intensity can be define by the approximate expression

$$H \cong \frac{H_0}{t_0}\, t\,(1-\frac{1}{l_0}\int_0^t u(t_1)\,dt_1). \qquad (14)$$

Taking into account condition (11), the velocity of the plate motion can be defined in the first approximation by the expression

$u = a\,t$,

here a is an acceleration of medium.

The integral in expression (14) is equal approximately to

$$\int_0^t u(t_1)\,dt_1 \approx a\frac{t^2}{2}. \qquad (15)$$

Substitution of (15) to (14) and further on into (10) gives the equation related to medium acceleration:

$$a \approx \frac{\mu H_0^2}{2\rho L}\left(\frac{t}{t_0}\right)^2\left(1-\frac{a t^2}{l_0}\right). \qquad (16)$$

In the last expression we have neglected the term

$$a^2\,\frac{t^4}{4 l_0^2}.$$

Equation (16) leads to obtaining the approximate time dependence for acceleration:

$$a = \left(\frac{\mu H_0^2\, t^2}{2\rho L t_0^2}\right)\left(1+\frac{\mu H_0^2\, t^4}{2\rho L t_0^2\, l_0}\right)^{-1}. \qquad (17)$$

Using correlation (17) integral in the expression (9) can be written in the following approximation:

$$\int_0^t u(t_1)\,dt_1 \approx \frac{l_0}{4}\ln\left(1+\frac{\mu H_0^2\, t^4}{2\rho L\, t_0^2\, l_0}\right). \qquad (18)$$

The expression for the magnetic field intensity has been obtained after substitution of (18) to (9), and the expression below is correct for the interval of space $0 \le x \le l\ (t)$, including the border surface of conducting plate at the end of this interval:

$$H(0,t) = H_0\frac{t}{t_0}\left[1-\frac{1}{4}\ln\left(1+\frac{\mu H_0^2\, t^4}{2\rho L t_0^2\, l_0}\right)\right] \approx$$

$$\approx H_0\frac{t}{t_0}\left(1-\frac{\mu H_0^2\, t^4}{8\rho L t_0^2\, l_0}\right). \qquad (19)$$

To calculate the magnetic field intensity inside the plate, the known formula of mathematical theory of diffusion [4, p. 57] can be used:

$$H(x',t) = \int_{x'/2\sqrt{D_M t}}^{\infty} H\left(0,\ t-\frac{x'^2}{4D_M\,\varepsilon^2}\right)e^{-\varepsilon^2}\,d\varepsilon =$$

$$= \int_{x'/2\sqrt{D_M t}}^{\infty}(H_0/t_0)\left(t-\frac{x'^2}{4D_M\,\varepsilon^2}\right)\times$$

$$\times\left[1-\frac{\mu H_0^2}{8\rho L t_0^2 l_0}\left(t-\frac{x'^2}{4D_M\,\varepsilon^2}\right)^4\right]e^{-\varepsilon^2}d\varepsilon. \qquad (20)$$

Distribution of the field intensity inside the plate can be calculated in (20) using the numerical methods.

The skin depth for the magnetic field intensity can be calculated using the known expression

$$\delta_H = \frac{1}{H(0,t)}\int_0^L H(x'',t)\,dx'', \qquad (21)$$

with substitution of $H\,(0,\,t)$ from (19) and $H\,(x',\,t)$ from (20).

The magnetic field intensity can be expressed using the velocity of Alfven's waves

$$v_A = H_0\sqrt{\frac{\mu}{\rho}},$$

in the following form:

$$H(0,t) = H_0\left(\frac{t}{t_0}\right)\left(1-v_A^2\frac{t^4}{8L l_0 t_0^2}\right);$$

$$H(x',t) = \int_{x'/2\sqrt{D_M t}}^{\infty}\left(t-\frac{x'^2}{4D_M\,\varepsilon^2}\right)\times$$

$$\times\left[1-\frac{v_A^2}{8L\,l_0 t_0^2}\left(t-\frac{x'^2}{4D_M\,\varepsilon^2}\right)^4\right]e^{-\varepsilon^2}\,d\varepsilon. \qquad (22)$$

