UDC 656.7.052:551.508.85(045)

[1]**I. M. Braun**
[2]**R.B. Sinitsyn**, Cand. Sci. (Eng.)
[3]**F.J. Yanovsky**, Dr. Sci. (Eng.)

ASSESSMENT OF RELIABILITY AND COMPARISON OF TWO ALGORITHMS FOR HAIL HAZARD DETECTION FROM AIRCRAFT

[1,3]NAU Institute of Information and Diagnostic Systems, e-mail: braun/yanovsky@nau.edu.ua
[2]NAU Institute of Electronics and Control Systems

This paper presents and analyzes two algorithms for the detection of hail zones in clouds and precipitation: parametric algorithm and adaptive non-parametric algorithm. Reliability of detection of radar signals from hailstones is investigated by statistical simulation with application of experimental researches as initial data. The results demonstrate the limits of both algorithms as well as higher viability of non-parametric algorithm. Polarimetric algorithms are useful for the implementation in ground-based and airborne weather radars.

Introduction

Determination of zones in the troposphere where hailstones are concentrated is a task of great importance for air navigation and air traffic control. Polarimetric radar means both airborne and ground-based can be successfully used for this purpose.

Calculations and experiments have clearly shown that back-scattering on non-spherical particles depends on polarization of radar waveform and polarization properties of receive antenna.

Hence information about hydrometeors shape can be derived by changing polarization parameters of radar antennas and special processing of received signals. Additional possibilities appear to improve the quality of hydrometeor type identification and to enhance the accuracy of precipitation intensity estimate. The purpose of this paper is to present two algorithms that were proposed, and mainly to research them on the basis of measurements and modeling.

Comparison of different algorithms between themselves is done from the point of view of reliability at different situations.

Parametrical algorithm

Synthesis of parametrical detector of hail zones is done in the framework of binary multi-parametrical problem of dangerous weather phenomena detection [1]. On the basis of measuring information parameters, the hypothesis about presence of hailstones is considered against the alternative about absence of hail. The logarithm of likelihood ratio is used as a discriminate function

$$F(x,y) = \ln \frac{p\left(x, y / \omega_{hail}\right)}{p\left(x, y / \omega_{rain}\right)}, \quad (1)$$

where $p\left(x, y / \omega_i\right)$ is bivariate distribution of information parameter x and y in situations of hail ($i = hail$) and rain ($i = rain$).

Discriminant function is calculated for different models by using experimental data to find estimates of distribution parameters.

Fig. 1 shows examples of probability density of reflectivity Z and linear depolarization ratio L_{DR} as information parameter s x and y.
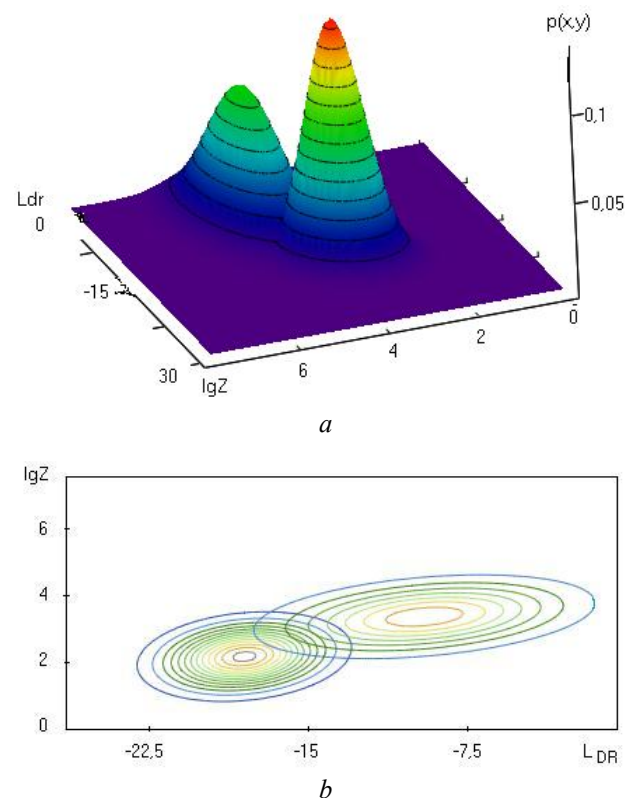

a

b

Fig. 1. Bivariate distributions of L_{DR} and lg$Z$ built on the basis of data as 3D image and (*a*) contour diagram (*b*)

Analysis has shown that these distributions can be described by the bivariate Gaussian models.

Explicit form of the discriminant function can be obtained by substitution of the mathematical models of the distributions to formula (1).
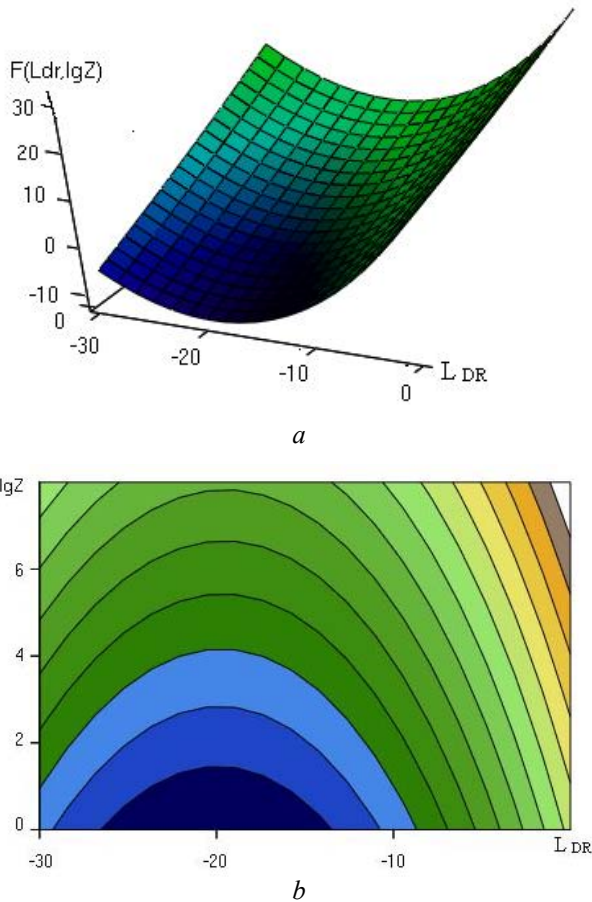
Plot of discriminant function is shown in fig. 2.

a

b

Fig. 2. Discriminant function $F(L_{DR}, \lg Z)$ as a surface (*a*) and a contour diagram (*b*)

Statistical algorithm of hail detection [2] is reduced to calculation of the value of discriminant function $F(x, y)$ and threshold test F_0:

$$\begin{cases} F \geq F_0 \Rightarrow Hail \\ F < F_0 \Rightarrow Rain. \end{cases}$$

The threshold F_0 is determined in accordance with a figure of merit, for example, to satisfy given probabilities of detection and false alarm.

When using a limited sample of measurements (for example, $x = L_{DR}$ and $y = Z$), sometimes unambiguous distinction between rain and hail classes (on sample histogram) can be reached.

However, the wings of conditional distributions $p(x, y/\omega_i)$ will cause errors that are investigated later on.

Graphically behavior of the discriminant function can be investigated if to reduce it to a function of single variable.