Substitution of (22) to (13) has resulted in the expression for the current density distribution inside conducting medium (plate under consideration):

$$j(x',t) = \frac{H_0 x'}{2D_M\, t_0}\int_{x'/2\sqrt{D_M t}}^{\infty}\left[1-\frac{5}{8}\frac{v_A^2}{L l_0 t_0^2}\left(t-\frac{x'}{4D_M\,\varepsilon^2}\right)^4\right]\times$$

$$\times\left(\frac{e^{-\varepsilon^2}}{\varepsilon^2}\right)d\varepsilon. \qquad (23)$$

It is important to take into consideration that in accordance with the obtained expression (22) for distribution of magnetic field intensity we have obtained electric current density distribution (23) with zero current density on the surface of conducting medium:

$$j_y(0,t) = 0.$$

Numerous calculations must be done to get the skin depth using formula (21). It is expedient to find the approximate value of the skin depth using a more simple way. On this way we shall consider that the field intensity inside the plate at depth x' is equal approximately to $\left[\text{ see (22) }\right]$

$$H(x',t) \cong \frac{H_0}{t_0}\left(t - \frac{x'^2}{4D_M}\right)\times$$
$$\times\left[1 - \frac{v_A^2}{8Ll_0t_0^2}\left(t - \frac{x'^2}{4D_M}\right)^4\right].$$

Approximation for determination of skin layer depth

According to the classic definition skin depth is a depth distance for the field intensity reduction up to 17% with respect to its value on the surface of medium. So it is possible to use the approximate correlation

$$(H_0/t_0)\left(t - \frac{\delta^2}{4D_M}\right)\left[1 - \frac{v_A^2}{8Ll_0t_0^2}\left(t - \frac{\delta^2}{4D_M}\right)^4\right] \approx$$
$$\approx 0{,}17\,H_0\,(t/t_0)\left(1 - \frac{v_A^2\,t^4}{8Ll_0t_0^2}\right). \qquad (24)$$

Simple, but painstaking calculations based on equation (24) have resulted in the correlation

$$\delta^2 \cong \quad 4\cdot 0{,}83\,D_M\,t\,\left(1 + \frac{\mu H_0^2\,t^4}{16\rho L l_0 t_0^2}\right).$$

Finally, the non-dimensional (normalized) skin layer depth for the pulsed magnetic field determined by the time dependence (7) has been written in the form

$$\delta/\delta_0 \cong 1{,}8\sqrt{1+\beta(t)}\,, \qquad (25)$$

where δ_0 is the classic value of the skin layer depth in the immovable medium:

$$\delta_0 = \sqrt{t/(\mu\sigma)}\,.$$

The time-dependent parameter included is

$$\beta(t) = \frac{\mu H_0^2\,t^4}{16\rho l_0 L\,t_0^2} = \frac{1}{8}\,\frac{p_{H0}}{\rho}\,\frac{l_0}{L}\left(\frac{t}{l_0}\right)^2, \qquad (26)$$

where the magnetic pressure of the field source is

$$p_{H0} = p_{H0}(t) = \frac{\mu H_0^2}{2}\left(\frac{t}{t_0}\right)^2.$$

There are presented also the ratio of initial gap value to the thickness of plate ($\gamma = l_0/L$) as well as normalized time $\tau = t/t_0$ and some basic value of velocity $V_b = l_0/t$, calculated via initial distance between the plate surface and the surface of the external field intensity application. In the other designations, the parameter $\beta(t)$ can be written as

$$\beta(t) = (1/16)\,\gamma\,\tau^2\,Al\,, \qquad (27)$$

here the known magnetic criterion of Alfven [5] has been used:

$$Al = \frac{\mu H_0^2}{\rho V_b^2}.$$

The amplitude value of magnetic field intensity H_0 and basic velocity V_b govern this criterion.

In the expression (26) the non-dimensional value l/l_0 can be included, it can present here the relative distance between the surface of medium under consideration and field source. In such a case the criterion of Alfven can be written with real mean velocity of plate $V = l(t)/t$ instead of V_b.

The graphical interpretation of dependence (25) is given in fig. 2.