In fig. 3 one can see calculation results of discriminant function $F(L_{DR}, \lg Z)$ versus one of two arguments of function at the fixed value of other argument.
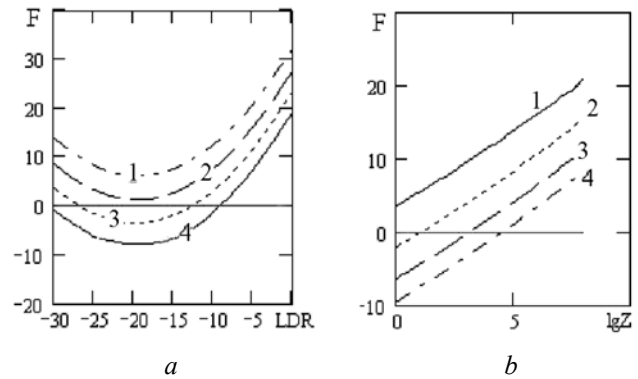

a   *b*

Fig. 3. Discriminant function at $\lg Z = $ const (*a*) and $L_{DR} = $ const (*b*)
a: *1* – $\lg Z = 8$; *2* – $\lg Z = 6$; *3* – $\lg Z = 4$; *4* – $\lg Z = 2$;
б: *1* – $L_{DR} = $ -5; *2* – $L_{DR} = $ -8; *3* – $L_{DR} = $ -11; *4* – $L_{DR} = $ -14;

The straight lines conditionally indicate the threshold level $F_0 = 0$, which shows that at big enough values of one of the parameters, the hail situation is detected independently on values of the second parameter.

Non-parametrical algorithm

Probability density distribution of measurable variables of very complicated and dynamic targets as weather formations can be various both for different objects and for the same object at the different time. That is why signal analysis based on stable statistical models is not adequate to the problem. Adaptive or invariant to signal and noise models, non-parametric methods can be preferable.

In this section we consider a possibility to detect a hail zone without previous suppositions about statistical models of signals and noise, and information parameters now as before are measurable polarimetric parameters.

The essence of the method is the following. An estimate of multivariate probability density of information parameter of radar echoes is done for weather entire formation and for a local zone, which can contain hailstones potentially.

In case of two information parameter, the estimate of probability density obtained by using potential functions method [3] can be expressed as

$$\widehat{f}(x, y) = \sum_{X_s \in X} \kappa(x, y, X')$$

with $X' = x', y'$ as measured information parameter vector; X as an area where the estimate of probability density is done; $\kappa(\cdot)$ as a potential function, which is subjected by normalized limits; $s = 1$ in case of hail situation and $s = 0$ for non-hail situation.

Multivariable Gaussian probability density with constant of proportionality providing the normalization condition is used as a potential function.

The estimates $\widehat{f}(x,y)$ are done for the local region of weather formation where hailstones are supposed to be present as well as for the rest of weather formation. These estimates are used to build the decision function $L\left(\widehat{f}_1(X'),\widehat{f}_0(X')\right)>C$, where $\widehat{f}_1(X'),\widehat{f}_0(X')$ are estimates of probability density for local region and entire weather formation. Likelihood ratio is used as decision function however some other functions can be also used. The decision is made by comparison with threshold C.

Thus the proposed detection algorithm considers hail zone as inhomogeneity on the background of the rest "homogeneous" part of weather formation. This algorithm is implemented as two-step procedure applied to the test statistics [3].

Let us assume that processing is done on two parameters x_1, x_2.

That means that polarimetric informative parameters are considered as bivariate vector random value variable ξ_1, ξ_2.

Assume that a functional transformation $\varphi(\xi_1, \xi_2)$ exists.

This transformation reduces a random variable ξ_1, ξ_2 with arbitrary distribution to the random value η_1, η_2 that is characterized by uniform distribution. In accordance with probability theory such transformation $\varphi(\xi_1, \xi_2)$ can be implemented by vector function

$\eta_1 = W_{\xi_1}(\xi_1)$;

$\eta_2 = W_2(\xi_2 \mid \xi_1)$,   (1)

where $W_{\xi_1}(x_1)$ is cumulative distribution function of random variable ξ_1; $W_2(x_2 \mid x_1)$ is conditional cumulative distribution function of random variable ξ_2 under the condition that $\xi_1 = x_1$.

The functions $W_{\xi_1}(x_1)$ and $W_2(x_2 \mid x_1)$ are unknown at signal processing. They are substituted by their estimations $\widehat{W}_{\xi_1}(x_1, \vec{x})$ and $\widehat{W}_2(x_2 \mid x_1, \vec{x})$, which can be obtained by analysis of samples $\vec{x}$. One can assume that when a sample size increases the estimation converges to the sought cumulative distribution function. The use of an empirical distribution function as preliminary functional transformation allows to achieve a similarity of the detection procedure. That means a stable probability of false alarm.

Non-parametric properties of the detection procedure strongly depend on the quality of estimation. Cumulative distribution function (as well as probability density function) in this problem is convenient to be estimated with help of potential functions [3]. Estimation of bivariate probability density function $w(x,y)$ is presented as a sum of kernels $\kappa_i(x,y)$ (potential functions):

$$\kappa_i(x,y) = \frac{1}{n}\omega\,(x-x_i, y-y_i)\,,$$

where $\omega(x,y)$ is some probability density function, particularly Gaussian; (x_i, y_i) is i-th sample that serves as a basis of the estimation.

In order to estimate $\widehat{W}_\xi(x)$ – one-dimensional cumulative distribution function the kernels are

$$P_i(x) = \frac{1}{n}\int_{-\infty}^{x}\int_{-\infty}^{\infty}\omega(u-x_i, v-y_i)dudv\,,$$

and to estimate $\widehat{w}_\xi(x)$ – one-dimensional probability density function the kernels are described by the formula

$$d_i(x) = \frac{1}{n}\int_{-\infty}^{\infty}\omega(x-x_i, v-y_i)dv\,.$$

By using probability theory, the estimation of the conditional cumulative distribution function can be obtained as

$$\widehat{W}_2(y \mid x) = \frac{1}{w_{1\xi}(x)}\int_{-\infty}^{y}\widehat{w}(x,v)dv =$$

$$= \frac{\sum_{i=1}^{n}\int_{-\infty}^{y}\kappa_i(x,v)dv}{\sum_{i=1}^{n}d_i(x)} = \frac{\sum_{i=1}^{n}Q_i(x,y)}{\sum_{i=1}^{n}d_i(x)}\,,$$

where

$$Q_i(x,y) = \int_{-\infty}^{y}\kappa_i(x,v)dv\,.$$

The procedure of hail zone detection as inhomogeneous areas is based on the hypothesis checking about probability density function of informative parameters of the reflected signal. Neumann-Pearson criterion is used. Samples of n two-dimensional independent random variables (x_1, x_2) are used as input data. Assume that probability density function is different in two situations: when medium contains hailstones and when it does not contain them. Then hail detection is reduced to the hypothesis checking about the kind of distribution. Functional transformation (1) of the samples is necessary to provide the property of similarity of the detection procedure (stability of the first kind error).

A distribution of the transformed statistics

$y_1 = \widehat{W}_{\xi_1}(x_1)$;

$y_2 = \widehat{W}_2(x_2 \mid x_1)$

tends to be uniform at sample size increasing in case of absence hailstones. In other words, a distribution the transformed statistics (y_1, y_2) that corresponds to hypothesis H_0 "No hail" is asymptotically uniform.