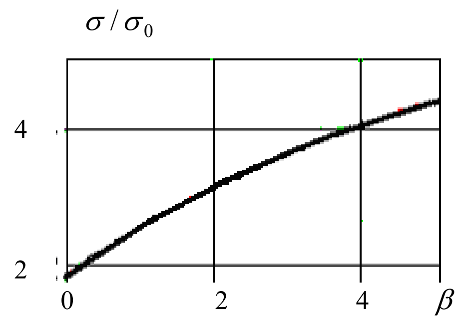

Fig. 2. Dependence of a normalized depth of skin layer (25) on the correcting parameter β given by expression (26)

The obtained mathematical dependence (25) displays a certain degree of similarity with the dependence for elastic motion of conducting medium given in the paper [1] at the constant value of the field intensity H jump on the border. Alternatively, in the result obtained in this paper the correcting term under the root sign in the expression for δ/δ_0 is defined only by the magnetic criterion of Alfen, while in the result presented in the paper [1] the correcting term was defined by the Alfven's number and magnetic Reynolds' number. As it is seen in (26) a correction to the classic value of skin layer depth has been presented here by the term which depends on the time as proportional to t^4.

Additional factors γ and τ in (27) reflect the peculiarities of the applied field, taking into consideration increase of the field intensity at the initial interval of time and its decrease as a result of conducting plate surface runaway.

Conclusion

In the paper presented the theory of one-dimensional magnetic field penetration into conducting medium (flat plate) has been developed taking into account the motion of plate under the action of magnetic field at the linear growth of field in the time on the external border of conductor. The process is defined by the equation of diffusion with convection term together with the equation of medium motion under the specific boundary condition. The last includes the integral expression for the connection of magnetic field intensity with velocity of medium motion, while the velocity of medium in this expression is found by the equation of motion.

By combination of interconnected magnetic, mechanical and geometrical conditions described above, the normalized skin layer depth has been derived on condition that it is less than the thickness of plate. The depth of skin layer has increase in comparison with the classical case of immovable medium for two physical reasons: 1) as a result of the field reduction on the border of conducting medium caused by motion of conducting medium away from the external field source; 2) as a result of reaction to the medium motion away from the field source due to the action of electrical field induced by motion. Both reasons can be considered as connected with one another, but each of them leads to the similar increase of skin layer depth independently.

The result obtained here on the basis of diffusion theory can be considered as a new demonstration of skin layer depth dependence on the velocity of conducting medium motion, in addition to presented earlier in [1].

The correcting term that causes difference in comparison with immovable medium is time-dependent as $\sim t^4$. It has been determined via the magnetic criterion of Alfven, normalized time and normalized geometric data.

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Уточнене визначення скінової глибини під час дифузії імпульсного магнітного поля в рухоме електропровідне середовище

Розроблено теорію одновимірного проникнення магнітного поля в електропровідне середовище (плоску пластину) з урахуванням руху пластини під дією магнітного поля за умови лінійного зростання напруженості поля в часі на зовнішній межі провідника. Процес був описаний рівнянням дифузії, що враховує конвективний перенос, і рівнянням руху середовища за спеціальних граничних умов. Розв’язок отримано для часового інтервалу, в якому товщина скінового шару залишається меншою, ніж товщина пластини. Скінова глибина при цьому збільшується порівняно з класичним випадком нерухомого середовища. Цей результат пояснюється уповільненням зростання поля на поверхні середовища в результаті переміщення межі провідника та безпосередньо впливом руху середовища.

В.Т. Чемерис, А.И. Райченко

Уточненное определение скиновой глубины при диффузии импульсного магнитного поля в движущуюся электропроводящую среду

Разработана теория одномерного проникновения магнитного поля в электропроводящую среду (плоскую пластину) с учетом движения пластины под действием магнитного поля при линейном росте напряженности поля во времени на внешней границе проводника. Процесс был описан уравнением диффузии с учетом конвективного переноса и уравнением движения среды при специальных граничных условиях. Решение получено для временного интервала, в котором толщина скинового слоя остается меньшей, чем толщина пластины. Скиновая глубина при этом возрастает по сравнению с классическим случаем неподвижной среды. Этот результат объясняется замедлением роста поля на поверхности среды в результате перемещения границы проводника и непосредственно влиянием движения среды.