Estimations of probability density functions obtained from real samples of reflection from rain before and after the transformation are shown in fig. 4.
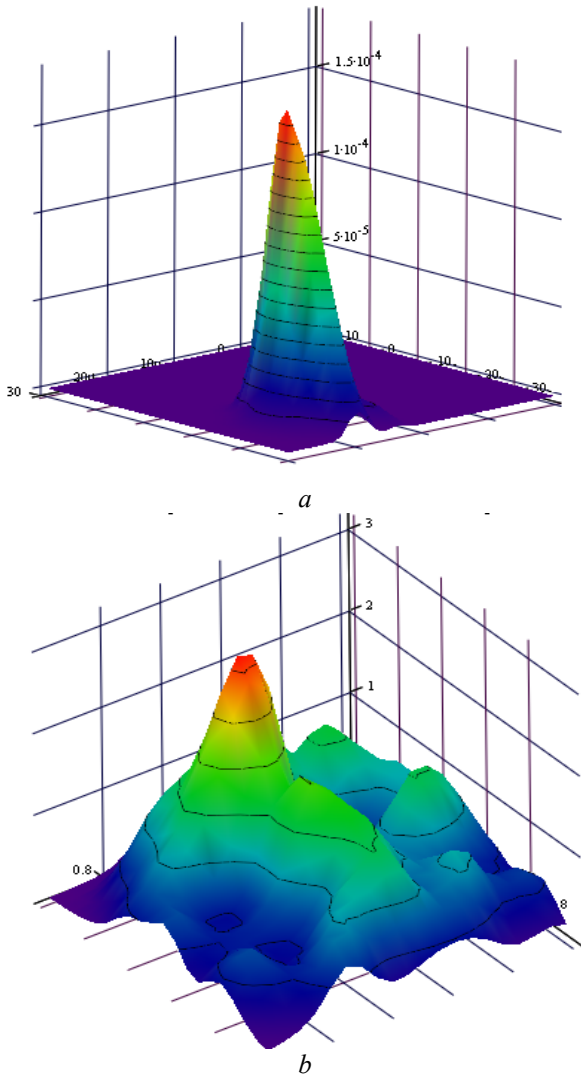

a

b

Fig. 4. Bivariate estimations of probability density functions of returns from rain before (*a*) and after (*b*) functional transforamation (4)

Thus, hypothesis checking about hail presence is reduced to hypothesis checking about uniformity of transformed statistics $\mathbf{y} = (y_1, y_2)$ distribution, and likelihood ratio is reduced to likelihood function of the same statistics. The likelihood function is substituted by its estimation, which is obtained with help of potential function method when kernels are:

$$l(\mathbf{y}) = \prod_i^n \hat{w}_\eta(\mathbf{y}_i), \tag{2}$$

$$\hat{w}_\eta(\mathbf{y}_i) = \sum_{j=1}^m \kappa_i(\mathbf{y}_i),$$

where m is sample size of the checking statistics, which are obtained on the reflected signal from objects with hailstones. Logarithmically we obtain from (2) the final formula of the decision rule for hail detection:

$$\lambda(\mathbf{y}) = \sum_{i=1}^n \ln(\hat{w}_\eta(\mathbf{y}_i)) \geq C.$$

Statistical modeling and comparison

Models of signal probability density in situations with hail is given by bivariate normal law with correlation coefficient $r = 0{,}5$ and variance $W_0 = \sigma^2$.

A hail cloud is characterized by probability density of signals, which are formed as a sum of reflection from rain cloud (as interference) and reflection from hail with variance W_1 and correlation $r = 0$. That is the statistical model correspondent to the situation "Hail" can be written as

$$f(x_1, x_2, q) = \frac{\exp\left\{-\dfrac{x_1^2 + x_2^2 - 2\dfrac{r}{(1+q)}x_1 x_2}{2(1 - \dfrac{r^2}{(1+q)^2})\sigma^2(1+q)}\right\}}{2\pi\sigma^2(1+q)\sqrt{1 - \dfrac{r^2}{(1+q)^2}}},$$

where q is ratio of variances which actually characterizes a signal to interference ratio:

$$q = \frac{W_1}{W_2}. \tag{3}$$

The estimates of detection probability D and false alarm probability F were obtained by Monte Carlo method. Non-parametric algorithm was simulated (values D and F were calculated) at different signal to interference (3) $q = W_1/W_0$. Different pairs of information parameter were used from the three: $\lg Z$, Z_{DR}, Z_{dBZ}. The results for $x_1 = L_{DR}$ and $x_2 = Z_{dBZ}$ together with curve $D = f(F)$ for parametric algorithm are shown in fig. 5.
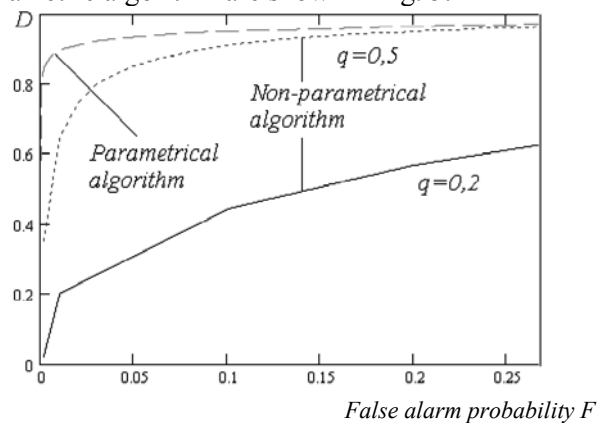

Fig. 5. Comparison of parametric and non-parametric algorithms

Naturally, parametrical algorithm provides higher detection probability (if accepted models are adequate that always is doubt). Reliability of non-parametric algorithm increases when a signal to interference ratio q increases.

However, it is unlikely to find information parameter for which q is more than 0,5.

Conclusion

Parametrical algorithm is characterized by the most complete use of a priori information. That is why it is the most reliable under the condition of full correspondence of accepted statistical model to the reality.

However, parametrical algorithm is sensitive to the difference of real statistics to the models, which where used at the synthesis. Hence, the reliability of the parametrical algorithm can decreased in some real situations.

Non-parametrical algorithm, which was created on the basis of potential functions, does not require a priori information about statistical characteristics (probability density functions) of the informative parameters of reflected signals from the objects. It requires estimating the parameters of real reflected signal and provides the stability of detection as well as stable probability of the error of first kind (false alarm) due to the constant threshold of decision making.

Workability of the test procedure of nonparametric algorithm is confirmed with a method of statistical modeling; however, quantitative results of an estimation of reliability of nonparametric algorithm demand the further researches with application of large volume of experimental data and modeling of various situations.

When SNR increases and or false alarm probability increases, the detection probability of hail by the non-parametric algorithm tends to the detection probability by the parametric algorithm.

Literature

1. *Яновский Ф.И.* Локализация опасных метеорологических явлений с борта воздушного судна. – К.: Знание, 1991. – 28 с.

2. *Яновський Ф.І.* Метеонавігаційні радіолокаційні системи повітряних суден. – К.: НАН, 2003. – 302 с.

3. *Sinitsyn R.B., Braun I.M., Yanovsky F.J.* Nonparametric Detection of Inhomogeneous Areas in Clouds // Conf. proc. The 11th conf. on Microwave Technique, COMITE 2001. – Sept. 18–19, 2001, University of Pardubice, Czech Republic. – P. 77–81.

І.М. Браун, Р.Б. Синіцин, Ф.Й. Яновський

Оцінка достовірності та порівняння двох алгоритмів виявлення градової небезпеки з борту літака

Проаналізовано два алгоритми, розроблені для виявлення зон граду в хмарах і осіданнях: параметричний алгоритм і адаптивний непараметричний алгоритм. За допомогою статистичного моделювання з використанням результатів експерименту як початкових даних досліджено достовірність виявлення радіолокаційних сигналів від сукупності градин. Результати дослідження показують можливості обох алгоритмів і більш високу стійкість непараметричного алгоритму. Поляриметричні алгоритми рекомендовано для наземних і бортових засобів метеорологічного спостереження для забезпечення авіації.

И.М. Браун, Р.Б. Синицын, Ф.И. Яновский

Оценка достоверности и сравнение двух алгоритмов обнаружения градовой опасности с борта самолета

Проанализированы два алгоритма, разработанные для обнаружения зон града в облаках и осадках: параметрический алгоритм и адаптивный непараметрический алгоритм. С помощью статистического моделирования с использованием результатов эксперимента в качестве начальных данных исследована достоверность обнаружения радиолокационных сигналов от совокупности градин. Результаты исследования показывают возможности обоих алгоритмов и более высокую устойчивость непараметрического алгоритма. Поляриметрические алгоритмы рекомендованы для наземных и бортовых средств метеорологического наблюдения для обеспечения авиации.